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Weibull and Gamma distributions for Wave Parameter Predictions

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ABSTRACT

Distribution of maximum wave height follows the model derived on the concept that the individual zero up-crossing wave heights follow the Weibull law. Better results are obtained when a depth factor included to accommodate shallow water attenuation effects. Certain wave height parameters such as mean maximum wave height, most frequent maximum wave height, extreme wave height, return period of an extreme wave and probability of realising an extreme wave in a time less than the designated return period are estimated and compared with computed values giving reliable results. Predicted significant wave heights are comparable with the computed value with maximum deviation of 0.21m. Theoretical and empirical supremacy of Weibull is re-established. A 100.0% empirical support is obtained in the simulation of zero up-crossing wave periods by Gamma at 0.05 level of significance. A Weibull-Gamma (with non-zero correlation) joint distribution simulates more effectively the real 3D plot.

INTRODUCTION

Unlike in deep water, the wave climate in shallow water exhibits spatial variations owing to complex transformation processes. Coastal engineering projects require reliable wave data for design and implementation.

In order to understand the wave phenomenon correctly, it is necessary to carry out field measurements and then try to express the physical processes in terms of mathematical equations. If the properties of waves at a given location are available, then it is possible to select the suitable mathematical models to describe them. The model thus selected may help in predicting the statistical wave characteristics at a given location. Such simulated wave characteristics are becoming an accepted practice in coastal and ocean engineering.

Of the various theoretical models available for modelling ocean waves, only the Rayleigh distribution is derived mathematically on the basis of physical processes governing the wave heights (Longuet-Higgins 1952). Yet we need a model that can meet the twin objectives of a) accommodating the Rayleigh distribution whenever the basic assumptions that justify it are satisfied, b) fitting data situations under more general conditions. Thus we arrive at Weibull model for simulating the wave height distribution. Another possible explanation for the Weibull curve was offered in terms of the intensity function (Muraleedharan, Unnikrishnan Nair & Kurup 1993).

A modified Weibull distribution which includes the depth factor to accommodate the shallow water attenuation effects is used to fit the shallow water maximum wave height distribution. Significant wave heights computed from zero up-crossing wave heights are utilised to calibrate the depth factor.

The Erlang model is suggested for simulating the distribution of zero up-crossing wave periods.Various wave period statistics are predicted and empirically validated (Unnikrishnan Nair, Muraleedharan & Kurup 2003).

For a complete description of wind generated random waves, it is necessary to consider wave height and period simultaneously. Serious consideration must be given to the combined effect of height and period. Wave data measured in the ocean often show that the height and period of incident waves are not statistically independent. All the existing joint distribution models of wave heights and periods are derived on the basis that there exists zero correlation between them. Many researchers have pointed out that there exists a significant correlation between them. The Weibull-Gamma joint distribution including correlation effects is recommended for the simulation of surface plots of real wave situations of wave height-period distributions.

MATERIALS AND METHODS

Long-term visually estimated wave heights and periods (deep water wave data) off Goa (NPOL Atlas 1978) and recorded shallow water wave data off Alleppey and Valiathura (CESS Wave Data 1984)are considered in this study. Monthly averaged visually estimated wave height distributions off Goa (Grid.9, 13^0-17^0N, 73^0-77^0E, 4^0x4^0, NPOL Atlas)) are fitted to Rayleigh, Weibull, Gumbel and Exponential models. The offshore of Goa is enclosed by Grid.9.

The visually estimated(appro.=H_s, Grid.17, NPOL Atlas) and recorded off Valiathura (May-October, CESS Data) significant wave height distributions are utilised to validate the simulation capability of the Weibull model.

Long-term recorded shallow water maximum wave height distributions off Alleppey(8^0-12^0N, 75^0-77^0E, water depth = 5.5m) from May-October (1981-84) are simulated by a modified Weibull model including a depth factor to accommodate the shallow water attenuation effects.

As a case study individual zero up-crossing shallow water wave heights obtained from wave recorder charts by the zero up-crossing technique for the month of January 1981 off Valiathura (water depth=5.5m) by using a digitising software package ACECAD are also analysed to compute the significant wave height(H_s) and are then compared with the estimated value obtained from the parametric relation derived from the Weibull distribution giving reliable accuracy.

The zero up crossing wave period distribution of the active south-west monsoon season off Valiathura (May to October, 8^0 26′ N, 76^0 54′E, Water depth = 5.5m, CESS Data) are tried to be simulated by available theoretical models such as Gamma, Rayleigh, exponential and Bretschneider. The simulation capability of the various models are tested using root mean square relative error (e_{rms}) and relative bias (e_{mean}) (Roelvink 1993). The supremacy of Gamma over other competing models is established.

Then various significant wave periods($T_{1/3}$,$T_{1/4}$,.......$T_{1/10}$,.....) are computed from the above zero up-crossing wave period data and are compared with estimated values from the expression derived from Erlang(special case of Gamma for its shape parameter equal to the nearest integer, considered for its simplicity) giving acceptable accuracy.

Then the data of the visually and monthly averaged wave periods(appro.= T_s) from Grid.17 (5^0-9^0N, 73^0-77^0E, 4^0X4^0, NPOL Atlas) enclosing Valiathura coastal station are used for validating the Erlang model(or Gamma). This model is suggested by mathematical logic for simulating the significant wave period distribution. The validation results show its applicability.

Modified Weibull model

The modified Weibull model with depth factor for maximum wave height distribution is given as (Muraleedharan, Unnikrishnan Nair & Kurup 2001)

$$[F(h)]^n = \left[1 - \ell^{-\left(\frac{h}{a}+\frac{\bar{h}}{d}\right)^b}\right]^n \quad \text{.......... ....(1)}$$

where F(h)-Weibull distribution function, a scale parameter, b-shape parameter, n-sample size, $\bar{h}$-is the maximum wave height, d- is the water depth, h-mean maximum wave height. The various design wave height parameters derived were (Muraleedharan, Unnikrishnan Nair & Kurup 1999)

Most frequent maximum wave height (H_{mfm})

The most frequent maximum wave height is

$$H_{mfm} = a\left[\left(1-\frac{1}{nb}\right)^{\frac{1}{b}} - \frac{\bar{h}}{d}\right] \text{..................................(2)}$$

Mean of maximum wave height

$$\bar{H}_{max} = \left(\frac{a}{b}\right)\left[\frac{n\Gamma\left(\frac{1}{b}\right)}{1!} - \frac{n(n-1)\Gamma\left(\frac{1}{b}\right)}{2!\times 2^{\frac{1}{b}}} + \cdots + \frac{(-1)^{r+1}\Gamma\left(\frac{1}{b}\right)}{n^{\frac{1}{b}}}\right] \text{......................(3)}$$

r = 1,2,3......

Significant wave height

$$H_s = a\times\left[(\ln 3)^{\frac{1}{b}} - DF - SC\right] + \frac{a}{b}\times 3\times I_{\ln 3}\left(\frac{1}{b}\right) \text{ .... (4)}$$

where $I_{(x)}(P) = \int_x^{\infty} \ell^{-t}\times t^{p-1}\times dt$, is the incomplete Gamma function, DF-depth factor(ratio of mean wave height to water depth), SC-shoaling coefficient (= 1.15). (Kurian 1987).

The accuracy of the predicted design wave parameters is evaluated by root mean square relative error (e_{rms}) and relative bias (e_{mean}) (Roelvink 1993) i.e

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root mean square relative error $\varepsilon_{rms} = \dfrac{\sqrt{\frac{1}{N}\sum [P - C]^2}}{\frac{1}{N}\sum C}$

relative bias (mean error) $\varepsilon_{mean} = \dfrac{\sum [P - C]}{\sum C}$

where ,

N = *total number of data.*

P = *predicted value*

C = *computed value*

Prediction of various significant wave periods

The prediction of various significant wave periods is given as (Unnikrishnan Nair, Muraleedharan & Kurup 2003) :

The probability density function of the Erlang model is given to be

$$f(t) = \lambda^{\alpha} \cdot t^{\alpha-1} \frac{\ell^{-\lambda t}}{(\alpha-1)!}, \; t > 0, \; \alpha, \lambda > 0 \quad \text{.......................... (5)}$$

as a model for wave period. The distribution function is obtained as,

$$F(t) = 1 - \sum_{i=0}^{\alpha-1} (\lambda t)^{i} \frac{\ell^{-\lambda t}}{i!}, \quad t \geq 0, \quad \text{....................... (6)}$$

$$m(t) = E\left(T \middle/ T > t\right) = t + \left(\frac{1}{\bar{F}(t)}\right) \times \int_t^{\infty} \bar{F}(t) \times dt$$

λ and α are the scale and shape parameters respectively.

$$\text{where, } \bar{F}(t) = \frac{\sum_{i=0}^{\alpha-1} (\lambda t)^{i} \times \ell^{-\lambda t}}{i!}$$

$$\text{and } \int_t^{\infty} \bar{F}(t) \times dt = \int_t^{\infty} \frac{\sum_{i=0}^{\alpha-1} (\lambda t)^{i} \times \ell^{-\lambda t}}{i!} \times dt$$

$$= \sum_{i=0}^{\alpha-1} \frac{\lambda^{i+1}}{\lambda \times i!} \times \int_t^{\infty} t^{(i+1)-1} \times \ell^{-\lambda t} \times dt$$

$$= \sum_{i=0}^{\alpha-1} \frac{1}{\lambda} \times \left[1 - \int_0^{t} \frac{\lambda^{i+1}}{i!} \times t^{(i+1)-1} \times \ell^{-\lambda t} \times dt \right]$$

$$= \sum_{i=0}^{\alpha-1} \frac{1}{\lambda} \times \left[1 - \left(1 - \frac{\sum_{j=0}^{i} (\lambda t)^{j} \times \ell^{-\lambda t}}{j!} \right) \right]$$

$$= \sum_{i=0}^{\alpha-1} \sum_{j=0}^{i} \frac{1}{\lambda} \times \frac{(\lambda t)^{j} \times \ell^{-\lambda t}}{j!}$$

Hence

$$m(t) = t + \left[\lambda^{-1} \frac{\sum_{i=0}^{\alpha-1} \sum_{j=0}^{i} \frac{(\lambda t)^{j}}{j!}}{\sum_{i=0}^{\alpha-1} \frac{(\lambda t)^{i}}{i!}} \right] \text{....... (7)}$$

The various predicted significant wave periods ($T_{1/3}$, $T_{1/4}$,…$T_{1/10}$….) derived from the expression (7) are compared with the computed values from the recorded wave data off Valiathura during the southwest monsoon seasons and its accuracy is estimated using e_{rms} and e_{mean}.

Distribution model for significant wave period.

An Erlang distribution model is suggested theoretically for the significant wave period distribution from the mathematical logic (Unnikrishnan Nair, Muraleedharan and Kurup 2003)

$$\bar{F}(t) = 1 - F(t) = \exp\left[-\int_0^{t} \left(\frac{m`(t)}{m(t) - t} \right) \right] dt, \text{.......(8)}$$

where, $m`(t)$ *is the derivative of* m*(t)*

F*(t) is the distribution function*

Validation of the Model

Visually estimated wave period data for Grid.17 compiled for ten years(NPOL 1978) are utilised to validate the simulation capability of the Gamma model for significant wave period distribution. Valiathura recording station is located in this grid.

Weibull – Gamma joint distribution function

Considering the marginal distributions, Weibull for wave heights and Gamma for wave periods makes the Weibull–Gamma joint distribution. The frequency of occurrence of a particular wave height and period can be predicted including correlation effect by the expression

$$F(x_1, x_2) = F_1(x_1) \times F_2(x_2) \times [1 + \theta \times [1 - F_1(x_1)] \times [1 - F_2(x_2)]] \text{ ...(9)}$$

Where $F_1(x_1)$, $F_2(x_2)$ are the univariate distribution functions and θ is a constant; ($-1 \leq \theta \leq 1$)is a bivariate function. As the range of the Karl Pearson's correlation coefficient (r) is the same as that of θ, it is taken as the correlation coefficient for a given set of data for which the joint distribution function is fitted.

In this case, univariate distribution functions are Weibull for wave height and Gamma for wave period.

$$F_1(x_1) = F(h) = 1 - \ell^{-\left(\frac{h}{a}\right)^b}$$

$$F_2(x_2) = F(t) = \left(\frac{1}{\alpha.\Gamma(\alpha)}\right)(\lambda t)^{\alpha}.\ell^{-\lambda t}\left[1+\left(\frac{\lambda t}{\alpha+1}\right)+\left(\frac{(\lambda t)^2}{(\alpha+1)(\alpha+2)}\right)+..................\right]$$

RESULTS AND DISCUSSION

Weibull model for deep water wave heights

The visually estimated significant wave height distribution off Goa (Grid.9, 13^0-17^0N, 73^0-77^0E, 4^0x4^0,NPOL data) clubbed over a period of 10 years (1960-1969) are simulated by the available long-term distribution models, Weibull, Rayleigh, exponential and Gumbel for both rough monsoon and calm conditions. Instrumentally recorded maximum wave height distribution from May to October averaged over a period of 3 years (1981-84) off Alleppey (water depth = 5.5m, CESS Data) is used for predicting the mean maximum wave heights, most frequent maximum wave height, extreme wave height, return period of an extreme wave height and probability of realising an extreme wave height in a time less than the designated return period by including a depth factor $\frac{\overline{H}_{max}}{depth}$ in the modified Weibull model for maximum wave height distribution. The differences in the computed and predicted wave parameters are estimated by the root mean square relative error (e_{rms}) and relative bias (e_{mean}).

Weibull model shows an empirical support in 100% cases after appropriate grouping by c^2-test at 0.05 level of significance for significant wave height distribution off Goa (e.g. Fig.1).

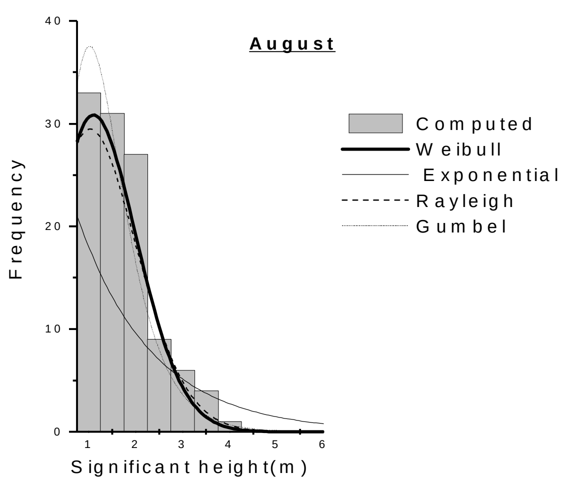

Figure 1. Visually estimated and theoretical significant wave height distribution off Goa.

Gumbel and Rayleigh fail to explain the higher side of the observed wave height distribution. Weibull gives good results even during the monsoon season when surface waves are high. The Weibull model was successfully used for wave height predictions by the method of characteristic function using the redefined significant waves(Muraleedharan, Unnikrishnan Nair & Kurup 1999). This study also reveals that the Weibull model effectively simulates the significant wave height distributions(Muraleedharan 1991).

A modified Weibull model for shallow water maximum wave height distribution and predictions.

The maximum wave height distributions off Alleppy during monsoon seasons (May-October) are simulated after accommodating a depth factor (water depth = 5.5m) in the modified Weibull density function to accommodate the shallow water wave transformation effects(Muraleedharan, Unnikrishnan Nair & Kurup 2001) (e.g. Fig.2).

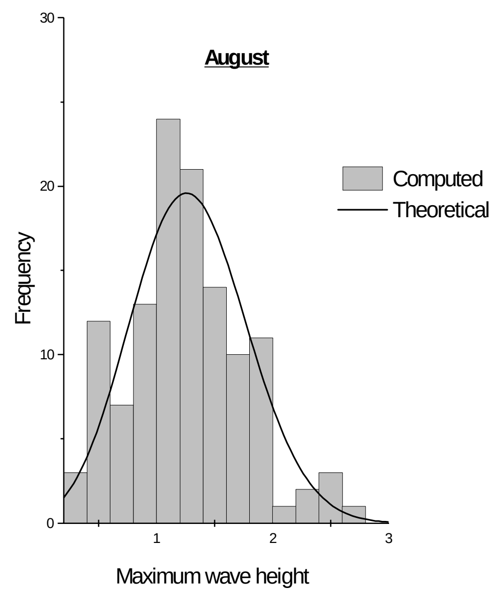

Figure 2. Computed and theoretical maximum wave height distributions off Alleppey

Frictional, shoaling and refraction coefficients show little effect on maximum wave height distributions. A 100% empirical support is obtained for shallow water maximum wave height distributions (after appropriate grouping) by χ^2-test at 0.05 level of significance.

Design wave parameters predicted are shown in Table.1. Maximum and minimum deviations of most frequent maximum wave height, extreme wave height and mean maximum wave height are respectively 0.56m and 0.06m, 0.63m and 0.32m, 0.01m and 0.00m.

Table 1. Computed and predicted design wave parameters off Alleppey at shallow water (water depth = 5.5m)

Wave height		*May* (m)	*June* (m)	*July* (m)	*August* (m)	*September* (m)	*October* *(m)*	ε_{rms}	ε_{mean}
$H_{extreme}$	*Com* *Pre*	2.30 2.27	2.70 3.10	2.70 2.69	2.70 2.82	2.10 2.73	1.05 1.10	0.136	0.082
H_{mfm}	*Com* *Pre*	0.90 0.96	1.70 1.15	1.60 1.04	1.19 0.98	0.60 0.74	0.40 0.46	0.317	-0.165
$\bar{H}_{max}$	*Com* *Pre*	1.17 1.40	1.56 1.89	1.35 1.64	1.25 1.54	0.93 1.19	0.49 0.49	0.223	0.202

Com ⇔ Computed; Pre ⇔ Predicted

Table 2. Probability of realising an extreme wave height in a time (m-years) less than the designated return period (R_p-years)

Month	m (years)	R_p	Probability (%)
May, June August, September, October	*1*	*4*	*29.29*
	2		*50.00*
	3		64.64
July	1	3	30.66
	2		51.93

Prediction of significant wave height in shallow water

Distribution of digitised zero up-crossing wave heights from wave recorder charts for the month of January '81 off Valiathura are simulated by the depth factor accommodated Weibull model. It fits in all cases by appropriate grouping at 0.05 level of significance. The significant wave heights are predicted using the expression (4). The results are given in table.3.

It can be seen that the predicted significant wave heights are comparable with the computed values, in some cases even for two decimal places. A maximum deviation of 0.21m and a minimum deviation of 0.00m are obtained. An average of the shoaling coefficient(Kurian 1987) is also included in the prediction formulae for estimating the significant wave height to give more accurate results. This stems from the fact that since significant wave height is the average of the one-third highest waves, the higher side of the wave height distribution is influenced by both depth factor and shoaling coefficient.

Table 3. Computed and predicted significant wave heights off Valiathura (water depth = 5.5m)

Date	Time (hrs)	Hs *(m)* *Com*	*Pre*	ε_{rms}	ε_{mean}
1/22/81	0900hrs 1200hrs 1500hrs	0.60 0.46 0.58	0.65 0.50 0.58	0.068	0.055
1/23/81	0900hrs 1200hrs 1500hrs	0.47 0.45 0.72	0.64 0.53 0.51	0.298	0.024
1/24/81	0900hrs 1200hrs 1500hrs	0.46 0.49 0.51	0.63 0.55 0.55	0.219	0.185
1/25/81	0900hrs 1200hrs 1500hrs	0.74 0.65 0.77	0.63 0.63 0.88	0.126	-0.009
1/27/81	0900hrs 1200hrs 1500hrs	0.54 0.53 0.62	0.58 0.56 0.49	0.143	-0.036
1/28/81	0900hrs 1200hrs 1500hrs	0.44 0.42 0.49	0.47 0.50 0.60	0.179	0.163
1/29/81	0900hrs 1200hrs 1500hrs	0.42 0.47 0.44	0.41 0.53 0.52	0.131	0.098
1/30/81	0900hrs 1200hrs 1500hrs	0.51 0.60 0.36	0.45 0.59 0.54	0.224	0.075

Zero up-crossing wave period distribution

The zero up-crossing wave period distributions (recorded) for the periods of May-October (active south-monsoon season) off southwest coast of India (Valiathura) recorded by Center for Earth Science Studies Thiruvananthapuram from 1980-1984 are simulated using the available competing theoretical models, Gamma, exponential, Rayleigh and Bretschneider. The empirical validations of these models suggest an empirical support in 100% cases at 0.05 level of significance for Gamma distribution by Chi-square test, which establish the supremacy of the model for zero up-crossing wave period distribution simulation (e.g. Fig. 3).

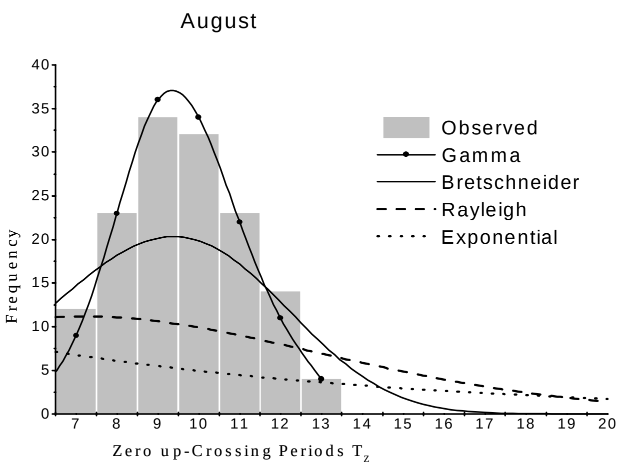

Figure 3. Comparisonof observed (histograms, recorded) and theoretically simulated (curves) zero up-crossing wave periods distributions

Most of the observed wave period distributions are multi peaked and range from 6.5 sec. to 13.5 sec., the narrow band 5 to 15 sec. period waves are important for coastal engineering studies which are dominant during rough monsoon seasons. The distribution is almost normal. The Gamma curves take the normal shape and more or less effectively simulates the observed wave period distribution and peak frequencies.

Prediction of significant wave periods

The computed and predicted significant wave periods using (7) for the active monsoon seasons (for the above data) are compared (e.g. Fig. 4).

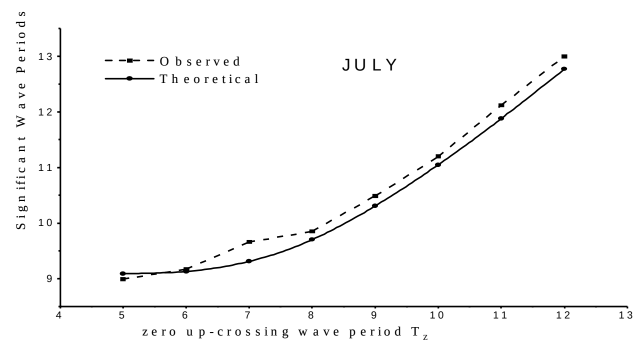

Figure 4. Comparison of various computed and predicted significant wave periods.

The comparability is attained by the root mean square relative error (ε_{rms}) and relative bias (ε_{mean}) with a maximum error of 4.5% and 4.4% respectively (Table 4).

Table 4. Root mean square relative error (ε_{rms}) and relative bias (ε_{mean})

Month	ε_{rms}	ε_{mean}
May	0.025	-0.021
June	0.043	-0.032
July	0.023	-0.021
August	0.045	0.044
September	0.021	-0.018
October	0.017	-0.014

This shows that the various predicted significant wave periods are good estimations.

Erlang distribution model for significant wave periods

The model for significant wave period distributions is derived from the expression (7) by mathematical logic(Unnikrishnan Nair, Muraleedharan & Kurup 2003). It is found that F(x) is Erlang distribution function. As Erlang is obtained from approximating the shape parameter of the Gamma model to the nearest integer, the Gamma model is the appropriate distribution for the significant wave period distribution.

Validation

The validation of the Gamma model for simulating the significant wave periods is made by using the visually estimated wave periods (Grid.17) (NPOL atlas 1978) (e.g. Fig. 5).

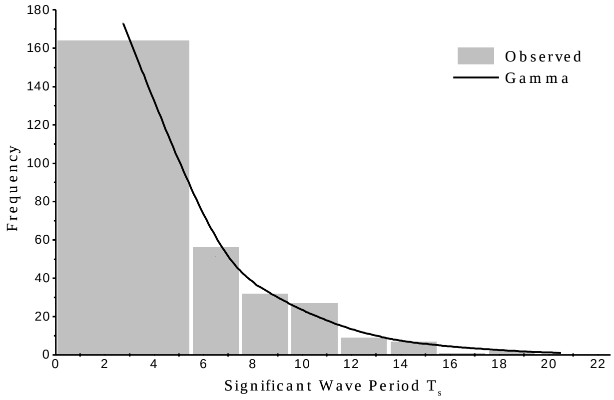

Figure 5. Comparison of observed (histograms, visual) and simulated (Gamma curve) significant wave period distributions.

The observed (histogram) and the simulated (curve) distributions show exponential behaviour throughout the year. The Gamma curve very well follows the observed distributions and also estimates the peak frequencies accurately. It is to be noted that the visually estimated wave heights and periods are the significant wave heights and periods.

The model fits in all cases at 0.05 level of significance by chi-square test after appropriate grouping.

Joint distribution

Visually averaged joint distribution of significant wave heights and periods over a period of ten years for Grid.17 (5°-9°N,73°-77°E, 4°x4°, NPOL data) is simulated by Bretschneider and Gluhovskii with zero correlation (Baba and Harish 1985) and Weibull-Gamma (with non-zero correlation) joint distribution models. The joint distribution of significant wave heights and zero crossing periods (recorded, CESS data) clubbed over a period of four years (1981-84) off Valiathura, south-west coast of India are also tried to be simulated by the same joint distribution models. (e.g. Fig.6).

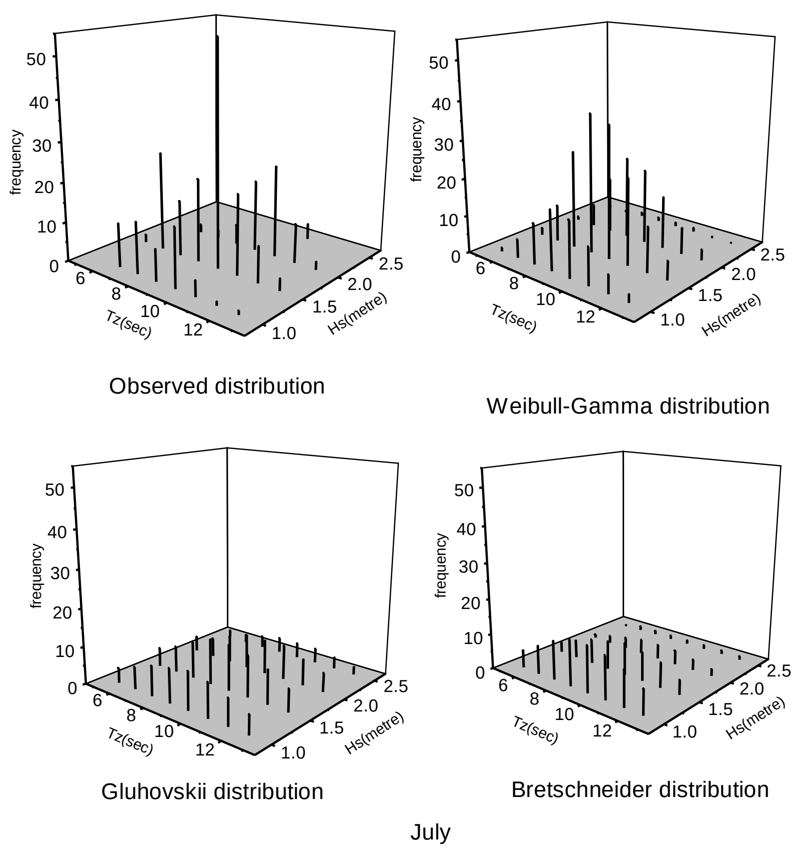

Figure 6. Computed and simulated surface plots off Valiathura in shallow water

The marginal distributions of wave heights and periods follow Weibull and Gamma laws respectively with goodness of fit of 83.3% and 100% by c^2-test at 0.05 level of significance. Since there exists significant correlation between the wave heights and periods (Table 5), the Weibull-Gamma joint distribution accommodating correlation effects simulates the significant wave height-zero crossing period surface plots, but it slightly underestimates the peak. The other two models completely fail to explain the observed fields.

Table 5. Correlation coefficient (ρ) of significant wave height and zero crossing period in shallow water off Valiathura

Depth	*Month*	ρ
Shallow water	May	-0.05
	June	0.25
	July	0.36
	August	0.30
	September	0.16
	October	0.30

The analysis of significant wave heights and significant wave periods are carried out by comparing the theoretically simulated 3-D plots. A considerable correlation is found to exist between the H_s and T_s (Table 6) and the Weibull-Gamma joint distribution including correlation effects is found to simulate more effectively than the Bretschneider and Gluhovskii joint distributions (e.g. Fig.7).

Table 6. Correlation coefficient (ρ) of significant wave height and period in deep water (Grid.17)

Depth	Month	ρ
Deep water	January	0.44
	February	0.38
	March	0.28
	April	0.63
	May	-0.46
	June	0.42
	July	0.43
	August	-0.47
	September	0.44
	October	0.44
	November	0.15
	December	0.22

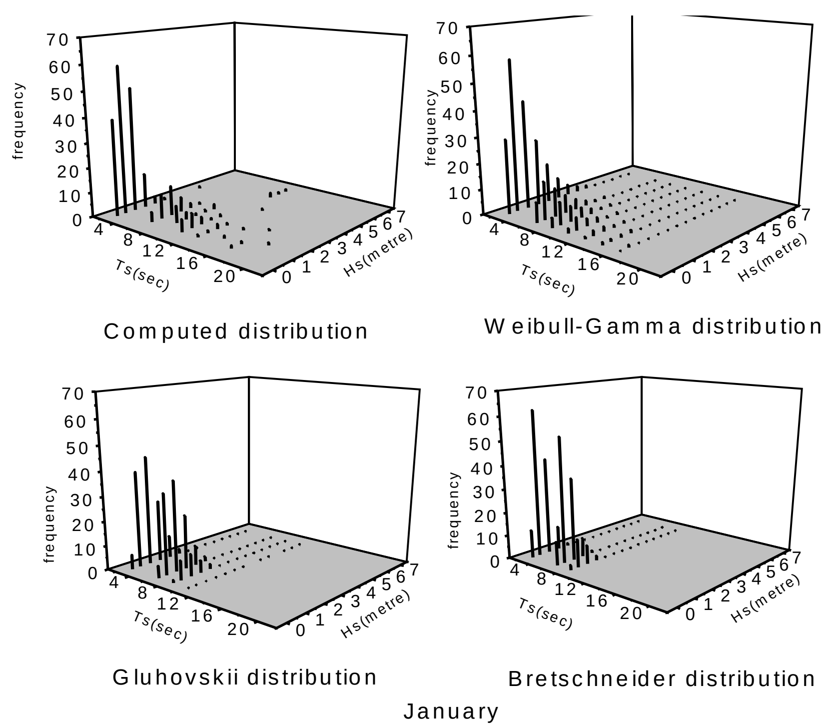

Fig.ure 7. Computed and simulated surface plots off Valiathura

Table 7. Relative bias error in shallow water for Weibull-Gamma joint distribution

Depth	Month	Relative bias (mean error, ε_{mean})
Shallow Water	May	-0.0738
	June	0.1238
	July	0.1455
	August	0.0642
	September	-0.0119
	October	0.1197

Table.8. Relative bias error in deep water for Weibull-Gamma joint distribution

Depth	Month	Relative bias (mean error, ε_{mean})
Deep Water	January	0.0600
	February	0.1311
	March	0.0945
	April	0.2227
	May	-0.2490
	June	0.1801
	July	0.1973
	August	-0.2774
	September	0.1704
	October	0.1341
	November	0.0669
	December	0.0827

Cases in which mean error exceeds 20% are rejected (Tables 7 & 8). It can be seen that the model fits in 83.3% cases affirming empirical support.

CONCLUSIONS

The long-term distribution of significant wave height is more effectively simulated by the Weibull model than the other competing models. The design wave parameters predicted have reliable accuracy and hence the parametric relations derived from the modified Weibull model for maximum wave height distribution, after accommodating a depth factor to include shallow water wave transformation effects, could be used for estimation in coastal water.

The significant wave height predicted also is sufficiently accurate suggesting that the prediction formulae derived from the Weibull model are effective in shallow water.

The application of the model Gamma for simulating the zero up crossing wave period distribution is reaffirmed empirically. Since the distribution function of this model is an infinite power series, any further mathematical treatment will lead to complexity. Hence the shape parameter of this model is approximated to the nearest integer to arrive at Erlang distribution model and various prediction formulae.

Therefore the expression derived for predicting various significant wave periods from Erlang model is considered for further analyses. The comparability of the distribution of the theoretically simulated various significant wave periods and the computed are made by ε_{rms} and ε_{mean} giving promising results. The maximum errors are respectively 4.5% and 4.4%. From this study the suggestion of Gamma law for zero up crossing wave period distribution is again re-established.

The validation of the Gamma distribution for simulating the observed significant wave period distributions are carried out using ten years visually estimated wave period data for deep waters giving fit for 100.0% cases at 0.05 level of significance by Chi-square test. Here the Weibull and Gamma are found to be empirically and logically the marginal distributions of Hs - T_s and H_s - T_z. The Weibull - Gamma joint distribution accommodating correlation effects simulated the surface plots of H_s - T_s and H_s - T_z more effectively than the Bretschneider and Gluhovskii joint distribution models with zero correlation.

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