

A short note on the application of Singular Spectrum Analysis for Geophysical Data processing

*Rajesh Rekapalli and R.K.Tiwari

CSIR-NGRI, Hyderabad-500007, India

AcSIR-NGRI, Hyderabad-500007, India

*Corresponding Author: rekapalli@gmail.com

ABSTRACT

Singular Spectrum Analysis (SSA) is a popular time series analysis tool. Numerous studies have proven its efficacy in processing the data contaminated with various kinds of noises. As the geophysical field observations are highly non-linear and contains random as well as coherent noises, the analysis of data using SSA provides the significant information in terms of Eigen properties of the system under investigation. Unlike standard Fourier and wavelet methods, the basis functions of the SSA are completely data adaptive (Eigenvectors of trajectory matrix). Such data adaptive basis functions enable the self similarity of time series in data gap filling and noise suppression. Here we made an effort to briefly discuss the principal component analysis, frequency filtering, noise suppression and data gap filling of SSA and their application in time domain geophysical data processing.

INTRODUCTION

Geophysical field data symbolizes an amalgamated response of physically interpretable signal of concern and certain amount of noise (unwanted signal). Researchers have developed several filtering techniques to separate signal from the field data with random noise as it appears as a flat spectrum in frequency domain (Yilmaz, 2001; Canales, 1984; Abma and Claerbout, 1995; Karsli et al 2006). The stratification of different earth layers has been occurred at different times in the geological history. Hence the geophysical depth series always stands as a synonym for the geological time series. Thus the analysis of the geophysical depth series using time series analysis tools enable the detection of basic properties of the underlying system that has engendered the depth series.

The singular spectral analysis (SSA) is one of the popular and powerful time series analysis tools, which has been invariably used to identify the unknown or partially known dynamics of the underlying systems (Vautard and Ghil, 1989; Vautard et al, 1992). Unlike the classical spectral methods, SSA signal decomposition and reconstruction employs completely data-adaptive basis functions(i.e., the Eigen modes of the trajectory matrix) , rather than the fixed sines, cosines and mother wavelets used in the other methods (Ghil and Taricco, 1997; Ghil et al, 2002). The geophysical data are often non linear in nature and also contains unavoidable data gaps. So, the analysis of such signals with discontinuities via classical methods invoking fixed basis functions (ex. Fourier, Wavelet etc.,) may not be more appropriate (Dimri, 1992; Bansal and Dimri, 2001, 2005; Rajesh et al., 2014). Such abrupt jumps generally symbolize the boxcar or seesaw-shapes. The precise reconstruction of such boxcar or seesaw-shapes could be possible using a single pair of

eigen triples in SSA rather than involving many harmonics in conventional methods (Ghil and Taricco, 1997). Hence, it is creditable analysing geophysical data in terms of the Eigen properties of independent principal process using SSA. Even though the applicability of SSA extended from astronomical data processing to stock market prediction, in the present communication we restrict our self to discuss its applicability (like principal component analysis, data gap filling, and Frequency filtering and S/N ratio enhancement by noise reduction) to geophysical data.

METHODOLOGY

The brief mathematical description of the SSA methodology is given bellow following Golyandina et al (2001).

Embedding:

We begin with embedding the trajectory matrix (**T**) of size $L \times (N-L+1)$ from time series $Y(t) = \{y_1, y_2, \dots, y_N\}$ using an appropriate window length L given by

$$\mathbf{T}_{L \times K} = [\mathbf{Y}_1, \dots, \mathbf{Y}_K] \longrightarrow (1)$$

Here $\mathbf{Y}_i = \{y_i, y_{i+1}, \dots, y_{i+L}\}^T$ indicates the vector of length L where $K = N-L+1$.

The window length is the crucial parameter, which dictates the separability of different Eigen modes. The classical limit for window length is $2 \leq L \leq N/2$ (Golyandina et al., 2001). If the data contains low frequency component of frequency $f_{\min}$, then we can choose $1/f_{\min}$ as the window length. Alternatively, window length can also be computed from the lagged auto correlation of the data. The half of the difference between the lags corresponding to any two successive points of same phase on the auto correlation plot can be chosen as the Window length.

Singular Value Decomposition (SVD):

The trajectory matrix was decomposed using SVD to obtain the eigenvectors and eigenvalues. The periodicity and the contribution of the principal components can be seen from the eigenvectors and eigenvalues respectively. The decomposition of T is given by

$$T = \sum_{i=1}^d \sqrt{\lambda_i} U_i V_i^T$$

$$= T_1 + T_2 + \dots + T_d \quad \text{--- (2)}$$

Where, λ_i is the i^{th} eigenvalue corresponding to the i^{th} eigenvector U_i of TT^T and d is the number of nonzero Eigen values. The triple $(\sqrt{\lambda_i}, U_i, V_i)$ is called the i^{th} eigen triplet. The SVD is a key process of SSA, which helps to identify the periodicities of different Eigen/ principal modes along with their significance and contribution. In general, the eigen components/ principal components of low variance present in the tail of the eigen spectrum corresponds to the unwanted signal (noise), (Trickett, 2003). Hence it is possible to identify the eigen components correspond to noise.

Grouping and Reconstruction:

In the next step, the eigen triplets obtained in the above SVD process are grouped by the periodicity of Eigenvectors and by dropping the insignificant eigen triplets corresponds to low eigenvalues as they resemble the unwanted noise. For example, If there exist two such groups (satisfying the requirement of periodicity and eigen values) given by $G_1 = \{T_1, T_2, T_4, T_5\}$, $G_2 = \{T_3, T_7\}$ among d triplets, then $T = G_1 + G_2 + G_r$ where $G_r = \{T_8, T_9, \dots, T_d\}$ is the residual signal representing the noise. The reconstructed trajectory matrix (T_r) could be computed from the identified groups G_1 and G_2 as follows. $T_r = G_1 + G_2 = \sum_{i=1}^7 \sqrt{\lambda_i} U_i V_i^T$

$$T_r = \begin{bmatrix} y_{(1,1)} & \dots & y_{(1,K)} \\ \vdots & \ddots & \vdots \\ y_{(L,1)} & \dots & y_{(L,K)} \end{bmatrix} \quad \text{--- (3)}$$

For different objectives the grouping scheme will be different. In frequency filtering the triplets with the eigenvector periodicities satisfying the cut off frequency constraints will be grouped to reconstruct the trajectory matrix. For principal component analysis we will group the individual or the pairs of (when they are of the same periodicity) eigentriples to reconstruct the trajectory matrix. For denosing we consider the eigentriples of significant eigenvalues for reconstruction.

Diagonal averaging:

Finally, we deduce the reconstructed time series of length N by diagonal averaging of T_r . Let us denote the reconstructed series by $Y_r = \{g_1, g_2, g_3, \dots, g_N\}$. The elements of Y_r can be computed as follows.

We define $L^* = \min(L, K)$, $K^* = \max(L, K)$ and let $y_{ij}^* = y_{ij}$ if $L < K$ and $y_{ij}^* = y_{ji}$ otherwise

$$g_k = \frac{1}{k+1} \sum_{m=1}^{k+1} y_{m, k-m+2}^* \quad \text{for } 1 \leq k < L^*$$

$$= \frac{1}{L^*} \sum_{m=1}^{L^*} y_{m, k-m+2}^* \quad \text{for } L^* \leq k < K^*$$

$$= \frac{1}{N-k} \sum_{m=k-K^*+2}^{N-K^*+1} y_{m, k-m+2}^* \quad \text{for } K^* \leq k < N \quad \text{--- (4)}$$

And $g_1 = y_{(1,1)}$

PRINCIPAL COMPONENTS AND THEIR SEPARATION

The principal component analysis is the basic and important utility of the SSA technique to reveal the dynamical components of the time series data for physical interpretation and prediction (Ghil et al., 2003; Serita, 2005; Tiwari and Rajesh, 2014, Rajesh and Tiwari, 2014, Tiwari et al., 2014). The identification is possible through Eigen analysis of trajectory matrix. As stated in methodology, the accuracy of the results depends on the selection of proper window length. The smaller window length than the optimal window length leads to the poorly resolved singular spectrum. This problem can be resolved by choosing the proper window length using above mentioned lagged auto correlation method. Finally, the separability among the principal components can be computed using the formula of Weighted correlation (W_c) to avoid artifacts (Ghil and Taricco, 1997).

$$W_c = \frac{(Pc^1, Pc^2)_w}{\|Pc^1\|_w \cdot \|Pc^2\|_w} \quad \text{--- (5)}$$

Where $\|Pc^1\|_w = \sqrt{(Pc^1, Pc^2)_w}$ and $(Pc^1, Pc^2)_w = \sum_{k=1}^N w_k \cdot Pc_k^1 \cdot Pc_k^2$, w_k and L is greater than or equals to $N/2$. The components are said to be well resolved if the W_c is nearer to zero.

The individual principal components can be reconstructed from the corresponding eigentriple. An example of principal component analysis of Global, Southern and Northern hemispheric Sea Surface Temperature (SST) annual average data from 1850 to 2012 is presented in the

Fig.1 (Data source: www.metoffice.gov.uk/hadobs.). The raw data singular spectrum, SSA reconstructed trend (first principal components) and second principal components of Global, Southern and Northern hemispheres using window length 30 are shown in Fig.1a, Fig.1b and Fig.1c respectively. The mismatch between the nonlinear trend as well as the second eigen mode of northern and southern hemispheric SST shown in the Fig.1 apparently evident for the hemispheric asymmetry reported by other researchers (Stouffer et al, 1989; Goosse et al., 2004; Friedmann et al., 2013; Neukom et al., 2014).

FREQUENCY FILTERING

Several researchers have demonstrated the time domain frequency filtering using SSA (Harris and Yan, 2010; Golyandina and Zhigljavsky, 2013; Rajesh et al, 2014). Accordingly, we can calculate the periodicity from the eigenvectors obtained in the SVD process for grouping and reconstruction stages to perform filtering operation. If we want to perform low pass filtering with a cut off frequency f_{LC} , we have to ignore the Eigen triplets corresponds to eigenvectors with periodicity less than $1/f_{LC}$.

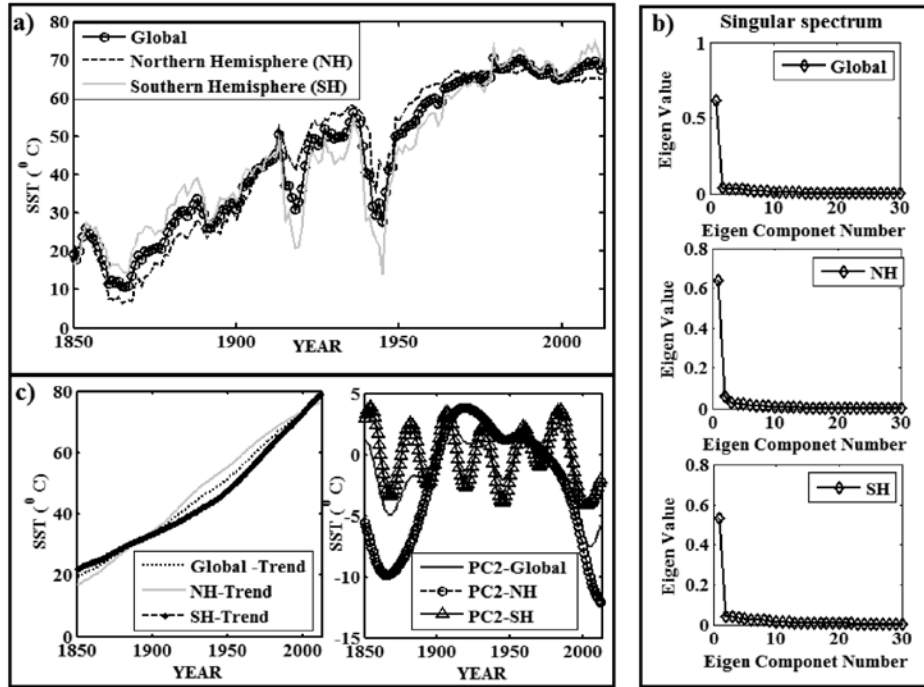


Figure 1: Principal component analysis of Global, Northern and Southern hemispheric SST data :(a) SST annual average data sets b) Singular spectrum using window length 30 and c) Non linear trend and 2nd principal component of global, northern and southern hemispheres.

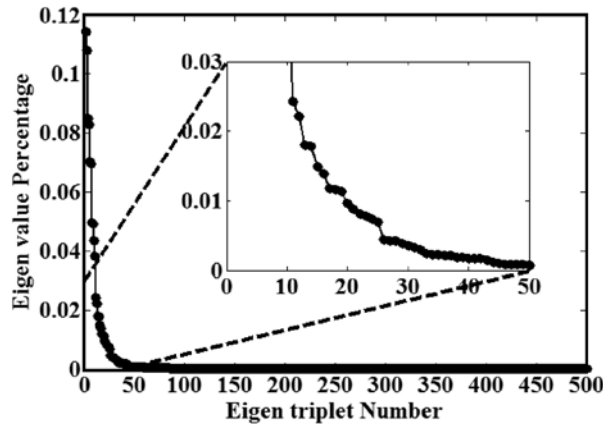


Figure 2. Eigen spectrum/ Singular spectrum of the reflection trace data.

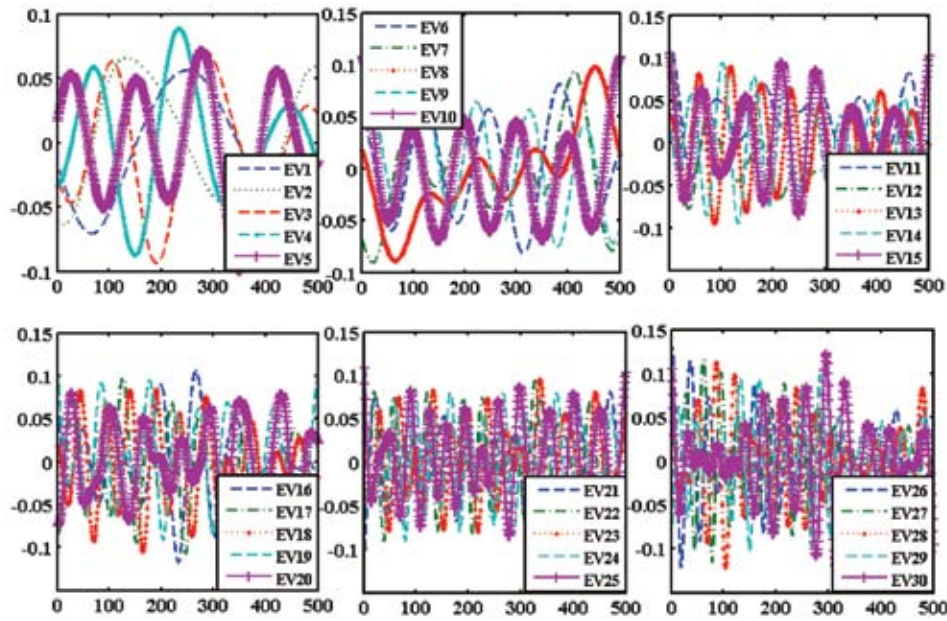


Figure 3: First 30 Eigenvectors of seismic trace used to identify the periodicities in filtering operation.

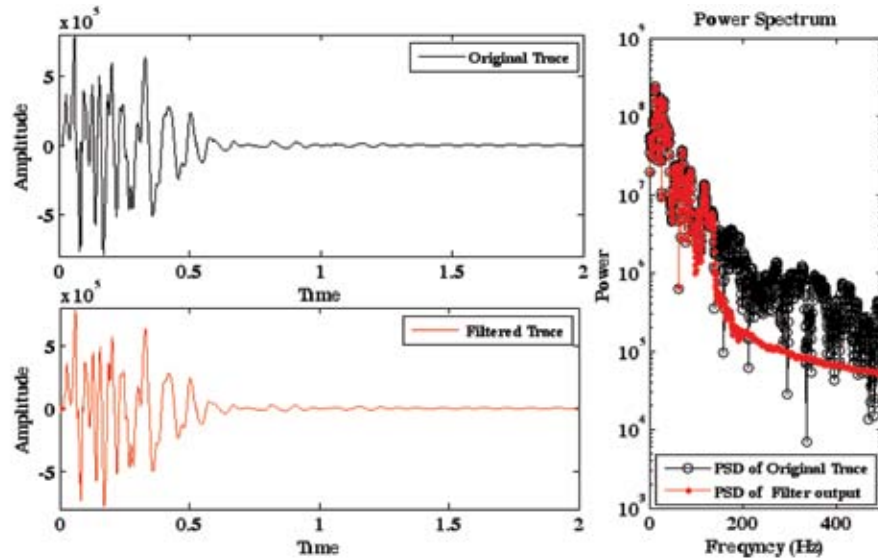


Figure 4: Original seismic trace, filtered trace using SSA low pass filtering and their respective power spectral densities.

For high pass filtering with a cut off frequency f_{HC} we have will consider the eigentriplets corresponds to eigenvectors with periodicity greater than $1/f_{HC}$. In band pass case we will select the eigentriplet having the eigenvector periodicity between $1/f_{LC}$ and $1/f_{HC}$. Here f_{LC} , f_{HC} are the low and high frequency cut off values of the pass band.

APPLICATION OF SSA FILTERING ON SEISMIC TRACE

The frequency filtering using SSA is demonstrated here by applying the method on single seismic trace containing

8000 samples with 0.25mS. Figure 2 and Figure 3 respectively shows the Eigen values percentages (Singular spectrum) and first 30 eigenvectors (EVs) of the data. As mentioned above, we will calculate the periodicity of EVs to filter the data.

Figure 4 shows the original and reconstructed traces using first 30 Eigen triplets as the rest are of the frequency ($1/\text{period of Eigenvector}$) greater than 135Hz (f_{LC}) which is the required upper frequency limit of low pass filter. The power spectral density of filtered data shown in the Fig.4 apparently indicates that the spectral components with frequency greater than 135 Hz are suppressed in the SSA

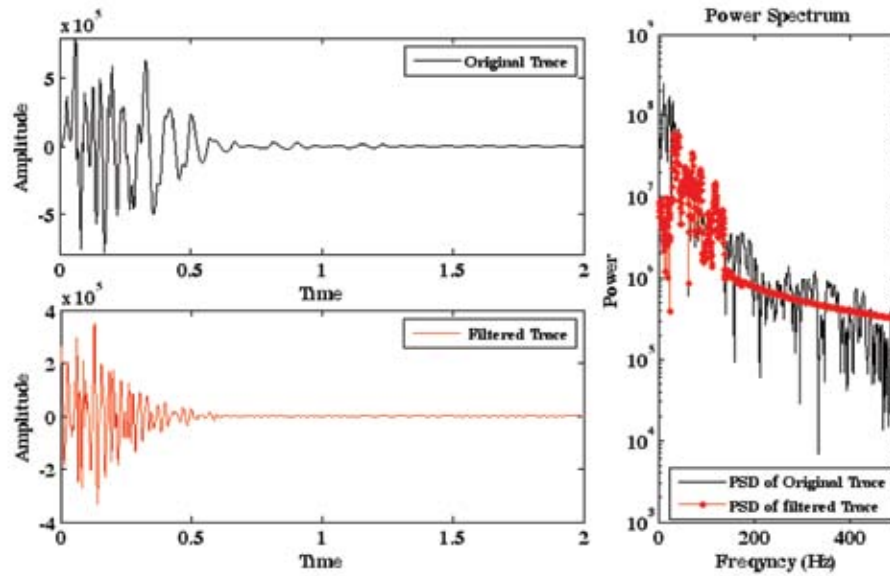


Figure 5: Original seismic trace, SSA band pass filtered trace along with the respective power spectrum.

Low pass filtering operation.

Finally the original seismic trace, band pass filtered data reconstructed using Eigen triplet group G(6,9,10 and 13 to 20) along with the respective spectral densities are shown in Fig.5. The periodicities observed from the eigenvectors in above triplet group are within the pass band limits (1/30, 1/135).The operation of band pass filtering includes the low pass and high pass filtering. Hence, from the above bandpass filtering one can understand the SSA high pass filtering operation.

NOISE SUPPRESSION AND DATA GAP FILLING

It is simple to define noise as an unwanted signal, but is difficult to hunt for such unwanted signal amalgamated with the data. In attempt to suppress the random noise, several SSA based techniques have been developed by researchers (Rajesh et al., 2014; Rajesh et al., 2012; Oropeza and Sacchi, 2011; Sacchi, 2009; Tricket, 2003; Allen and Smith, 1997; Vatuard et al., 1992). According to Tricket (2003), the components with maximum variance in the eigen spectrum represents the signal whereas the incoherent energy mapped on to the rest. Even though the insignificant coherent noise present within the data, it can be clearly identified in the Eigen spectrum with low eigenvalues in the tail of the spectrum. The random noise is more clearly visible with randomly fluctuating, structure less eigenvectors. Thus the eigen/singular value analysis of SSA enables the identification of such noises present in the data. Final, signal reconstruction by plummeting those noise components allow the suppression of noise. As the noise and data gaps increases the rank of the trajectory matrix,

the denoising is a kind of rank reduction in the SSA based denoising algorithms.

In the first example of the noise suppression, we have generated a synthetic profiling data, to which we have added 30% complex noise generated using the following equation (Rajesh et al., 2014, 2012) (Fig.6).

$$x_{n+1} = \mu \cdot x_n \cdot (1 - x_n) \dots \dots \dots (6)$$

Where μ is chosen as 3.7 ($0 \leq \mu \leq 4$) and $x_1 = 0.1$

The smooth anomaly observed in the pure data was clearly perturbed by the complex noise. One can see that the SSA de-noised output reconstructed from the first 3 eigen triplets replicates the smooth anomaly present in pure data (Fig.6).

Finally, we have provided a comparison of Space Lagged SSA (SLSSA) (Rajesh et al., 2012) denoising with wavelet three step decomposition of the seismic signals. The synthetic data generated using Ricker wavelet convolution with model reflectivity series was contaminated with 30% complex noise generated using equation (6). The pure synthetic data (Fig.7a), noisy data (Fig.7b), SLSSA (Fig.7c) and Wavelet de-noised outputs (Fig.7d) are shown in Fig.7. One can apparently see that the noise suppression is good in the SLSSA output compared to Wavelet denoising. Thus it is clear that the singular spectral methods alleviates suppresses the complex noise also.

In general, the signal of unwanted frequency present in the data could also be referred as noise. For example, in the case of seismic reflection data, we have considered the 35 to 135 Hz frequency components as signal and the rest as noise. However, it can also be noticed from the Figs 4, 5, the SSA is capable of separating the noise from the signal

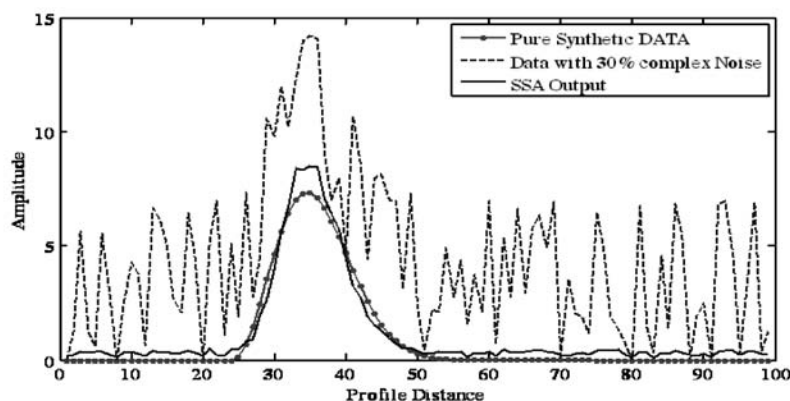


Figure 6: Pure synthetic data representing a smooth anomaly (Solid line with marker), synthetic data with 30% complex noise (dashed line) and its SSA reconstructed denoised output (solid line).

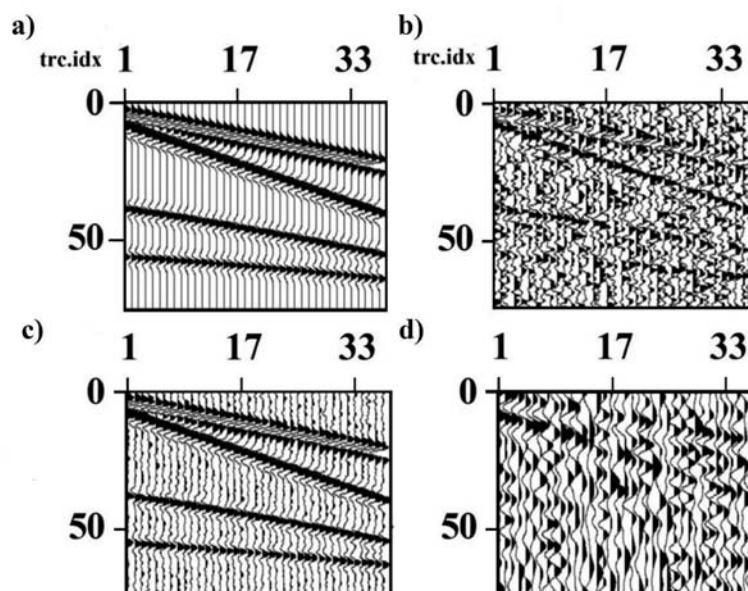


Figure 7: An example of data denoising (a) Pure synthetic data (b) Synthetic data with 30% complex noise (c) Space Lagged SSA denoised output of the noisy data (d) Wavelet denoised output of the noisy data.

using the frequency characteristics of signal and noise. In the low pass filtering operation we have suppressed the high frequency noise. Similarly, in the band pass filtering we have suppressed the low frequency noise (ground roll and other) along with high frequency noise.

Data gap filling is another unique feature of SSA (Kondrashov, et al, 2010; Kondrashov and Ghil, 2006; Schoellhamer, 2001). The gaps in the geophysical data are obvious due to many unavoidable reasons like sudden changes in the topography, malfunctioning of instruments etc. For precise data interpretation, we require to fill the gaps in the processing. The SSA signal reconstruction involves the eigen properties of the time series, more precisely the eigenvectors. Hence the data gap filling in SSA methodology is purely eigen analysis based recovery. The pre normalization and post multiplication by division

and multiplication of the data by the maximum is a kind of small matrix completion process in the methodology. So clearly it is a true preservation of eigen process and different from interpolation technique. Thus the validity of filled data is more reliable compared to averaging, interpolation techniques. Figure 8 illustrates the data gap filling efficacy of SSA using a simple smoothly varying synthetic signal. Top panel image of Fig.8 is the original data and in the middle panel data we have created the gaps (DG1, DG2 and DG3) by setting them to zero. The last panel of Fig.8 shows the SSA reconstructed data of middle panel data. Even though SSA can be used for data gap filling, one should keep in mind the data gap must be always smaller than that of window length. Otherwise the gaps are considered as features of data leading to wrong reconstruction.

In another example of data gap filling, we have

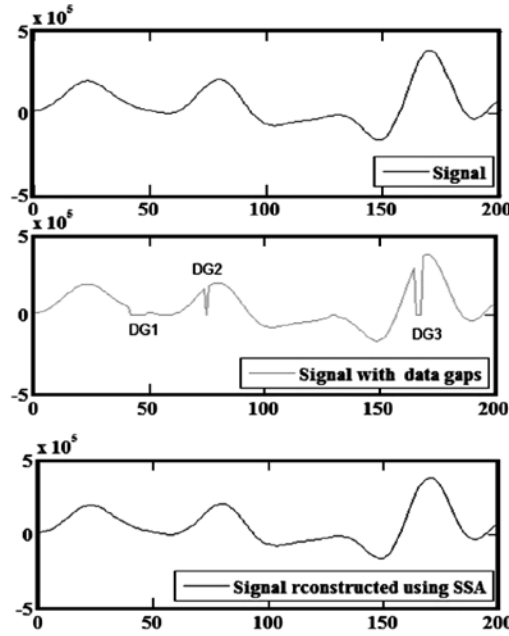


Figure 8: Example illustrating data gap filling efficacy of SSA.

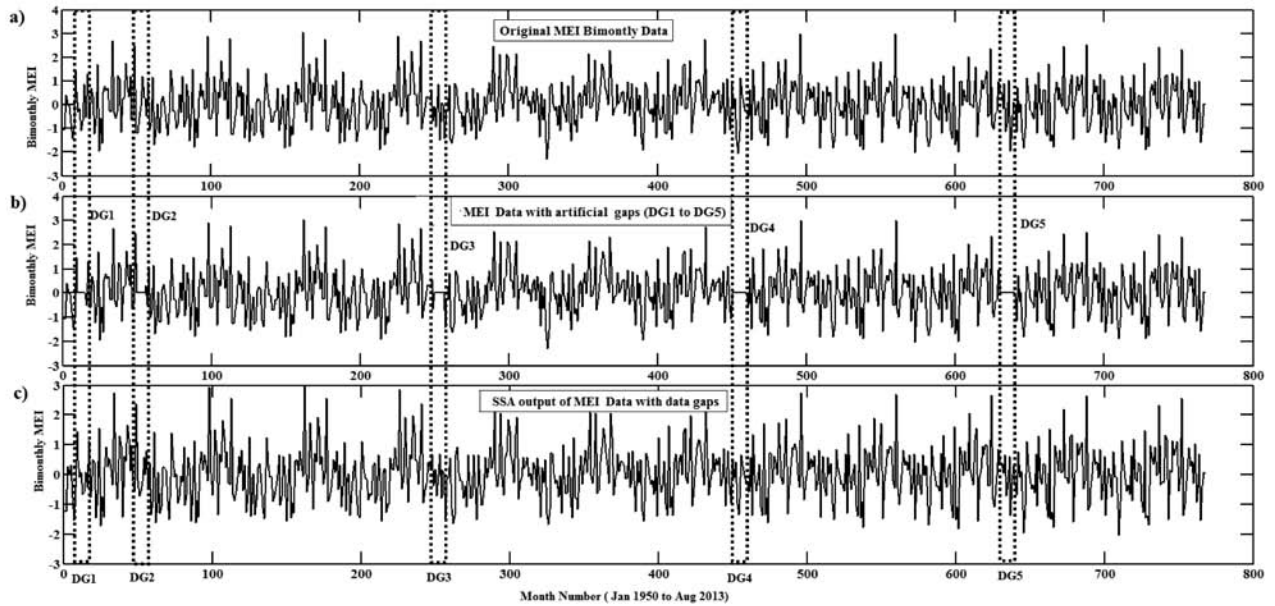


Figure 9: An example of Data gap filling: a) Original MEI bimonthly data b) MEI data with artificial data gaps c) SSA data gap filling output of the MEI data with data gaps.

generated artificial data gaps (DG1 to DG5) in the bimonthly Multi variate ENSO Index (MEI) data set downloaded from NOAA website (Wolter, 1987; Wolter and Timlin, 1993) for the period Dec 1949/Jan 1950, to July 2013/Aug 2013 as shown in the dashed rectangles in the Fig. 9. The original MEI data, MEI data with artificial data gaps and data gap filling using SSA are shown in Fig. 9a, 9b and 9c respectively. One can apparently see that the reconstructed data shown in Fig. 9c at artificial data gaps

generated in the MEI data (DG1 to DG5 in Fig. 9b) are matching with the original data (Fig. 9a).

Finally, we have compared the SSA data gap filling with conventional interpolation methods like Nearest Neighbour Interpolation (NNI), Piece wise Cubic Hermite Interpolation (PCHI) and Spline interpolation methods using the MEI data with artificial data gaps. Figures 10a and 10b respectively represents original MEI data, MEI data with artificial data gaps. The data recovered using NNI,

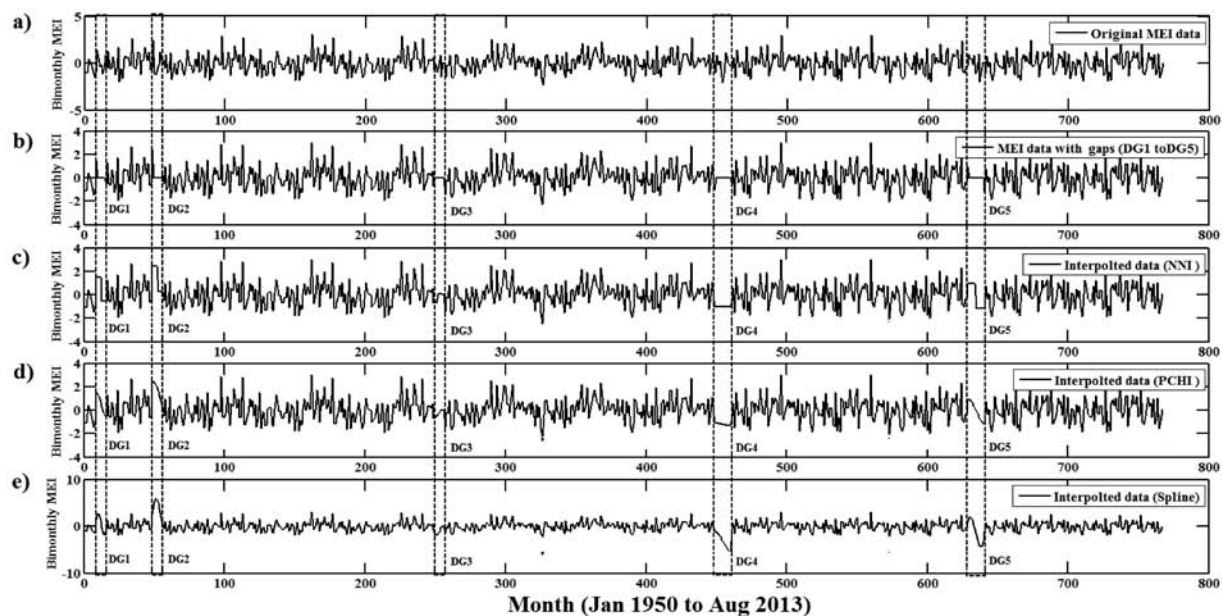


Figure 10: An example of Data gap filling: a) Original MEI bimonthly data b) MEI data with artificial data gaps c) data gap filling using NNI d) data gap filling using PCHI e) data gap filling using spline interpolation.

PCHI and Spline methods are shown in Fig.10c, Fig.10d and Fig.10e respectively. It can be noticed from the dashed rectangular regions of the Fig.10, the interpolation methods failed to recover the data. Hence the data gap filling using SSA is more robust than that of the conventional methods.

CONCLUSION

In conclusion, the SSA is a good choice for analyzing geophysical data for principal component identification, frequency filtering, noise suppression and data gap filling. The usage of data adaptive basis functions enables the self-similarity, a characteristic feature of many geophysical data sets, in de-noising and data gap filling. We have explained principal component analysis by applying the underlying method on SST hemispheric data sets. The mismatch between the first two eigen modes/ principal components demonstrates the inter hemispheric climatic disparity reported by several researchers. We have demonstrated the SSA frequency filtering (low pass, high pass and band pass filtering) using seismic reflection data. The SSA stands as the robust de-noising technique even for highly complex seismic data as it involves the eigen analysis for noise identification. Finally, we have shown that the SSA data gap filling efficiency in comparison with conventional interpolation techniques.

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Mr. Rajesh Rekapally received his M.Sc. degree from Andhra University, Visakhapatnam. He is currently working as a Senior Research Fellow at CSIR-NGRI and pursuing Ph.D with AcSIR-NGRI. His research interest includes the development and application of modern spectral techniques to process geophysical data and modeling of the earth and climate processes. He is passionate about the multi-disciplinary research problems.



Prof. R.K. Tiwari received M.Sc. and Ph.D. degrees from Banaras Hindu University, Varanasi, India. He is currently working as Chief scientist at CSIR-NGRI, professor and Ph.D. coordinator of Academy of Scientific and Innovative Research (AcSIR)-NGRI. He is well known for spectral analyses and nonlinear time series modelling and forecasting of geophysical and geological processes. His research interest includes the development and application of modern spectral techniques, concept of fractal and chaos physics for understanding the evolutionary history of the earth's processes. Prof. Tiwari was awarded the prestigious Krishnan Gold medal of the Indian Geophysical Union in 1987 for his outstanding contributions to the theoretical geophysics, National Mineral Award in 2006 by the Ministry of Mines, Government of India and Decennial Medal of Indian Geophysical Union (IGU) 2009. He is a guest lecturer for several universities on the subject of "Advanced Spectral and Nonlinear techniques of geophysical data processing".