

A quasi-numerical solution for 3-D meso-scale lee wave associated with meso-scale baroclinic dry mean flow across the Assam-Burma hills in India

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ABSTRACT

In this study an attempt has been made to obtain a 3-D meso-scale lee wave solution associated with a meso-scale flow across the Assam-Burma hills (ABH), following a quasi-numerical approach. To obtain the solution a laminar, Boussinesq, non-rotational 3-D mean flow with realistic vertical variation of wind and temperature across the ABH have been considered, where both buoyancy frequency (N) and basic flow (U) in the wind are realistically variant with height. ABH has been approximated by two 3-D elliptical barriers, separated by a valley of some finite width and is broadly north-south (NS) oriented. For more simplicity, the basic flow has been assumed to have only one component normal to the major ridges of the barriers. Two cases have been discussed. In every case, we have computed perturbation vertical velocities (w') (wave part) in the central plane at different heights, at different downstream locations. Vertical variation of w' at different downstream locations shows cellular structure. It is also found that the maximum updraft regions associated with lee waves are approximately 'horseshoe shaped', concave to down wind direction and spread laterally with vertical.

INTRODUCTION

It is well known that mountain lee wave is a potential aviation hazard because airflow over mountain regions is more disturbed than over plains. The cause of this is that vertical variation of static stability and wind of the mean flow (Dutta, 2005). Scorer (1949) first addressed the variation of wind and stability with height. He considered a frictionless, non rotating, steady and incompressible flow. He divided the atmosphere into two layers and obtained Scorer's parameter (l_s^2). He showed that no lee wave is possible for constant l_s^2 with height. He proved that the necessary condition for the occurrence of lee waves in a two layer atmospheric model $l_1^2 - l_2^2 \geq (\pi^2/4h^2)$, where l_1^2 and l_2^2 are the values of l_s^2 ($l_s^2 = (N^2/U^2) - (1/U) (\partial^2 U / \partial z^2)$), $N = N(z)$ is the Brunt-Väisälä frequency and $U = U(z)$ is basic-state wind flow, in the lower and upper layer respectively and h is depth of stable lower layer. Scorer and Wilkinson (1956) had studied three-dimensional mountain wave problem by synthesising an isolated 3-D hill by superposition of infinite ridges inclined at different angles and intersecting at a point, which was expressed mathematically by an integral. The lee wave pattern in their solution was approximately 'horseshoe shaped' with concave in the downwind direction. Wurtele (1957) approached the 3-D mountain wave problem, considering a stably stratified flow across a semi-infinite plateau with

narrow width. This predicted a region of updraft, which was approximately 'horseshoe shaped' with concave downwind, and was placed some distance downstream of the orographic barrier. Crapper (1959) presented a 3-D small perturbation approach of waves produced in a stably stratified air stream flowing over a mountain. He obtained the fundamental solution for a doublet disturbance in an air stream with Scorer's parameter remaining constant and then extended it to a disturbance caused by a circular mountain in the same air stream. He showed that circular mountain can give rise to waves which have greater amplitude than those produced by an infinite ridge in the same air stream. Crapper (1962) presented the air flow across a 3-D elliptical orographic barrier for two types of air stream. In one type the Scorer's parameter (l_s^2) was constant with height and in the other Scorer's parameter (l_s^2) falls exponentially with height. In both cases he found lee waves, which closely resembled bow wave of a ship and contained in a wedge. Sawyer (1962) studied gravity waves in the atmosphere as a 3-D problem. He solved the equation numerically for specified two or three layers atmosphere to determine possible wave lengths in the horizontal directions for lee waves. He got results for the cases when wind direction change with height as well as for the cases when wind direction remained same in the vertical. For the latter case he showed that lee waves are possible in 3-D, but as the wave number in y -direction increases, wave number

in x-direction also increases slowly. Das (1964) studied the influence of Himalaya, approximated as a layer 3-D circular mountain, using a linear baroclinic model which included the variation of Coriolis parameter (f) with latitude. He considered the effect of Coriolis force and showed that the nodal lines in his solution were a system of concentric circles. Sarker (1965) studied the theoretical mountain wave problem with respect to the Western Ghats. He expressed mathematically the E-W vertical cross section of Western Ghats and established the expression for vertical velocity and stream line displacement for a two dimensional flow during winter season when atmosphere was having sufficient stable stratification. Sarker (1967) has modified his analytical model for orographic rainfall over western Ghats with the aid of numerical methods, during southwest monsoon season when atmosphere is almost neutral with respect to pseudo adiabatic lapse rate. De (1970) showed lee waves as evidenced by satellite cloud picture over northeast India and observed wavelength of lee waves across Assam-Burma hills was between 17km and 34km. Smith (1980) studied linear theory of stratified flow past an isolated mountain. He showed that for a stability stratified hydrostatic flow over a 3-D isolated circular mountain, approximately 'horseshoe shaped' region of updraft with concave in downwind direction. Somieski (1981) derived a second order wave equation from the primitive equation including constant rotation and vertical wind shear of the mean flow. He solved the equation numerically stratified hydrostatic flow over 3-D circular mountain. Sinha Ray (1988) reported mountain waves over the western Himalayas as evidenced by satellite cloud imageries. Observed wavelength of mountain wave across western Himalayas was between 7km and 23km. Kumar et al (1995) have shown the effect of latent heat release on windward side of a mountain, due to precipitation, assuming wind speed changing with respect to height. They have considered a single profile based on actual Peshawar data for the analysis. Dutta et al (2002) have studied 3-D lee wave across the elliptical barrier in a stably stratified air stream and obtained transverse lee wave and divergent lee wave. Dutta (2003) obtained divergent lee wave with respect to the basic flow of 3-D lee wave in stably stratified barotropic atmosphere.

In India, the problems of lee waves across the Assam-Burma hills was first addressed by De (1970) and subsequently by De (1971), Farooqui and De (1974), Dutta and Naresh Kumar (2005). Farooqui and De (1974) used a two dimensional model to calculate the flow over a small obstacle (half-width 2km) and large obstacle (half-width 20km) across the Assam-Burma hills (200-300 km). De (1970, 1971) computed wavelength of lee waves over Assam-Burma hills using an approach, similar to Sarker (1966,1967). However, all the above studies on mountain wave across Assam-Burma hills are 2-D.

When air flows across an orographic barrier, then generally two types of waves are generated, viz., transverse lee waves and divergent lee waves. Transverse lee wave is mainly due to flow across the barrier and divergent lee wave is mainly due to flow around the barrier (Crappier 1959, 1962). However when this physical problem is modelled mathematically, following a 2-dimensional approach, basic flow around the barrier can't be incorporated and eventually the divergent lee waves are missed. Since any orographic barrier, existing realistically, does have a finite extension in the direction normal to the basic flow, it appears that it is better to have a 3-D treatment of the present problem of airflow across Assam-Burma hills. Accordingly, in this paper, a 3-D baroclinic lee wave model across the Assam-Burma hills has been developed for a basic flow, where, both stability and velocity in the basic flow are variant with height realistically, which has not been addressed so far.

DATA

As the ABH is north-south oriented, when it is approximated analytically by two 3-D elliptical barriers, separated by a valley, the major ridges become north south oriented. Hence the zonal component of the basic flow only interacts effectively with the ABH to give rise to lee waves. The only station on the upstream side is Guwahati (26.19°N Latitude and 91.73°E Longitude). Hence the RS/RW data of Guwahati for those dates, which corresponds to the observed two dimensional lee waves across ABH, as reported by De (1970, 1971), have been obtained from Archive of India Meteorological Department, Pune.

METHODOLOGY

In the present study an adiabatic, laminar, non-rotational, and steady state flow of a vertically unbounded stratified and Bousinesq fluid across a 3-D meso-scale orographic barrier has been considered. The present study is similar to the study of Dutta (2005) in most aspects, except for the lower boundary condition. It is assumed that the basic flow (U) is normal to the major ridges of elliptical barriers and it is realistic with height and the buoyancy frequency (N) considering the realistic vertical variation. The rectangular co-ordinate system in which x-axis point towards east, y-axis towards north and z-axis vertically upwards is considered. Using the method followed by Dutta(2005), we obtain the following vertical structure equation for perturbation vertical velocity ($\hat{w}$)

$$\frac{\partial^2 \hat{w}_1}{\partial z^2} + [f(k, l, z) - \kappa^2] \hat{w}_1 = 0 \quad \dots (1)$$

Where

$$f(k, l, z) = \left[\frac{N^2(k^2 + l^2)}{(Uk)^2} - \frac{1}{U} \frac{d^2 U}{dz^2} - \frac{1}{\rho_0} \frac{d\rho_0}{dz} \frac{1}{U} \frac{dU}{dz} + \frac{1}{4\rho_0^2} \left(\frac{d\rho_0}{dz} \right)^2 - \frac{1}{2\rho_0} \frac{d^2 \rho_0}{dz^2} \right] \quad \dots (2)$$

and

$$\kappa = \sqrt{k^2 + l^2}, \quad \hat{w}(k, l, z) = \sqrt{\frac{\rho_0(0)}{\rho_0(z)}} \hat{w}_1(k, l, z) \quad \dots (3)$$

and $\hat{w}$, $\hat{w}_1$ are the double Fourier transforms of $\hat{w}$, $\hat{w}_1$ respectively. $\rho_0(z)$ is the basic state density at the level z and k, l are wave number vectors. De(1973) found that in India, the strong mountain waves occur under the favourable condition during winter season over the Assam-Burma hills. Now, in winter season the atmosphere is stable respect to dry adiabatic lapse rate. So, the numerical computation is made for the specific case during winter season, we take $N^2 = g/\bar{T}(\gamma_d - \gamma)$, here γ is the environmental lapse rate and γ_d is the dry adiabatic lapse rate. At the ground surface ($z=0$) the flow is assumed to follow the terrain:

$$\hat{w}(x, y, 0) = U(0) \frac{\partial h(x, y)}{\partial x} \quad \dots (4)$$

Where $h(x, y)$ represents the terrain profile. The profile of Assam Burma hills (ABH) is double ridges with height ' h_1 ' and ' h_2 ' and half width ' a ' along the basic flow (along x-axis) and ' b ' across the mean flow (along y-axis), separated by a valley of finite distance (d_1). The mathematical expression of ABH profile in 3-D is given.

$$h(x, y) = \frac{h_1}{1 + \frac{x^2 + y^2}{a^2 + b^2}} + \frac{h_2}{1 + \frac{(x-d_1)^2 + y^2}{a^2 + b^2}} \quad \dots (5)$$

In the present study the values of a , d_1 , h_1 and h_2 are same as those in De (1971) and $b = 2.5a$ as in Dutta (2005), as in Prasanta Das et al (2013). Profile of ABH barrier (3) is given by Fig.1.

Therefore, we take $a=20\text{km}$, $b=2.5a$, $d_1=45\text{km}$, $h_1=0.9\text{km}$ and $h_2 = 0.7 \text{ km}$ and .

If $\hat{h}(k, l)$ be the double Fourier transformation of $h(x, y)$. The expression of $\hat{h}(k, l)$ given as

$$\hat{h}(k, l) = 2\pi ab(h_1 + h_2 e^{-ikd_1}) K_0(\sqrt{a^2 k^2 + b^2 l^2}) \quad \dots (6)$$

Where

$$K_0(\sqrt{a^2 k^2 + b^2 l^2}) \quad \dots (7)$$

is the Bessel function of second kind of order zero. The details of above derivation to see Appendix-I. Using the same methodology and same technique of Dutta (2005) to solve the equation (1) by using (3) and (4) for searching only lee wave part of vertical velocity $\hat{w}(x, y, z)$. In this similar way of Dutta (2005) the possible lee wave numbers are found out for a given divergent lee wave number. The total association of all possible lee waves for all divergent lee waves towards the $\hat{w}(x, y, z)$ is obtained by following Dutta (2005) as given below:

$$\hat{w}_{lee}(x, y, z) = -2\pi ab \sqrt{\frac{\rho_0(0)}{\rho_0(z)}} \sum_l \sum_{k_q} k_q U(0) \frac{\Psi(k_q, l, 0)}{\left[\frac{\partial \Psi(k, l, 0)}{\partial k}\right]_{k=k_q}} \left\{ \begin{array}{l} h_1 \sin(k_q x + ly) + \\ h_2 \sin(k_q x + ly - k_q d_1) \end{array} \right\} K_0(\sqrt{a^2 k_q^2 + b^2 l^2}) \delta l \quad \dots (8)$$

Where $\hat{w}_{lee}(x, y, z)$ is the lee wave part of $\hat{w}(x, y, z)$. The physical meaning of the symbols used in above equations has been illustrated in Appendix-II.

RESULT AND DISCUSSION

Using similar technique to Dutta (2005) we have found out the possible transverse lee wave numbers for a given divergent lee wave number and computed the perturbation vertical velocity ($\hat{w}$) at different heights and different downstream location across the Assam-Burma hills (ABH) for the different cases in winter season. Also we obtain the contours pattern of vertical velocity at the different heights for various cases. In the cases during winter season, the graphical relation between two wave number vectors k and l is shown, when atmosphere is strongly stratified for the occurrence of the lee waves. All the above results are discussed in order to case wise are given below:

CASE 1. (03.02.1967)

De (1973) investigated that the airstream characteristic across the ABH during winter season is favourable for the occurrence of the lee waves. The profiles of basic flow $U(z)$ and temperature $T(z)$ in the undisturbed flow are shown in Fig.2a. This is based on the average of 0000UTC and 1200UTC RS/RW data of ABH on this date during winter season. The Fig.2a shows that the profile of temperature $T(z)$ is constant lapse rate with vertical height and the vertical profile of basic flow $U(z)$ in the two layers between surface and 3km and between 3km and 5km, it is found that at both the layers the mean wind flow is approximately 2.75m/sec, where as in the lower layer $U(z)$ increases with height and in the upper layer $U(z)$ decreases with height, this is a most important sign for the satisfaction of Scorer's parameter (l_s^2) i.e. for occurrence of the lee waves.

Now the perturbation vertical velocities ($\hat{w}$) (i.e. wave part only) in the central plane at different heights and different downstream locations like as $x=0\text{km}$, $x=20\text{km}$, $x=50\text{km}$, $x=80\text{km}$, $x=130\text{km}$ are shown in the Fig.2b. From this Fig.2b we clearly see that at every point of downstream location the perturbation vertical velocity ($\hat{w}$) being alternatively positive and negative i.e. cellular structure and this is in conformity with the findings from earlier investigators. This evidence is very important for aviation. If we observe at the point of 80km on the lee side of two barriers, there is a maximum updraft at 9km level.

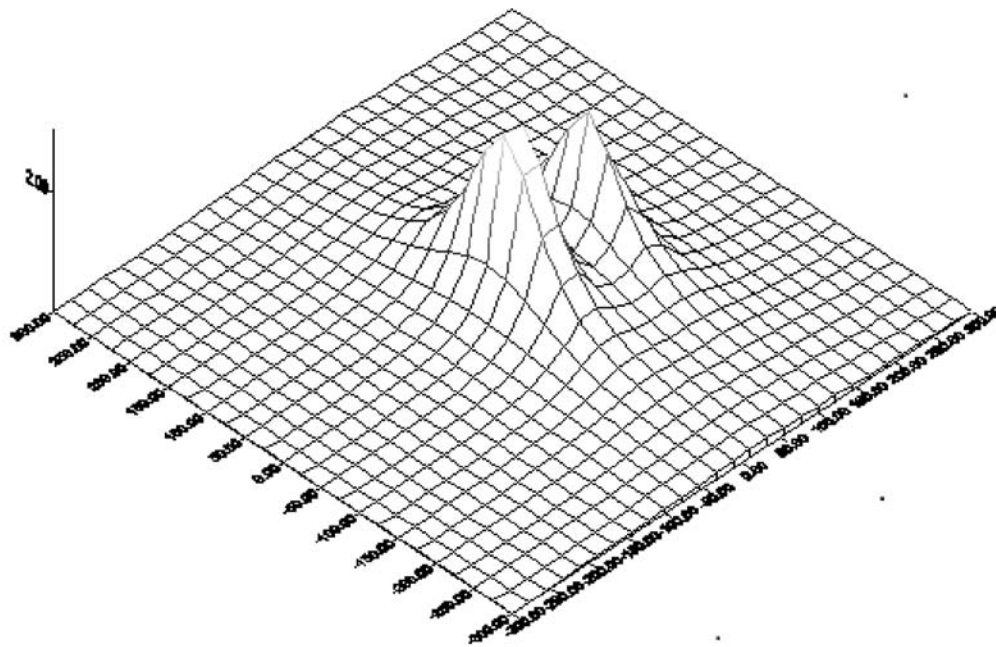


Figure 1. Profiles of Assam-Burma hills.

The maximum values of the $\bar{w}$ are 3.9 m/sec, 4.9 m/sec, 3.5 m/sec, 1.8 m/sec and 2.3 m/sec at the surface, 1.5km, 3km, 6km and 9km above mean sea level respectively.

Since, in the present 3-D study entire part of the mean wind doesn't flow across the barrier, rather one part flows across the barrier and another part flows around the barrier. As a result magnitude of computed vertical perturbation vertical velocity/displacement becomes lesser than that from a 2-D study.

The graphical relation between k and l as scatter diagram is shown in Fig.2c. by using the profile of $U(z)$ and $T(z)$ as input. From this Fig.2c clearly we see that a given divergent lee wave number (l) gives rise to a single transverse lee wave number (k) on that date across ABH and also k decreases with increases of $|l|$.

The contours of vertical velocity ($\bar{w}$) at surface, 1.5km, 3km, 6km and 9km above mean sea level, which approximately resemble to 900hPa, 850hPa, 700hPa, 500hPa and 300hPa respectively, are shown Fig.2(d-h). From these figures we see that the maximum updraft regions are approximately 'horseshoe shaped', concave to the down wind direction and lateral spreading with vertical, implying divergent lee wave, which is in qualitative conformity with finding from earlier results.

CASE 2. (08.01.1967)

The vertical profiles of wind $U(z)$ and $T(z)$ are shown in the Fig.3a, which are based on the average of 0000UTC and 1200UTC RS/RW data of ABH on that date during winter season. From this Fig.3a we see that the profile of

$T(z)$ in the undisturbed airflow is constant lapse rate. Now we consider the two layer between surface and 2km and between 2km and 3km. The mean wind of these two layers is approximately 3.7m/sec, where as in the lower layer $U(z)$ increases and in the adjacent upper layer $U(z)$ decreases. So Scorer's parameter (l_s^2) is higher in the lower layer and it is lower in the adjacent upper layer, a situation, which is conducive for the occurrence of lee waves.

Now, the vertical profile of wave part of the perturbation vertical velocities ($\bar{w}$) in the central plane at different heights and different downstream locations like as $x=0$ km, $x=20$ km, $x=50$ km, $x=80$ km, $x=130$ km, are shown in Fig.3b. The Fig.3b shows that at any level, the vertical velocity $\bar{w}$ in the central plane is cellular structure. This result is conformity with finding from earlier result. This is also a good sign of lee wave. If we see at the point of 80km and 130km are the lee side of two barrier, there are maximum updraft at about 0km level. This is very important information for aviation. The maximum values of vertical velocity $\bar{w}$ are 2.1m/sec, 2.3m/sec, 3.5m/sec, 2.7m/sec and 1.9m/sec at the surface, 1.5km, 3km, 6km and 9km, respectively.

Similar to case-I, in this case also the magnitude of computed value of perturbation vertical velocity may be attributed to the flow both across the barrier and over the barrier.

The graphical relation between (k , l) as a scatter diagram by using the profile of wind $U(z)$ and temperature $T(z)$ as input, is shown Fig.3c. Similar to previous case, in this case also, in general k increases with $|l|$ and there is a single transverse lee wave number (k) for a given divergent lee wave number (l).

Now, the contours of vertical velocity $\dot{w}$ are shown in Fig.3(d-h) at surface, 1.5km, 3km, 6km and 9km above mean sea level, which approximately resemble to 900hPa, 850hPa, 700hPa, 500hPa and 300hPa respectively. Here

we have seen that the maximum updraft regions are approximately 'horseshoe shaped', concave downwind and laterally spreading in the vertical, which is in qualitative conformity with finding from earlier results.

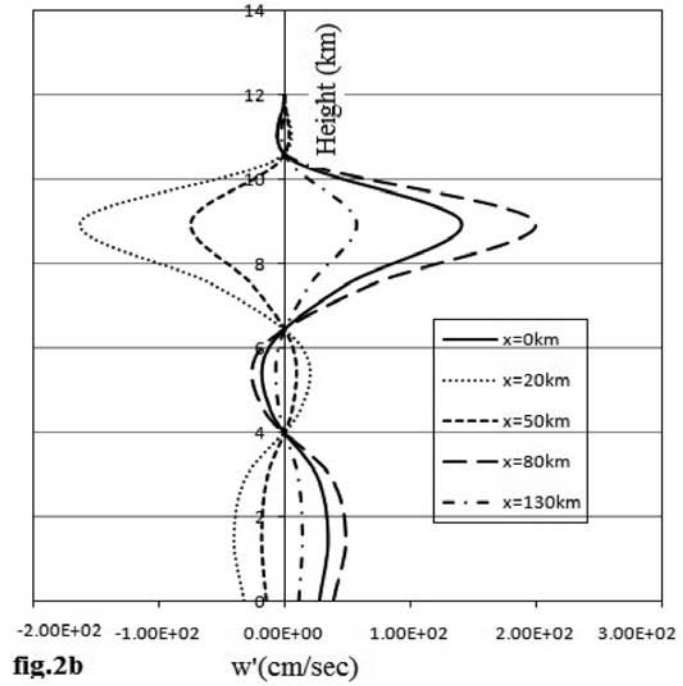
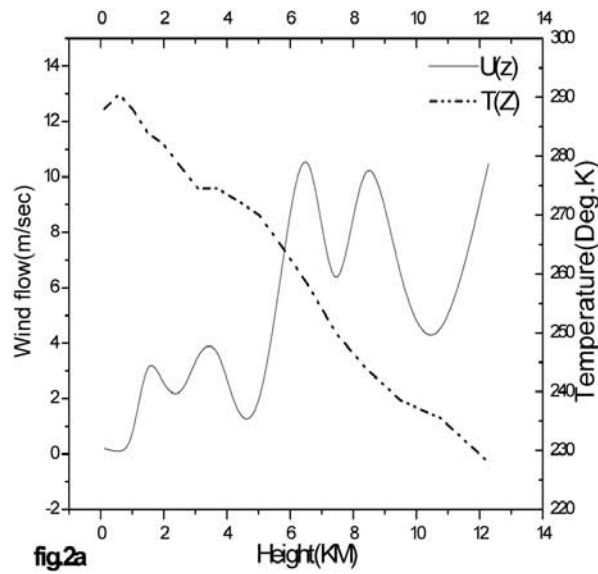
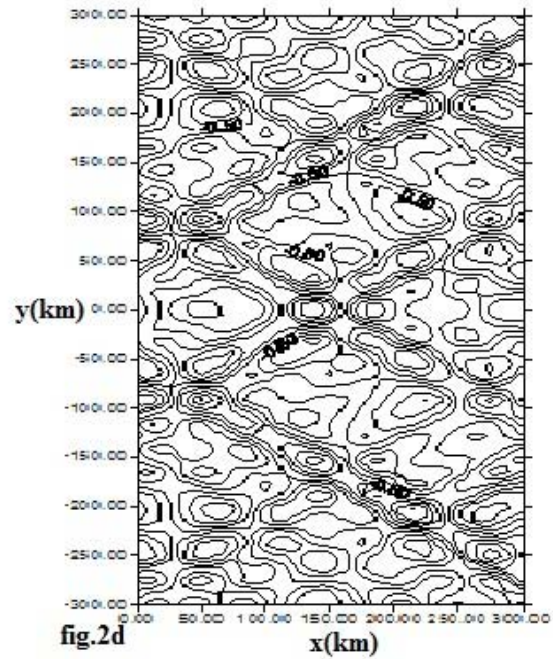
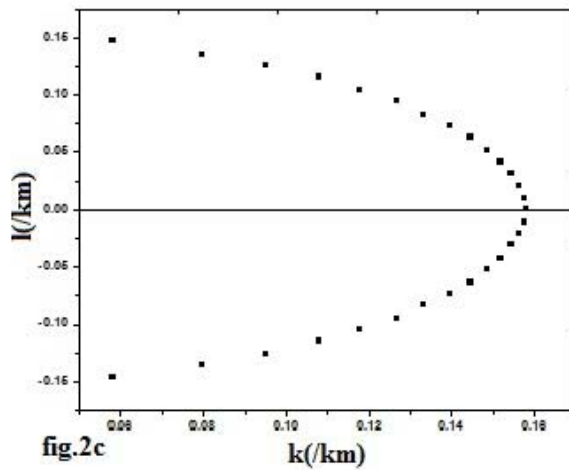


Figure 2a. Vertical profile of $U(z)$ and $T(z)$ across the ABH on 03.02.67 at ABH; **(b)** Vertical profile of w'_{lee} across the ABH on 03.02.67.



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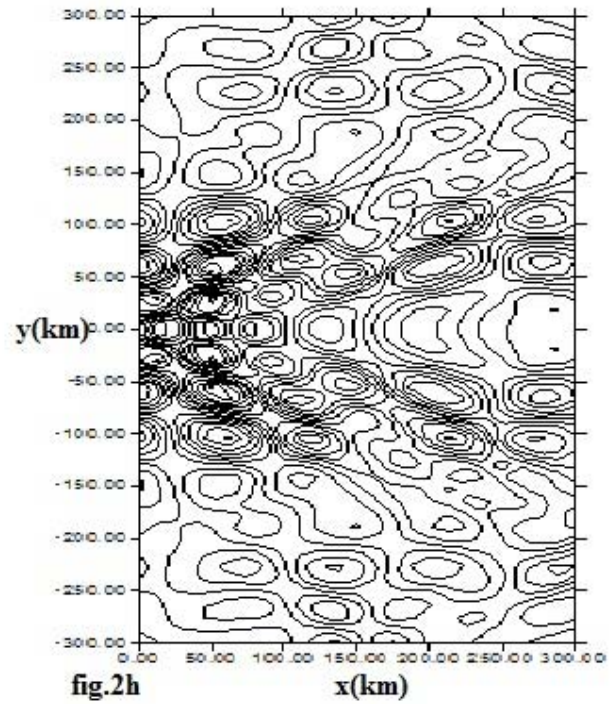
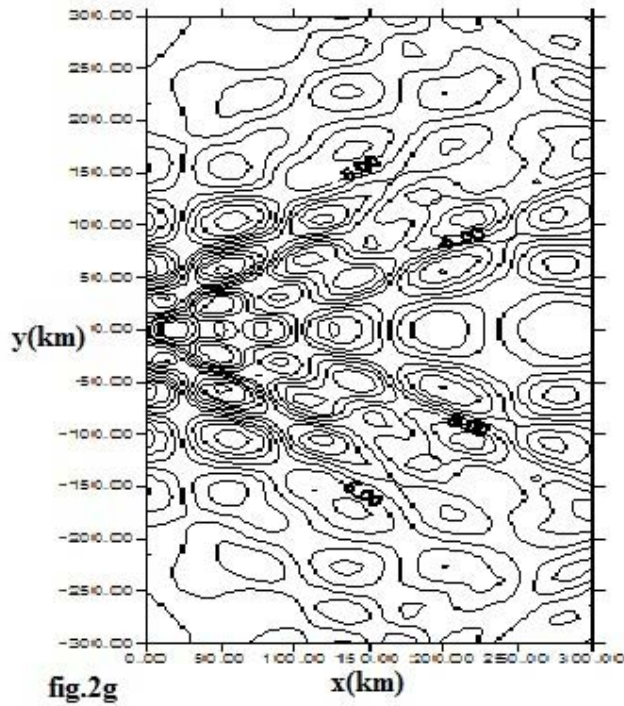
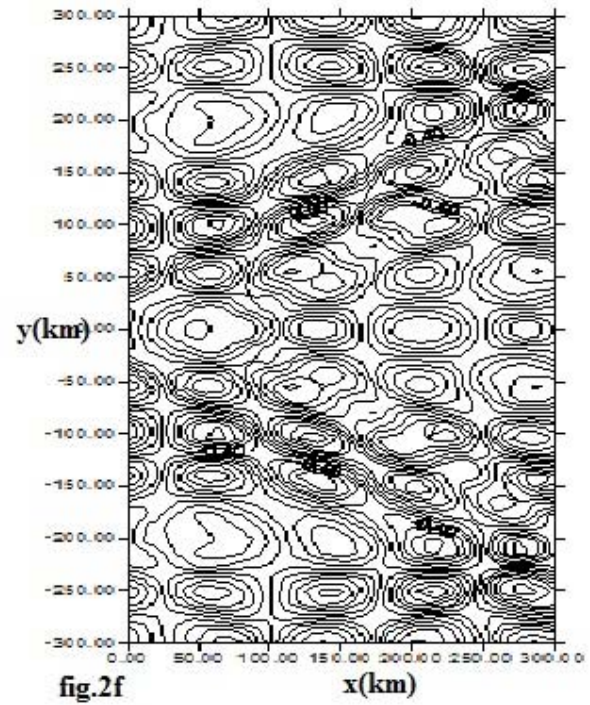
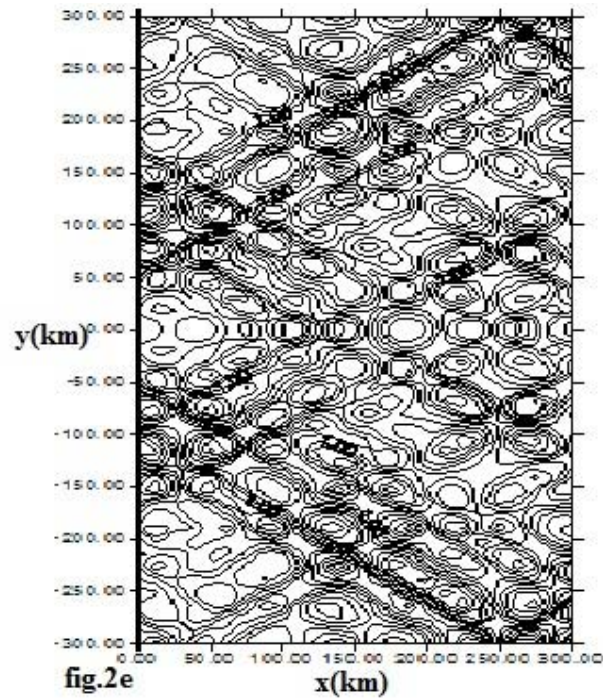


Figure 2c. Graphical relation between wave number vectors k and l on 03.02.67; 2(d-h) Contours of w_{lee} across the ABH on 03.02.67 at surface, 1.5km, 3km, 6km and 9km above mean see level respectively.

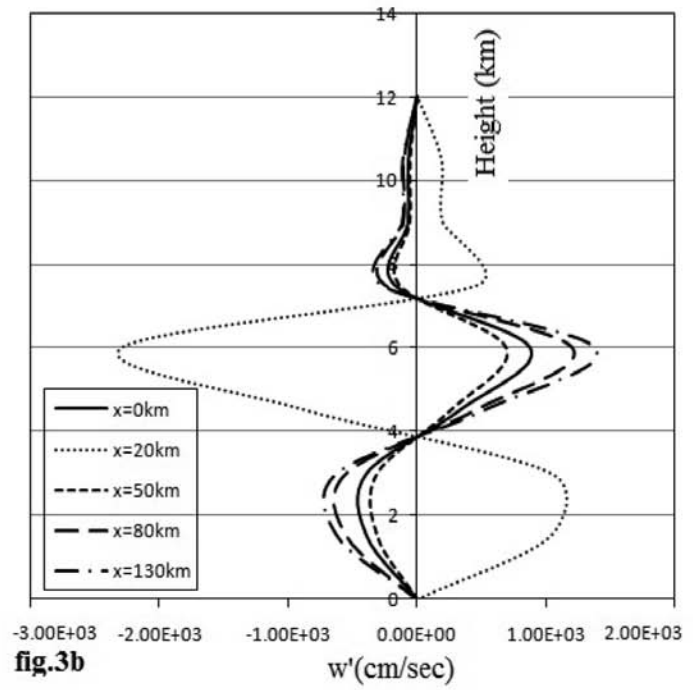
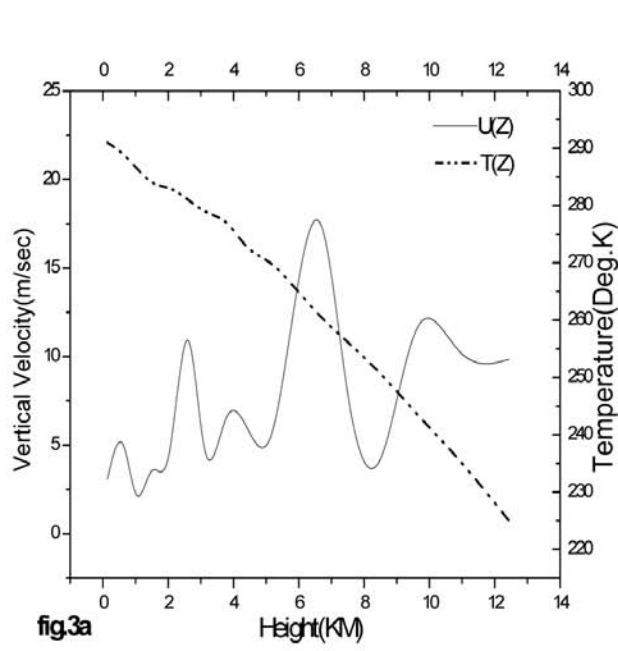
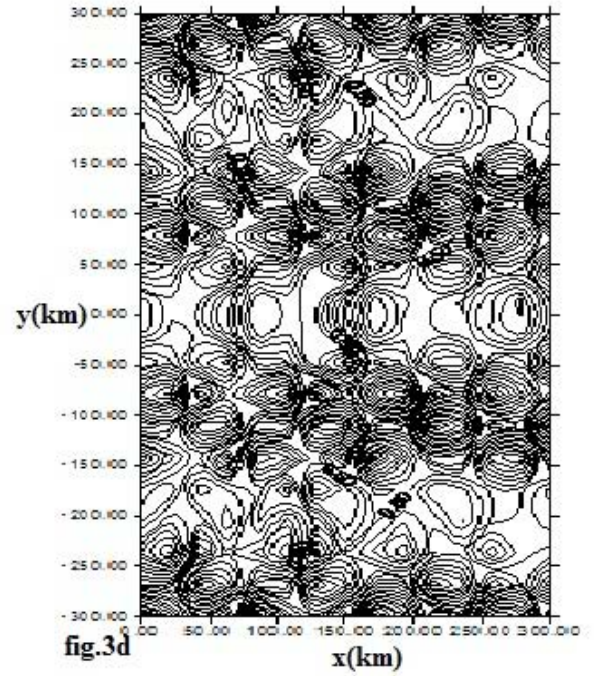
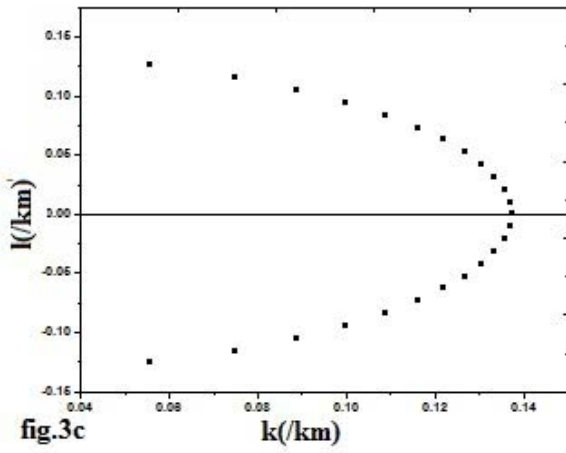


Figure 3a. Vertical profile of $U(z)$ and $T(z)$ across the ABH on 08.01.67; **(3b)** Vertical profile of w'_{lec} across the ABH on 08.01.67.



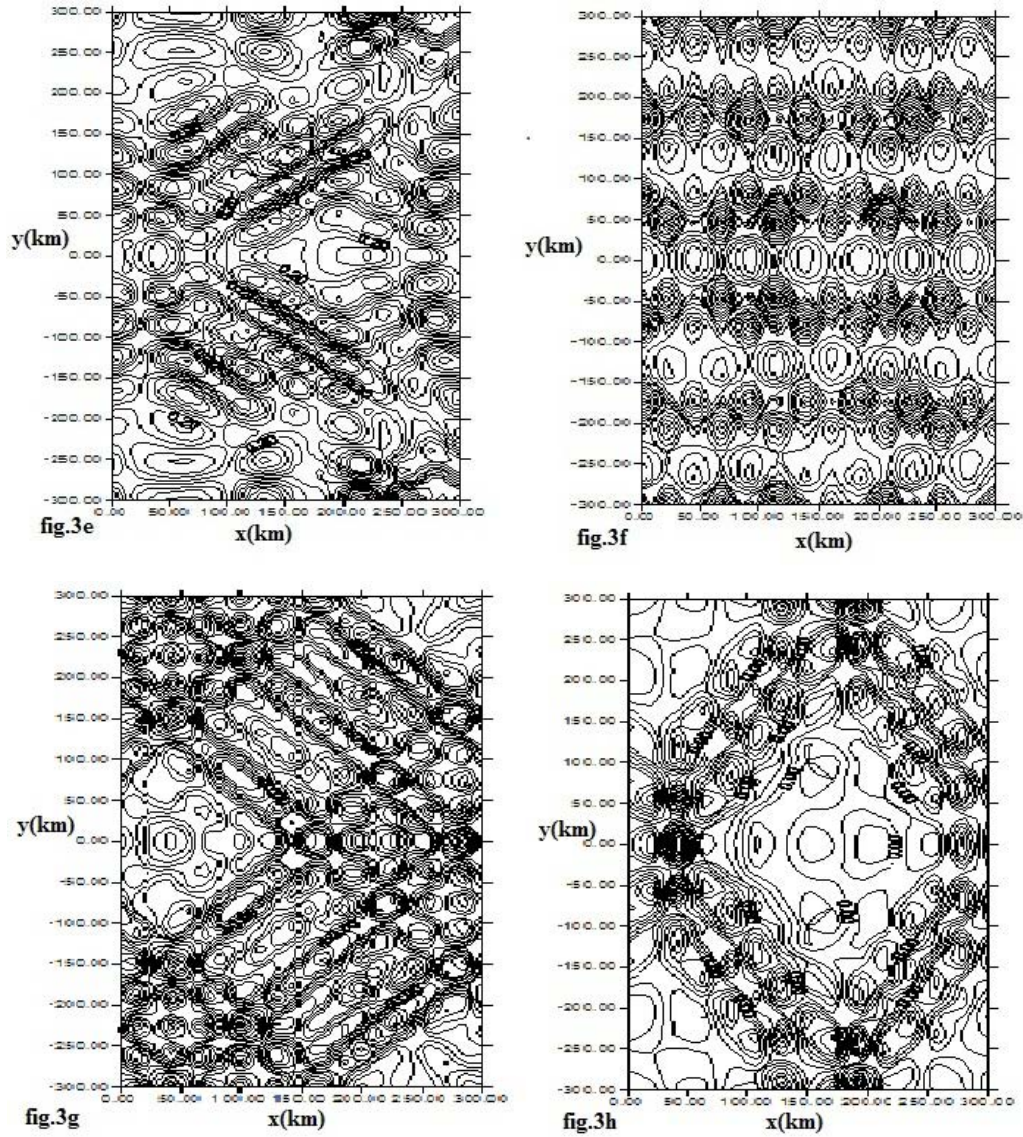


Figure 3c. Graphical relation between wave number vectors k and l on 08.01.67; **3(d-h)** Contours of $\bar{w}_{lee}$ across the ABH on 08.01.67 at surface, 1.5km, 3km, 6km and 9km above mean sea level respectively.

CONCLUSIONS

- (i) In all the above cases, the Scorer's parameter (l_s^2) is more in the lower layer than the adjacent upper layer. This situation is more conducive for the occurrence of the lee waves.
- (ii) The vertical velocities $\bar{w}$ (wave part) in the central plane, at different downstream locations across the Assam-Burma hills (ABH) have shown cellular structure in the vertical, which is in qualitative conformity with finding from earlier results.
- (iii) In all the above cases the transverse wave number (k) increases with divergent wave number $|l|$.
- (iv) In all the above cases the contours of the 3-D vertical velocities $\bar{w}$, at surface, 1.5km, 3km, 6km and 9km

above mean sea level, are approximately 'horseshoe shaped' updraft regions, concave to downwind direction and spread laterally with vertical, implying divergent lee wave, which is in qualitative conformity with finding from earlier results.

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APPENDIX-I

The Fourier Transform of the function $h(x, y) = \frac{h_1}{1 + \frac{x^2}{a^2} + \frac{y^2}{b^2}} + \frac{h_2}{1 + \frac{(x-d_1)^2}{a^2} + \frac{y^2}{b^2}}$ is given by

$$\begin{aligned} F\{h(x, y)\} &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x, y) e^{-i(kx+ly)} dx dy \\ \hat{h}(k, l) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ \frac{h_1}{1 + \frac{x^2}{a^2} + \frac{y^2}{b^2}} + \frac{h_2}{1 + \frac{(x-d_1)^2}{a^2} + \frac{y^2}{b^2}} \right\} e^{-i(kx+ly)} dx dy \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{h_1}{1 + \frac{x^2}{a^2} + \frac{y^2}{b^2}} e^{-i(kx+ly)} dx dy + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{h_2}{1 + \frac{(x-d_1)^2}{a^2} + \frac{y^2}{b^2}} e^{-i(kx+ly)} dx dy \end{aligned}$$

Putting $x = aX, y = bY$ for the first term and $x - d_1 = aX, y = bY$ for the 2nd term

And $k = \frac{k}{a}, l = \frac{l}{b}$ for the both terms and we get the following

$$\begin{aligned} \hat{h}(k, l) &= ab \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{h_1 e^{-i(kX+lY)}}{1 + X^2 + Y^2} dXdY + abe^{-i\frac{kd_1}{a}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{h_2 e^{-i(kX+lY)}}{1 + X^2 + Y^2} dXdY \\ \hat{h}(k, l) &= ab(h_1 + h_2 e^{-ikd_1}) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{e^{-i(kX+lY)}}{1 + X^2 + Y^2} dXdY \end{aligned}$$

Putting $X = r \cos \theta, Y = r \sin \theta$ and $k = \kappa \cos \alpha, l = \kappa \sin \alpha$ we get

$$\hat{h}(k, l) = ab(h_1 + h_2 e^{-ikd_1}) \int_0^{\infty} \int_0^{2\pi} \frac{e^{-ir\kappa \cos(\theta-\alpha)}}{1 + r^2} r dr d\theta$$

Now $\int_0^{2\pi} e^{-ir\kappa \cos(\theta-\alpha)} d\theta = 2\pi J_0(r\kappa)$ [taken from Dutta et al, (2002)]

And $\int_0^{\infty} \frac{r J_0(r\kappa)}{1 + r^2} dr = K_0(\kappa)$ [taken from Dutta et al, (2002)]

Where $J_0(x)$ and $K_0(x)$ are Bessel function of 1st and 2nd kind of order zero respectively.

Hence $\hat{h}(k, l) = 2\pi ab(h_1 + h_2 e^{-ikd_1}) K_0(\kappa)$

Now $\kappa^2 = k^2 a^2 + l^2 b^2$

Therefore

$$\hat{h}(k, l) = 2\pi ab(h_1 + h_2 e^{-ikd_1}) K_0(\sqrt{k^2 a^2 + l^2 b^2})$$

APPENDIX-II

List of symbols

x : Horizontal co-ordinate in the direction normal to the major ridge of the elliptical barrier.

y : Horizontal co-ordinate in the direction parallel to the major ridge of the elliptical barrier.

z : Vertical co-ordinate pointing towards local vertical.

U : Horizontal basic flow along x -axis.

N : Buoyancy frequency.

$\acute{w}$: Component of perturbation wind along z -axis.

$\rho_0(0)$: Density of the basic flow at ground surface.

$\rho_0(z)$: Density of the basic flow at level ' z '.

$\hat{w}$: 2-D Fourier transform of $\acute{w}$.

$\acute{w}_{lee}$: Lee wave part of $\acute{w}(x, y, z)$.

$\bar{T}$: Absolute temperature of the basic flow.

γ : Environmental lapse rate.

γ_d : Dry adiabatic lapse rate.

k : Horizontal wave number vector along x -axis.

l : Horizontal wave number vector along y -axis.

κ : Magnitude of horizontal wave number vector.

$h(x, y)$: Analytical expression for 3-D elliptical barrier.

$\hat{h}(x, y)$: 2-D Fourier transform of $h(x, y)$.

$$\hat{w}_1(k, l, z): \hat{w}(k, l, z) = \sqrt{\frac{\rho_0(0)}{\rho_0(z)}} \hat{w}_1(k, l, z)$$

$\psi(k, l, z)$: Arbitrary definite function, satisfying the same upper boundary condition as $\hat{w}_1(k, l, z)$.

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