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Design of Gravity energy filter to enhance signal-to-noise ratio of gravity measurements

Shib Sankar Ganguli*[,1,2], Aref A. Lashin[3,4], Nassir S. Al Arifi[5] and V.P. Dimri[1]

[1] CSIR-National Geophysical Research Institute (NGRI), Hyderabad - 500 007, India.
[2] Academy of Scientific and Innovative Research (AcSIR), CSIR-NGRI, Hyderabad - 500 007, India.
ganguli.ism@gmail.com, vpdimri@gmail.com
[3] King Saud University, College of Engineering, Petroleum and Gas Engineering Dept., PO Box 800, Riyadh 11421, Saudi Arabia.
[4] Benha University, Faculty of Science - Geology Department, P.O. Box 13518, Benha - EGYPT.
aref70@hotmail.com
[5] Geology and Geophysics Department, King Saud University, Riyadh, Saudi Arabia. nalarifi@ksu.edu.sa
*Corresponding Author: ganguli.ism@gmail.com

ABSTRACT

One of the significant objectives of geophysical data recording and processing is the enhancement of the signal-to-noise ratio, and in this perspective, the design of an optimum digital filter is pivotal. The well-known Wiener-Hopf filter has been successfully applied to attain this objective. In comparison, the output energy filter is a tool, by which one can attempt to enhance the signal-to-noise ratio by retrieving the signal at the output, producing a longer burst of energy in the time interval where the signal occurs. In the present work, we report the development of a digital filter, namely, gravity energy filter for the improvement of gravity signal-to-noise ratio, immersed in coloured noise. For the design of such filter, it is not obligatory to have explicit knowledge of the gravity signal shape; nevertheless, its performance level is not compromised. We demonstrated its applicability on synthetic data, generated by considering two spherical bodies at the subsurface, where in the gravity signal buried in coloured noise beyond visual recognition, is easily detected after filtering.

INTRODUCTION

One of the most significant steps in geophysical data recording and processing is filtering, and every form of the acquired data is filtered prior to its analysis and interpretation. Filtering helps to obtain geophysical information in which the noise is suppressed in relation to the signal which is considered as one of the fundamental problems in exploration geophysics (Andersen, 1974; Dimri, 1986; Dimri and Srivastava, 1990). With the improved and sophisticated digital methods of geophysical data processing aimed to obtain the maximum amount of information out of a geophysical time series, the improvement of signal-to-noise ratiois always demanding (Treitel and Robinson, 1969; Dash and Obaidullah, 1970; Dimri, 1992). Although the efforts to improve the signal-to-noiseratio have never been lacking, there have been few techniques to analyse the noise statistics that are associated with the signal (Dash and Obaidullah, 1970). One of the important studies to determine the noise statistics is carried out by Ostrander (1966), in which he used correlation theory to measure the power spectra of the noise from a seismic trace.

The design of a filter to improve the signal-to-noise ratio was first attempted by Smith (1954), but in a slightly different context. Later, Simpson (1955) derived a set of multi-channel filters to enhance the energy in the entire output due to the signal itself. Similarly, Claerbout (1963) provided a simplified derivation of this operator, both in time and frequency domain. Later, Treitel and Robinson (1969) further applied the output energy filter to raw seismic data to improve the seismic resolution.

It has always been known that the gravity interpretation is highly ambiguous, and hence rigorous research is desirable to reduce the uncertainty (Roy, 1962; Negi et al., 1973; Ganguli and Dimri, 2013). There are great many studies in the literature that describe methods to overcome such ambiguity. Such approaches can be broadly divided into three categories: gravity smoothing; approximating the long-wavelength component of gravity field with low-order polynomial; and linear digital filtering (Thurston, 1991).

In the present work, we developed an output energy filter, namely gravity energy filter, to obtain the threshold signal-to-noise ratio at the filter output, where we treat the input as a combination of signal and noise. The threshold signal-to-noise ratio can be obtained by the enhancement of the energy at the output mainly due to the signal contribution. This technique will help in discriminating the gravity anomalies of interest from the background noise, i.e. unwanted signal. We focus on the "output-energy" digital filter which was in prominence in seismic data processing in late 1960s (Robinson, 1967; Treitel, 1970). These filters seem to offer exciting, yet largely unexplored capabilities in many other field of earth sciences.

THEORY

An output energy filter is defined by the criterion that the energy at the output due to signal contribution is as large as possible and the energy output due to noise alone is small (Smith, 1954; Simpson, 1955; Robinson, 1967).

To derive the solution due to gravity energy filter, we assume a gravity signal S(x,y), fed into a filter with weighting or filter coefficient C(x,y), hence the output can be represented as

$$U(x,y) = \sum_t \sum_t C(x,y)\, S(x-x^1, y-y^1) \quad \dots (1)$$

We thus seek the weighting or filter coefficients ,which enhance the signal-to-noise ratio at the filter output. An input consists of a signal S(x,y)and an additive mixture of noise N(x,y) components can be represented by

$$J(x,y) = S(x,y) + N(x,y) \quad \dots (2)$$

Substituting eq. (2) in eq. (1), we obtain

$$U(x,y) = \sum_t \sum_t C(x,y)\, [S(x-x^1, y-y^1) + N(x-x^1, y-y^1)] \quad \dots (3)$$

We wish to maximize the output energy, i.e. the extremum value of $U^2(x,y)$subjected to the unit energy constraint (i.e., the sum of the filter coefficients is equal to unity) by means of the calculus of variations (Hildebrand, 1952). Thus, following Treitel and Robinson (1969) and using the method of Lagrange multipliers, we maximize

$$I = [U^2(x,y) - \lambda\, \{C^2(x,y) - 1\}] \quad \dots (4)$$

Where λ is the Lagrange multiplier. Substituting the value of U(x,y) from equation (3), we rewrite equation (4) as

$$I = \left[\sum_t \sum_t C(x,y)\, [S(x-x^1, y-y^1) + N(x-x^1, y-y^1)]\right]^2 - \lambda\, [C^2(x,y) - 1] \quad \dots (5)$$

Assuming signal and noise to be uncorrelated, and following Robinson and Treitel (1967), we can write

$$\emptyset_\tau = R_\tau + Q_\tau \quad \dots (6)$$

Here ϕ_τ, R_τ and Q_τ are the autocorrelation function of the input function, signal, and noise respectively.

At this point it will be more convenient to represent equation (6) in matrix form. Thus, differentiating 'I' with respect to each filter coefficients and setting the result equal to zero, we obtain following matrix equation,

$$\left\{\begin{bmatrix} R_0 & \cdots & R_m \\ \vdots & & \vdots \\ R_m & \cdots & R_0 \end{bmatrix} - \lambda \begin{bmatrix} Q_0 & \cdots & Q_m \\ \vdots & & \vdots \\ Q_m & \cdots & Q_0 \end{bmatrix}\right\} \begin{bmatrix} C_0 \\ C_1 \\ \cdot \\ \cdot \\ C_m \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix} \quad \dots (7)$$

This equation is defined as generalized eigenvalue or characteristic-value problem; values of λ for which nontrivial solutions exist are called eigenvalues (or characteristic values) of the matrix, and corresponding vector solutions are known as eigenvectors of the matrix (see e.g. Frazer, Duncan and Collar, 1938; Hildebrand, 1952). The latter provides the filter coefficients of the gravity energy filter. Since we desire the maximum value of signal-to-noise ratio, we select the eigenvectors corresponding to the maximum eigenvalue (λ_{max}) that constitute the desired memory function of the optimum output energy filter or gravity energy filter.

Following Treitel and Robinson (1969), equation (7) can also be written using summation

$$\sum_{s=0}^{m} [R_{t-s} - \lambda Q_{t-s}] C_s = 0, \quad t = 0,1,\dots,m. \quad \dots (8)$$

In this equation, R_t and Q_t are known quantities, while the parameters 'λ' and 'C_s' are undetermined. Since by definition our interest lies on λ_{max}, therefore, the eigenvector corresponding to the largest eigenvalue is computed to obtain the memory function of the output energy filter or gravity energy filter.

APPLICATION TO SYNTHETIC GRAVITY DATA

To check the efficacy of the derived gravity energy filter, we applied it on the synthetic gravity data computed by considering two spherical bodies buried at a depth of 3000m, with a radius of 2000m, and 800m with radius of 400m, respectively from the surface. The gravity response is estimated for these bodies at a distance of 35000m, and 70000m respectively from the initial point of the profile. The density contrast with the surroundings was chosen as 0.2 g/cc. The computed gravity response due to the buried spheres is shown in Figure 1.

Now, we consider an additive mixture of the gravity signal and coloured noise arriving at the detector, which is displayed in Figure (2). It shows that the signal from the shallower body is obscured.

Our goal is now to retrieve the energy at the output due to signal alone is large and the energy in the entire output due to the noise alone is small. Theoretical anomalies due to signal and noise sources were used to compute the signal autocorrelation and noise autocorrelation respectively. Equation (8) is, then applied to get the maximum eigenvalue (λ_{max}), which is found to be 1.814×10^5. This has been utilized to obtain the memory function of the gravity energy filter, depicted in Figure 3, since the eigenvectors associated with the maximum eigenvalue constitutes the filter coefficients. Finally, the output is obtained by convolving the filter coefficients with the input function (i.e., the sum of gravity signal and coloured noise) to the gravity energy filter.

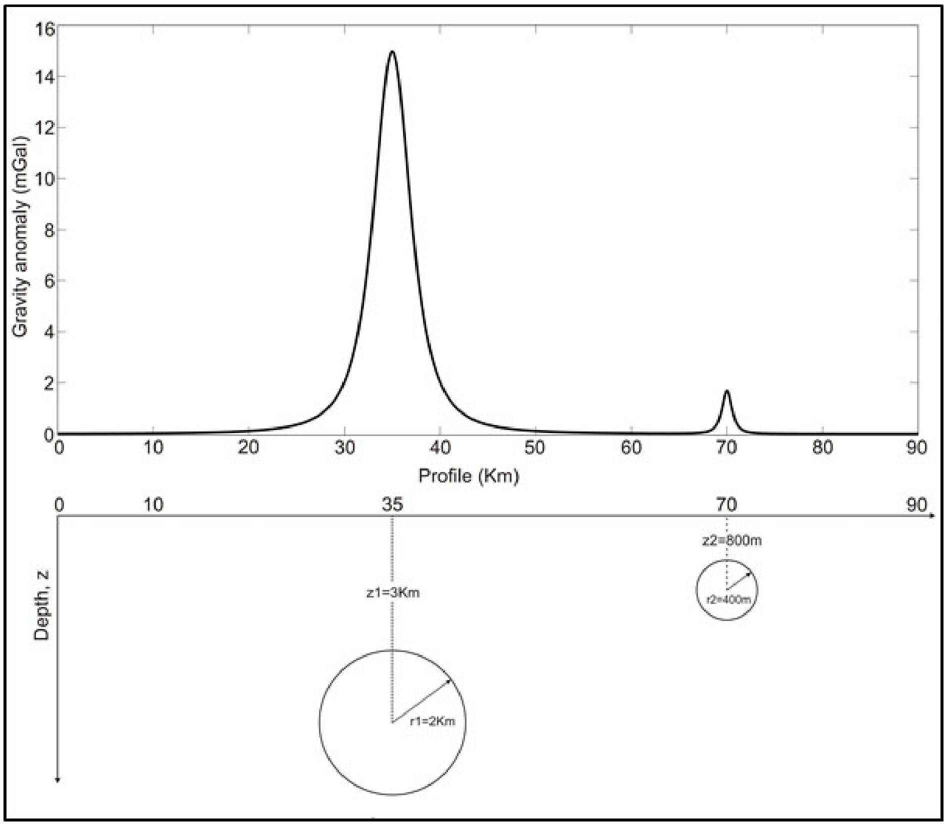

Figure 1. The gravity anomaly response due to a model consisting of two buried spheres at subsurface. The parameters used in the computation of the synthetic gravity data are mentioned in the text.

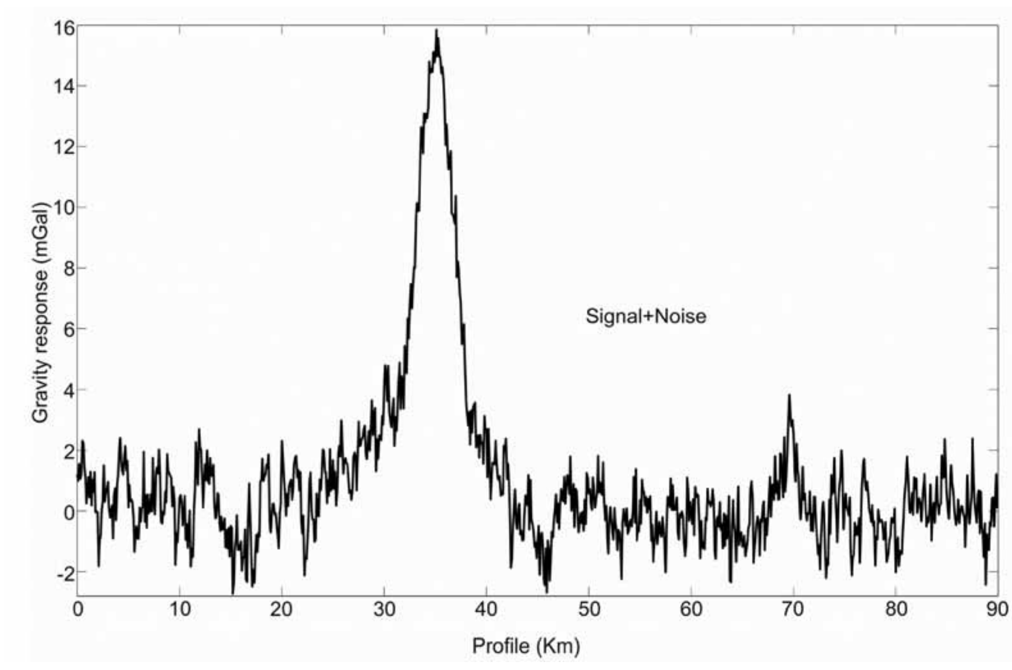

Figure 2. An additive mixture of gravity signal and coloured noise due to two buried spheres at subsurface. The signal due to the shallower spherical body immersed in the coloured noise is obscured and beyond the visual recognition.

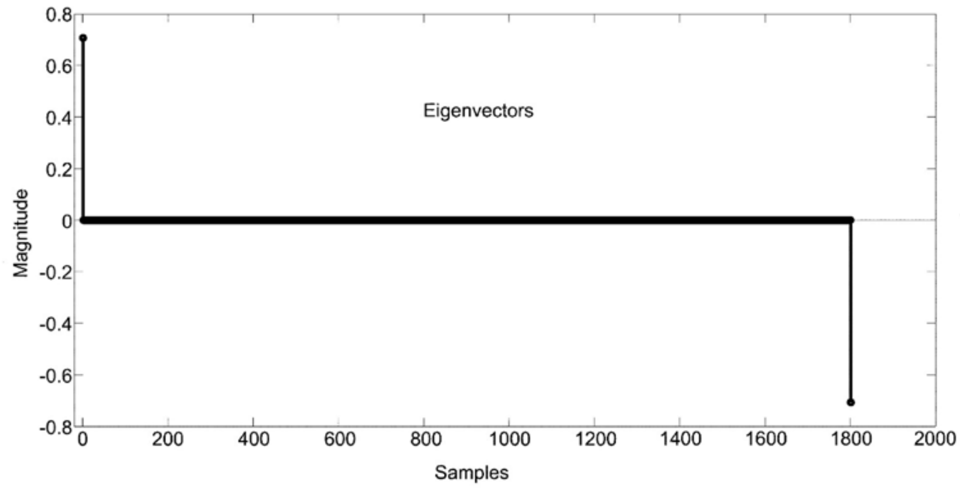

Figure 3. The illustration of eigenvectors corresponding to the maximum eigenvalue, which constitutes the memory function of the gravity energy filter. The magnitudes of all eigenvectors are plotted along y-axis against each samples.

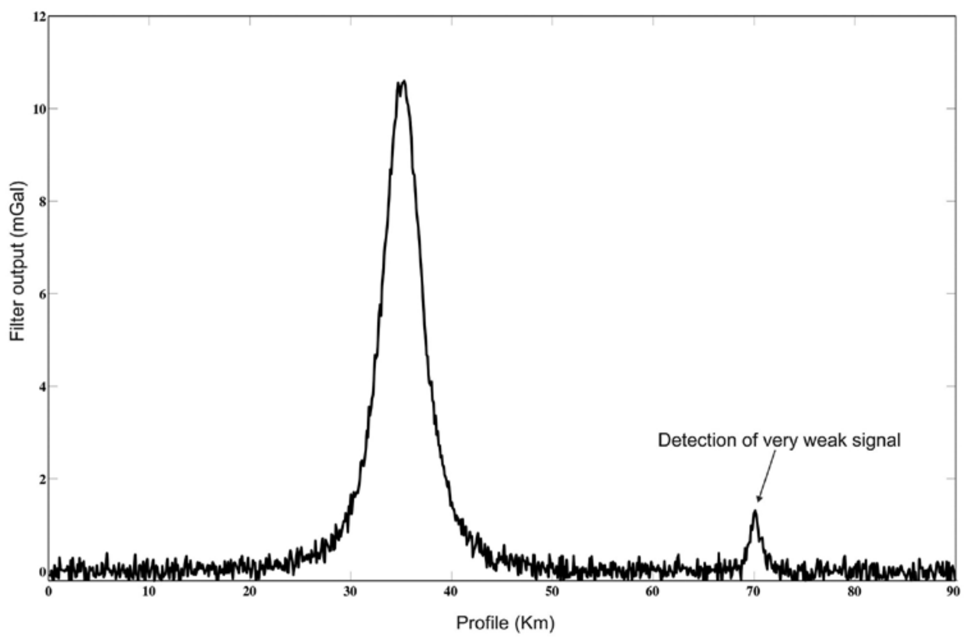

Figure 4. Results of the gravity energy filter acting on a weak signal, buried in coloured noise. It can be seen that the detection of very weak signal is also possible when it is immersed in the coloured noise.

Figure (4) illustrates the results from the gravity energy filter, which shows that the gravity response is fairly better even in the presence of coloured noise. We can clearly see that the filter is able to retrieve the signal at the output, which was earlier obscured when immersed in the noise beyond visual recognition. So we feel that the derived gravity energy filter could be helpful in enhancing the signal-to-noise ratio in the presence of coloured heavy noise. We also observe that the amplitude of the retrieved signal at the filter output is reduced compared to the original signal, prior to being immersed in coloured noise. Nevertheless, the shape of the signal resembles reasonably good to the original one..

CONCLUSIONS

We have developed gravity energy filter based on the criterion that the energy at the entire output due to the gravity signal alone is large, and the energy due to the noise is very less. We assume that the signal and noise are uncorrelated, and the energy of the filter is close to unity, satisfying the well-known unit energy constraint. The present study illustrates the theory with synthetic example of subsurface spherical body to facilitate the understanding the basic concepts. It has thus potential application to reduce the noise at the detector. It also reveals that the gravity energy filter is robust and a noteworthy tool to retrieve signal at the output.

We demonstrate that the application of gravity energy filter to the synthetic gravity data, will in general, increase the signal-to-noise ratio. We have chosen to develop the filter in the time domain since the definition of signal-to-noise ratio and the subsequent optimization was most easily accomplished in that domain. To check the robustness of the developed filter, it is desirable to apply it on real gravity data obtained from the field. This can be subsequently attempted. It is noteworthy that the gravity energy filter does not require the knowledge of shape of the signal wavelet, but only its autocorrelation function.

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Mr. Shib S. Ganguli is presently working as CSIR-Senior Research Fellow (SRF), and pursuing his Ph.D. from Academy of Scientific and Innovative Research (AcSIR)in CSIR-National Geophysical Research Institute, Hyderabad after receiving his M.Sc. (Tech) degree from Indian School of Mines, Dhanbad. His research interest includes development and application of modern methods to geophysical data, numerical modeling and simulation, geophysical inversion, and rock physics.

Dr.Aref A. Lashin is presently working as Associate Professor at King Saud University, Saudi Arabia, and Benha University, Egypt. He completed Ph.D. and M.Sc. in Exploration and Petrophysics from Benha University, Egypt. His research expertise includes reservoir characterization, petroleum system analysis, geothermal, AVO and seismic. He has more than 26 papers in peer reviewed national and international journals.

Prof. Nassir S. Al Arifiis presently working as Professor at King Saud University, Saudi Arabia. He completed Ph.D. from University of Manchester and Bachelor's degree from College of Science, King Saud University, Saudi Arabia. His research interests includes geothermal energy, geodesy, earthquake seismology and seismic hazard assessment studies, electrical resistivity and EM. He has published more than 56 papers in peer reviewed national and international journals.

Prof. Vijay Prasad Dimri served as a Distinguished Scientist of CSIR from October 26, 2010 to October, 2013. He served as the Director of NGRI from October 2001 to February 2010. He did Ph.D. and M.Sc. (Tech) from Indian School of Mines, Dhanbad. Dr.Dimri has more than 125 research publications in referred journals.Recently, he along with two co-authors published a book on "Fractal Models in Exploration Geophysics", Elsevier, 2012 & "Wavelets and Fractals in Earth System Sciences", Taylor & Francis Group, 2013.He is an author of the book entitled "Deconvolution and Inverse Theory" published by Elsevier, 1992. Dr.Dimri has edited two books on "Applications of fractal in Geosciences, Balkema, USA, 2000" and "Fractal Behaviour in Earth Science System, Springer, 2005". He has co-edited three other books published in 2008, 2010 and 2012.