J. Ind. Geophys. Union (April 2006)
Vol.10, No.2, pp.69-78

Tsunami warning systems for the hyperbolic (Pacific), parabolic (Atlantic) and elliptic (Indian) oceans

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ABSTRACT
It is shown that the Pacific, Atlantic and Indian Oceans have very different tsunami characteristics. Hence the numerical modelling of tsunamis in these three oceans has to be quite different, with particular relevance for the tsunami warning system. The Pacific and Indian Oceans generate ocean-wide tsunamis, but with one major difference. The Indian Ocean, being much smaller in geographical extent than the Pacific Ocean, has shorter tsunami travel times and in the Indian Ocean, reflected waves from the coast lines can interact with subsequent tsunami waves. Hence to determine the maximum tsunami amplitudes in the Indian Ocean, boundary reflections must be taken into account. In the much larger Pacific Ocean, the influence of boundary conditions is somewhat minimal. In the Atlantic Ocean, unlike in the Pacific and Indian Oceans, there are no converging tectonic plates, where tsunamigenic earthquakes could be generated. The mid-Atlantic Ridge is a diverging plate boundary. In the Atlantic Ocean, there are tsunamis only in the marginal seas at the edges. Since tsunamis travel slower in shallow water, Atlantic tsunamis are slow moving, somewhat like a diffusion or a Parabolic process. On the other hand, Pacific tsunamis, which are more influenced by initial conditions, are a Hyperbolic process. Indian Ocean tsunamis, which are strongly influenced by the boundary conditions, have to be modelled as an Elliptic process.

INTRODUCTION

The four global oceans (Fig. 1) are not simply connected, at least from the point of view of tsunami propagation. The Pacific, Atlantic and Indian Oceans are connected at the south, through what is generally referred to as the southern ocean. The Pacific and Atlantic Oceans are connected in the north to the Arctic Ocean. The Indian Ocean does not extend into the higher latitudes of the northern hemisphere, but terminates into the two marginal seas, namely the Bay of Bengal and the Arabian Sea.

Unlike the atmosphere, which is continuous, which makes it possible, at least in principle, to have a global weather forecasting system, since the oceans are not continuous, it is not scientifically feasible to have a global tsunami forecasting system. What is feasible is four separate tsunami early warning systems for the four oceans. There are also socio-economic reasons why each ocean needs its own warning system. Since different nations border different oceans, usually only the countries that have a coastline on the rim of a particular ocean will be interested in a tsunami warning system for that ocean.

At present, out of the four oceans, only the Pacific Ocean has a tsunami warning system in place since 1949, after the disastrous Aleutian earthquake tsunami of 1 April 1946. Until now, neither the Atlantic Ocean, nor the Indian Ocean have tsunami warning systems in place, because tsunami occurrences are rare in these oceans. At present, there is no priority for a tsunami warning system in the Arctic Ocean, mainly because of very low population density around the Arctic Ocean rim.

It will be shown below that the Pacific, Atlantic and Indian Oceans exhibit quite different tsunami characteristics, and for this reason, the numerical models for the tsunami warning systems for these oceans have to be different. It will be shown below that, specifically, in terms of the nature of the physical process, the Pacific is a hyperbolic ocean, the Atlantic, a parabolic ocean, and the Indian, an elliptic ocean, in the sense of the method of characteristics.

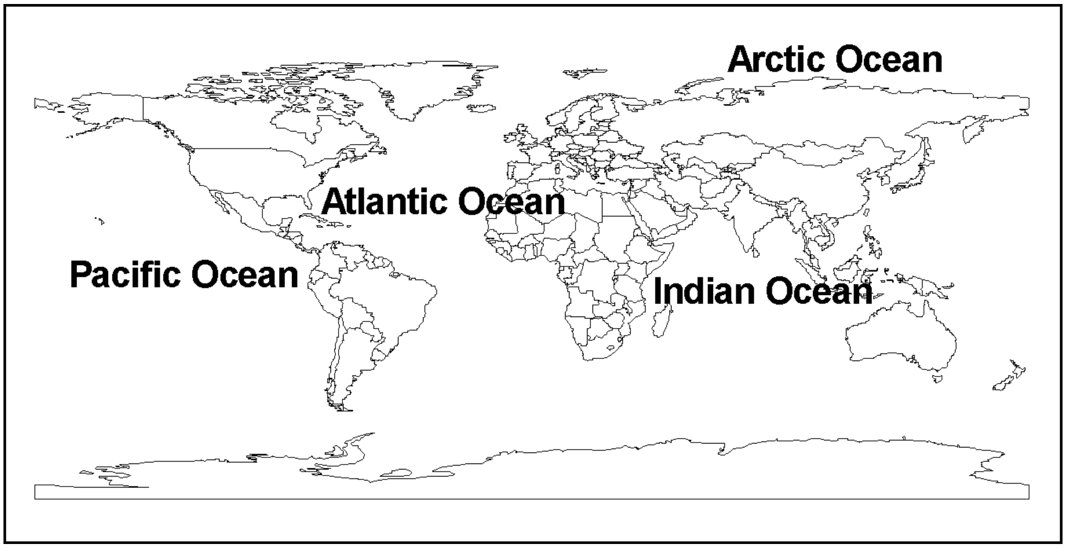

Figure 1. The four global oceans

TSUNAMIS IN THE PACIFIC OCEAN

As can be seen from Fig. 2, the Pacific Ocean contains some of the important global tectonic plates. Hence out of the four oceans, most tsunamigenic earthquakes occur in the Pacific Ocean. Some of the major tsunamis in the 20th century occurred in the Pacific Ocean, namely the Aleutian earthquake tsunami of 1 April 1946, the Kamchatka earthquake tsunami of 4 November 1952, the Chilean earthquake tsunami of 22 May 1960 and the Alaska earthquake tsunami of 28 March 1964. In addition to these major ocean-wide tsunamis, there were several significant local tsunamis, such as the one in the sea of Japan in 1983 and the one in Papua New Guinea in 1998.

Thus the Pacific Ocean gives rise to both trans-oceanic as well as local tsunamis. Because of the large geographical extent of the Pacific Ocean, tsunami travel times could be as high as 23 hours (Fig.3) and reflected waves from the boundaries do not play a significant role in the total water levels associated with tsunami waves.

It has been observed that tsunamis can travel vast distances in the Pacific Ocean without suffering too much dissipation through frequency dispersion. Observations of tide gauge records indicate that, usually up to 5 to 7 waves could be generated in a tsunami event and in some cases, even up to 10 waves. The first wave is usually not the highest generally in the Pacific Ocean, the wave with the highest amplitude lies between the third and the fifth.

TSUNAMIS IN THE ATLANTIC OCEAN

Unlike in the Pacific Ocean, there are no converging tectonic plates in the Atlantic Ocean, which can generate tsunamigenic earthquakes. The mid-Atlantic Ridge is a diverging plate boundary, which can create new ocean floor, but not tsunamigenic earthquakes. Compared to the Pacific Ocean, tsunami events are quite rare in the Atlantic Ocean.

In the Atlantic Ocean, there has been no ocean-wide tsunami in historical time, similar to Pacific tsunamis, even the 1755 Lisbon earthquake tsunami is not truly ocean-wide, outside of the genesis area, it might have propagated into the Caribbean Sea, but did not propagate in any significant manner onto the east coasts of Canada and U.S.A.

The marginal seas and the edges of the Atlantic Ocean did generate a few local tsunamis in historical time. A most noted one is the tsunami from the Grand Banks earthquake of 18 November 1929, which killed 28 people on the south coast of Newfoundland. Murty (1977) numerically modelled this tsunami and showed that most of the tsunami energy was directed towards the south coast of Newfoundland (Fig. 4) and very little tsunami energy reached Nova Scotia, because of the presence of extensive sand banks on the way. This tsunami reached the south coast of Newfoundland in about 3 hours and was amplified to 8 m in some coastal bays due to quarter wave resonance.

This tsunamigenic earthquake also generated ocean bottom turbidity currents that broke in a regular

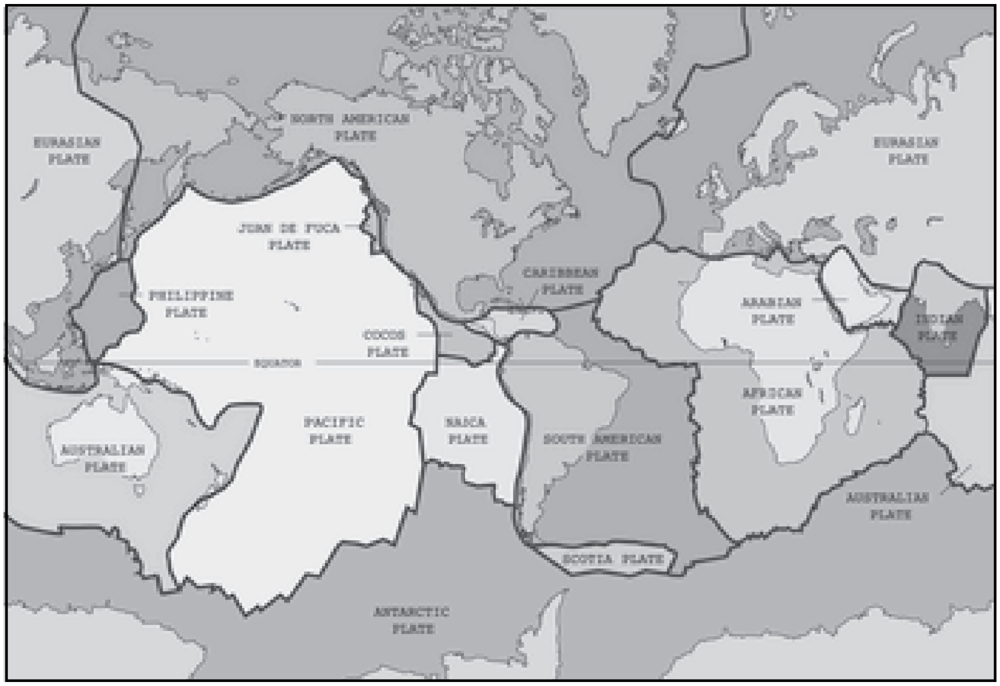

Figure 2. The global tectonic plates (from the website of Wikipedia)

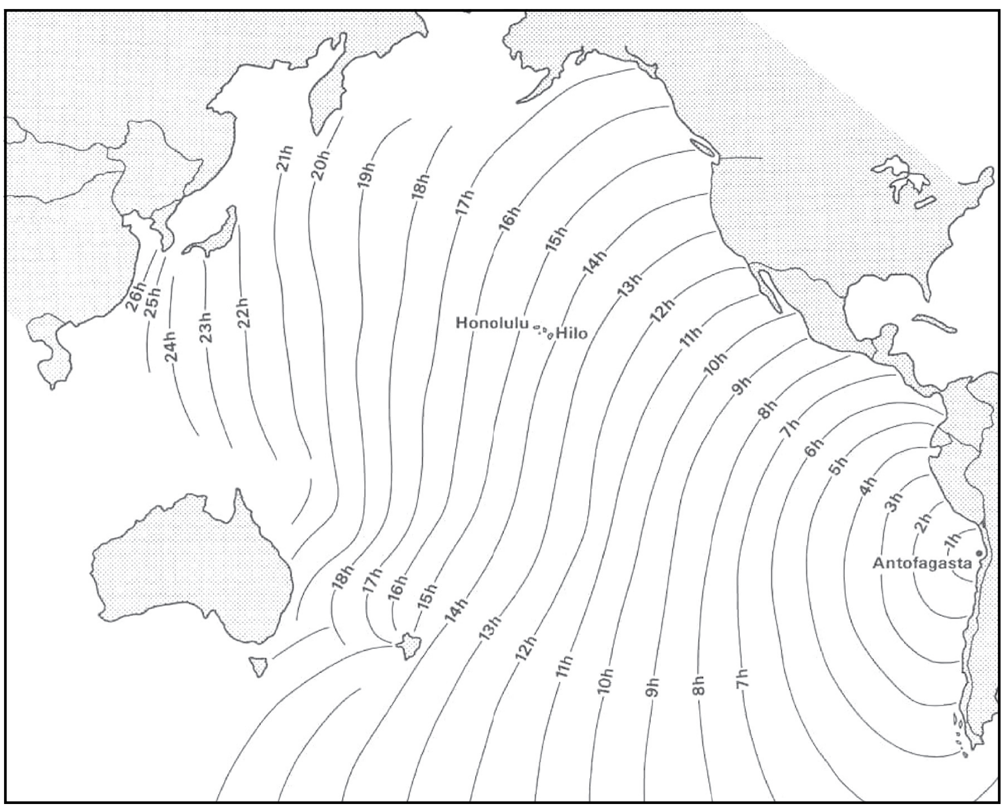

Figure 3. A tsunami travel time chart for the Pacific Ocean. The contours are in hours

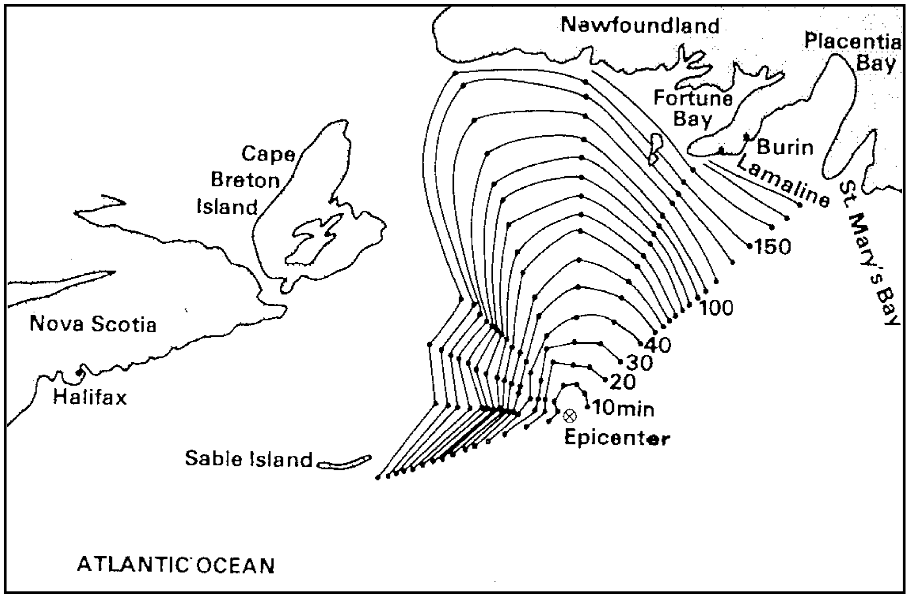

Figure 4. Tsunami travel times for the Grand Banks earthquake of 18 November 1929 (Murty 1977). The contours are in minutes.

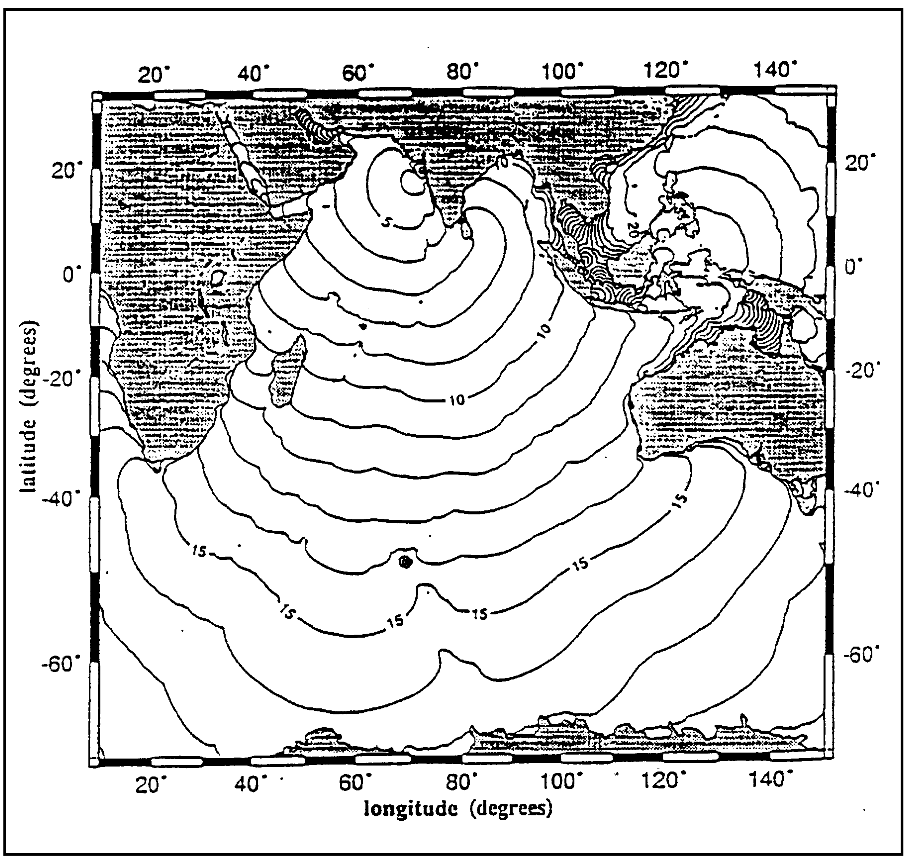

Figure 5. Tsunami travel time chart for Mumbai, Indian. The contours are in hours.

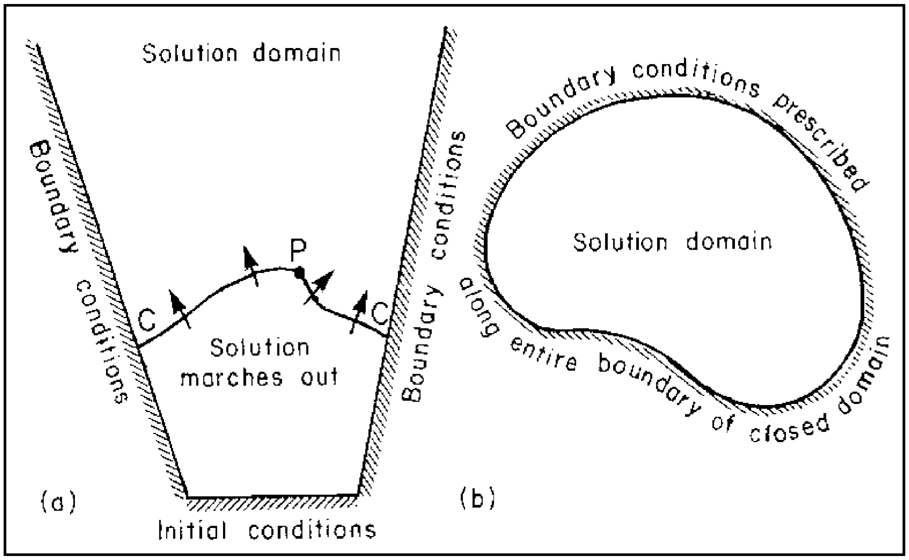

Figure 6. (a) Propagation problem is solved in an open domain; (b) Equilibrium problem is solved in a closed domain (Crandall 1956

sequence, the trans-Atlantic Ocean cables that were laid out previously to provide telephone service to Europe from North America.

TSUNAMIS IN THE INDIAN OCEAN

The Indian Ocean is somewhat similar to the Pacific Ocean in certain tsunami characteristics and also similar to the Atlantic Ocean in some other tsunami features. Similar to the Pacific, there are converging tectonic plate boundaries in the Indian Ocean, which give rise to tsunamigenic earthquakes. As far as the rarity of tsunami events in considered, the Indian Ocean is more like the Atlantic Ocean. However, this is where the similarity stops. There are no truly ocean-wide tsunamis in the Atlantic Ocean. Almost all tsunamis in the Atlantic are generally local and only have local impacts.

Unlike this, the Indian Ocean gives rise to both trans-oceanic as well as local tsunamis, and in the respect is similar to the Pacific Ocean. However, this is where the similarity with the Pacific Ocean also ends. If we consider only the populated coastlines of the Indian Ocean, the tsunami travel times are considerably less than those for the Pacific, and hence the available warning time is much less. Figure 5 shows a tsunami travel time chart for Mumbai, India.

There are also other important differences with the Pacific. The Pacific Ocean is so vast that tsunami waves reflected from coastlines do not contribute significantly to the water level distribution during a tsunami event in the Pacific Basin. However, this is not true for the Indian Ocean, since reflected waves play an important role in the tsunami water levels. For this reason, the numerical models for a tsunami warning system in the Indian Ocean have to be somewhat different from the models for the Pacific Ocean.

METHOD OF CHARACTERISTICS

One can distinguish between propagation (or marching) problems (Fig. 6a) and equilibrium (or jury) problems (Fig. 6b) (Crandall 1956).

Initial conditions must be specified throughout the domain, and boundary conditions must be specified at the open boundary at all times. (Generally speaking, boundary conditions must also be specified at the closed boundary at all time.) It will be shown below that the governing equations for propagation problems are either hyperbolic or parabolic whereas for the equilibrium problems, they are elliptic. Crandall (1956, p. 352) formulated the general propagation problem in the following manner:

The problem is to march out the solution of a governing system of partial differential equations of hyperbolic or parabolic type from prescribed conditions on an open boundary. The following is a pair of simultaneous first-order differential equations:

$$A_1 \frac{\partial u}{\partial x} + B_1 \frac{\partial u}{\partial y} + C_1 \frac{\partial v}{\partial x} + D_1 \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$A_2\frac{\partial u}{\partial x}+B_2\frac{\partial u}{\partial y}+C_2\frac{\partial v}{\partial x}+D_2\frac{\partial v}{\partial y}=0 \quad (2)$$

Here, u and v are the dependent variables, x and y are the independent variables, and the coefficients A_1, A_2, B_1, B_2, C_1, C_2, D_1, and D_2 are functions of u and v but not of x and y. Usually, the system of equations (1) and (2) is nonlinear, but since the coefficients are not functions of x and y, one can make it linear by treating u and v as the independent variables and x and y as the dependent variables. Hence, the system of equations (1) and (2) is referred to as a "reducible system".

Assume that the system of equations (1) and (2) is being solved in the domain shown in Figure 6a and that the solution up to curve CPC is known. At P, the continuously differentiable values of u and v along CPC are known, as well as all of their derivatives in the directions pointing towards the interior of the curve. The following question is asked: is the behaviour of the solution above P completely determined by the solution below CPC, or is additional information at the boundaries of C required? To answer this, consider the following argument.

Let S be a direction in which the distance is measured; then one can write

$$\frac{\partial u}{\partial S}=\frac{\partial u}{\partial x}\frac{\partial x}{\partial S}+\frac{\partial u}{\partial y}\frac{\partial y}{\partial S} \quad (3)$$

$$\frac{\partial v}{\partial S}=\frac{\partial v}{\partial x}\frac{\partial x}{\partial S}+\frac{\partial v}{\partial y}\frac{\partial y}{\partial S} \quad (4)$$

The above question can be reformulated: for a solution of equations (1) and (2), do the values of u and v along CPC uniquely determine the derivatives? Here, S measures the distance along CPC. Define

$$du \equiv \frac{\partial u}{\partial S}dS$$

$$dv \equiv \frac{\partial v}{\partial S}dS$$

$$dx \equiv \frac{\partial x}{\partial S}dS \quad (5)$$

$$dy \equiv \frac{\partial y}{\partial S}dS$$

The set of equations (1) – (4) written at P can be expressed in the following compact form:

$$\begin{bmatrix} A_1 & B_1 & C_1 & D_1 \\ A_2 & B_2 & C_2 & D_2 \\ dx & dy & 0 & 0 \\ 0 & 0 & dx & dy \end{bmatrix} \begin{bmatrix} \partial u/\partial x \\ \partial u/\partial y \\ \partial v/\partial x \\ \partial v/\partial y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ du \\ dv \end{bmatrix} \quad (6)$$

Since u and v are known at P, the coefficients A_1, A_2, D_1, and D_2 are also known. If the curve CPC is specified (i.e. its direction), then dx and dy are known. When u and v are known along CPC, then du and dv are also known. The system (equation (6)) constitutes a set of four simultaneous linear algebraic equations for the four unknowns $\partial u/\partial x$, $\partial u\partial/y$, $\partial v/\partial x$, and $\partial v/\partial y$. Two possibilities exist. If the determinant of matrix (6) = 0, there is an indefinite set of solutions; there may be discontinuities in the solutions on either side of CPC. If the determinant ≠ 0, there is a unique solution.

To find out under what conditions the determinant of this matrix can be zero, it is expanded to give

$$(A_1C_2-A_2C_1)(dy)^2-(A_1D_2-A_2D_1+B_1C_2-B_2C_1)dxdy+(B_1D_2-B_2D_1)(dx)^2=0 \quad (7)$$

One can consider this as a quadratic equation for the slope dy/dx. If the direction of CPC at P is such that it has a slope satisfying equation (7), then the derivatives of u and v are not uniquely determined by the values of u and v along CPC. Such a direction is called a characteristic direction.

Let the discriminant $(A_1D_2 - A_2D_1 + B_1C_2 - B_2C_1)^2 - 4(A_1C_2 - A_2C_1)(B_1D_2 - B_2D_1)$ be denoted by D; then the following is true. If D is positive, equation 7 gives two real slopes; the system of equations 1 and 2 is hyperbolic (there are two real characteristic directions at P). If $D=0$, equation 7 gives one real slope; the system of equations (1) and (2) is parabolic (there is only a single characteristics direction at P). If D is negative, equation (7) gives a pair of complex slopes; the system of equations (1) and (2) is elliptic (there are no real characteristic directions at P).

Similar analysis can be made for the following single second-order quasilinear equation:

$$a\frac{\partial^2\psi}{\partial x^2}+b\frac{\partial^2\psi}{\partial x\partial y}+c\frac{\partial^2\psi}{\partial y^2}=f \quad (8)$$

In which a, b, c, and f are functions of x, y, ψ, $\partial\psi/\partial x$, and $\partial\psi/\partial y$. The characteristic directions are determined from the following quadratic equation:

$$a(dy)^2-bdxdy+c(dx)^2=0 \quad (9)$$

Thus, if $b^2\text{-}4ac$ is positive, equation 8 is hyperbolic; if $b^2\text{-}4ac=0$, equation 8 is parabolic; if $b^2\text{-}4ac$ is negative, equation, equation 8 is elliptic.

A technique referred to as the method of characteristics, which will be dealt with in more detail

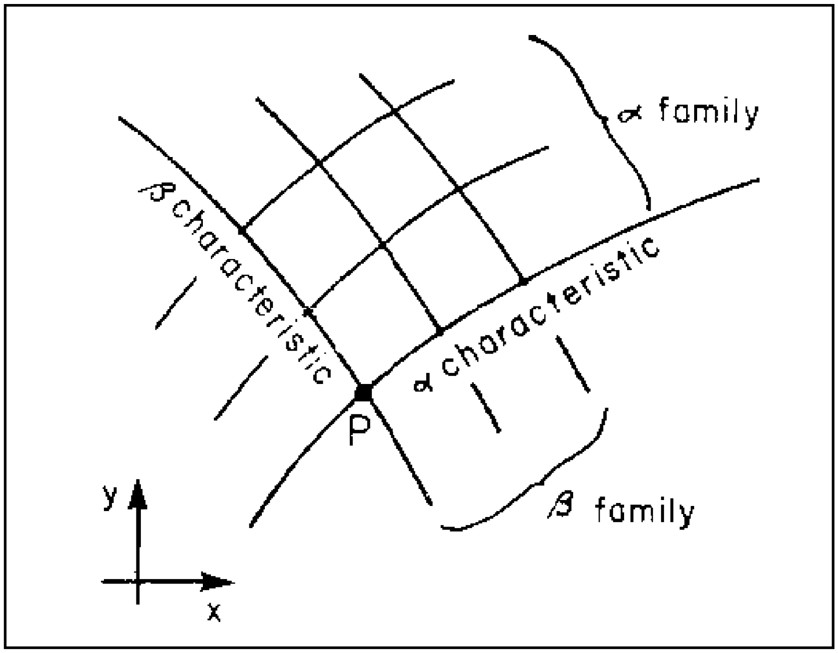

Figure 7. α and β characteristics in a given domain. The slopes of the characteristics are $(dy/dx)_a$, and $(dy/dx)_b$, respectively. (Crandall 1956)

in later sections, will be briefly outlined. Assume that the system given by equations (1) and (2) is hyperbolic in the domain under consideration. Thus, at every point there are two roots, $(dy/dx)_a$, and $(dy/dx)_b$, to the quadratic equation (7). A curve having a slope $(dy/dx)_a$ at each of its points is an a-characteristic and a curve with a slope $(dy/dx)_b$ is a β-characteristic. There are thus two families of characteristics filling the domain as shown in Fig. 7.

It has been shown that the characteristics are loci of possible discontinuities in the derivatives of a solution. In equation 6, if a characteristic direction is considered such that the determinant is zero, then, when the right-hand column is substituted for any column on the left-hand side, the resulting determinant must also be zero. Thus, replacing the fourth column on the left with the column on the right and equating the determinant to zero results in the following:

$$(A_1B_2 - A_2B_1)\, du + [(A_1C_2 - A_2C_1)\, dy/dx - (B_1C_2 - B_2C_1)]\, dv = 0 \quad (10)$$

From equation (7), one can obtain (dy/dx) as a root, and when this is substituted into equation (10), the latter becomes an ordinary differential equation for u and v along the a-characteristic. A similar equation can be obtained along the b-characteristic. Thus, for solving hyperbolic systems, one can first locate the characteristic curves and then integrate the ordinary differential equation (10) along these characteristics. This technique is referred to as the method of characteristics.

Richardson (1925) refers to the propagation and equilibrium problems as "marching problems" and "jury problems". Crandall (1956, p. 351) stated:

"In propagation problems, the solutions marches out from initial conditions guided in transit by side boundary conditions. In equilibrium problems, the entire solution is judged by a jury demanding simultaneous satisfaction of all the boundary conditions and all the internal requirements."

NUMERICAL MODELLING FOR TSUNAMI WARNING SYSTEMS

Based on all the above considerations, it is suggested that the following is an appropriate approach for numerical modelling for tsunami warning systems. For the Pacific Ocean which generates trans-oceanic tsunamis, but boundary reflections do not play a significant role, the hyperbolic method is relevant, in which the initial conditions are of paramount importance. Fig. 8 schematically illustrates this.

In the Atlantic Ocean, there are no major convergent tectonic plate boundaries to give rise to tsunamigenic earthquakes (Murty , Nirupama, Nistor, & Hamdi 2005b). Rather, the mid-Atlantic

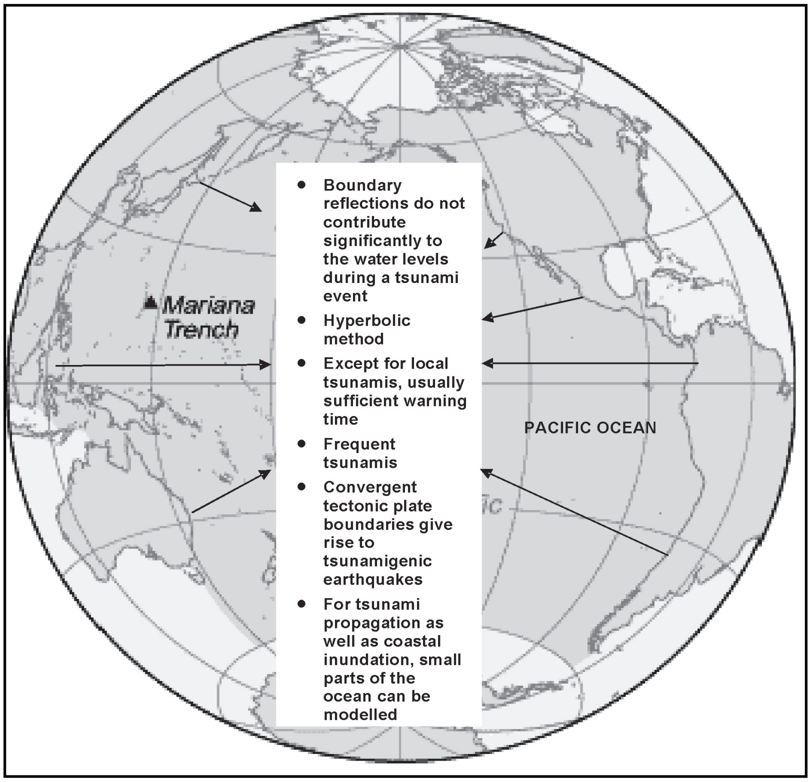

Figure 8. Schematic illustration of the tsunami numerical modelling concept for the Pacific Ocean

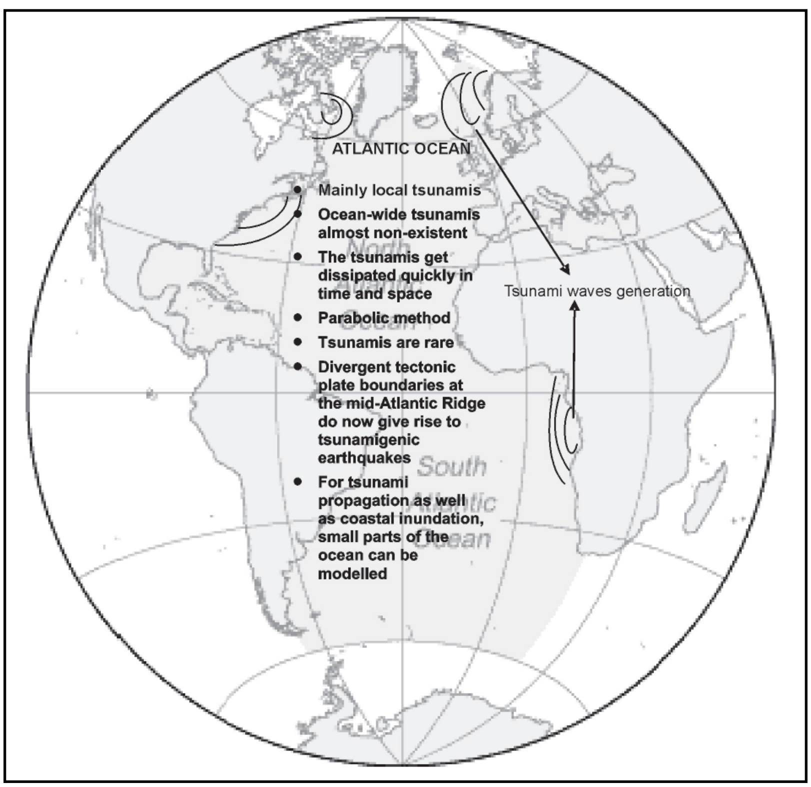

Figure 9. Schematic illustration of the tsunami numerical modelling concept for the Atlantic Ocean

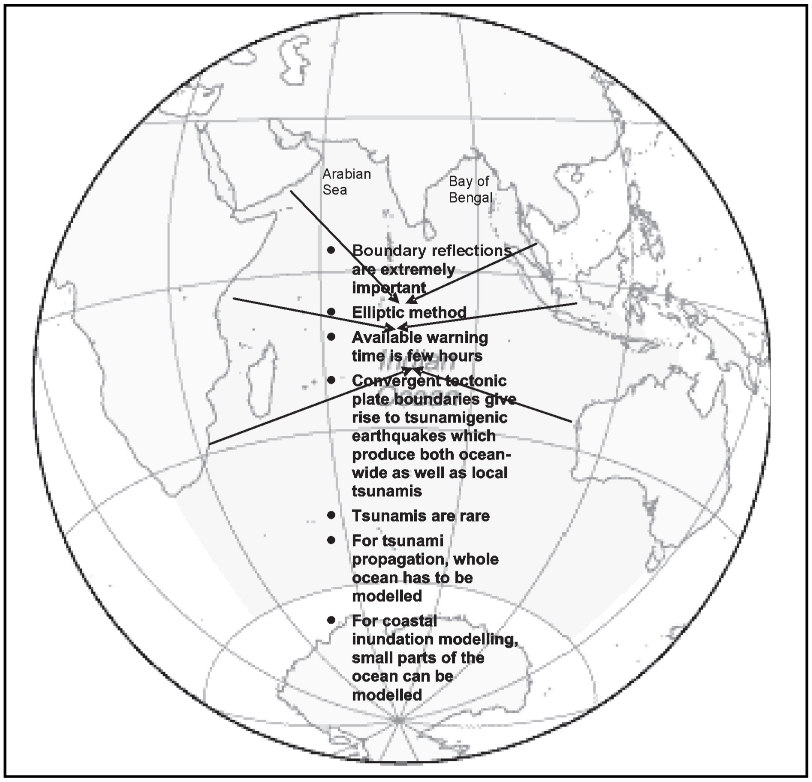

Figure 10. Schematic illustration of the tsunami numerical modelling concept for the Indian Ocean

Ridge is a divergent plate boundary. Hence tsunamis are rare here, and few that occur are almost always local and originate near the coastlines. Since tsunamis travel slowly in shallow water tsunami propagation in the Atlantic Ocean is more like a slow diffusion and hence the appropriate method is parabolic. This is schematically illustrated in Fig. 9.

In the Indian Ocean, boundary reflections contribute significantly to the water levels associated with the tsunami waves (Murty, Nirupama, Nistor. & Hamdi 2005a, Murty, Rao & Nirupama 2005c). For this reason the elliptic approach is most appropriate. This is schematically illustrated in Figure 10.

SUMMARY AND CONCLUSIONS

The Pacific and Indian Oceans have converging tectonic plate boundaries, which give rise to tsunamigenic earthquakes and ocean-wide tsunamis. On the other hand, in the Atlantic Ocean, the mid-Atlantic Ridge is a diverging plate boundary and does not give rise to tsunamigenic earthquakes or ocean-wide tsunamis. Thus whereas the Pacific and Indian Oceans produced both trans-oceanic as well as local tsunamis, the Atlantic Ocean mostly produces only local tsunamis.

One major difference between the Indian and Pacific Oceans is the role of boundary reflections in the water levels associated with tsunami waves. For the vast Pacific Ocean, boundary reflections do not play a significant role, while for the much smaller Indian Ocean, these are very important. Based upon these above considerations, it is suggested that the hyperbolic, parabolic and elliptic approaches should be used respectively for the Pacific, Atlantic and Indian Oceans.

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(Accepted 2005 December 15th. Received in original form 2005 November 15th)