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Hydraulic potentials due to finite-length line source over an anisotropic aquifer system with inclined bedding planes

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ABSTRACT

Water resources planning and management require estimation of seepage losses from surface water bodies like streams, rivers and canals to aquifer systems or recharge characteristics of aquifer systems. In such cases, numerical groundwater flow models are not quite capable of simulating flow in anisotropic aquifer systems with inclined planes of stratification. However, analytical results can be useful for simulating hydraulic heads/ flow in such systems. An analytical procedure for computing hydraulic heads in such a homogeneous anisotropic aquifer system due to a finite-length surface water source is presented. The procedure has been demonstrated using numerical experiments with the results as equipotential plots. Different coefficients of anisotropy, and orientation of bedding planes have been considered for the illustrations.

INTRODUCTION

Often, theoretical analyses of groundwater flow problems assume that the porous medium to be isotropic and homogeneous. However, field experience and laboratory tests show that most aquifers are anisotropic. The effect of anisotropic hydraulic conductivity on groundwater flow through certain geologic formations has been of concern to groundwater hydrologists as the directions of flow and of the hydraulic gradient in an anisotropic porous medium are not parallel to each other (Marcus 1962). Further, numerical groundwater flow models are designed with the assumption that principal axes of anisotropy coincide with the reference coordinate axes as in MODFLOW (Mc Donald & Harbaugh 1984). As such, popular groundwater flow models in use may not be able to simulate hydraulic heads in an anisotropic porous medium, where the planes of stratification of the soil bedding are inclined with respect to the ground surface. However, it is possible to use advancements made in the theory of exploration geophysics to formulate techniques for modelling anisotropic porous media.

Two attributes of the hydraulic conductivity viz., *heterogeneity* and *anisotropy*, are required to define the nature of an aquifer (Freeze & Cherry 1979). Anisotropy is the property of earth materials by virtue of which the hydraulic conductivity values vary with the direction of measurement at a given point whereas hydraulic conductivity variations through space within a geologic formation are termed heterogeneity. The directions in space at which the hydraulic conductivity, K attains its maximum and minimum values are termed as the *principal directions of anisotropy* and they are always orthogonal to one another (Freeze & Cherry 1979).

The theory of fluids flow through anisotropic porous medium is presented by(Marcus 1962; Scheidegger 1957; Polubaronova-Kochina 1962 and Harr 1962). Some investigations on the transformation of anisotropic medium to isotropic medium are available in Mishra 1972 and Strack 1989. Further, theory on geoelectric sounding provides analytical results for the computation of surface electric potentials in layered earth medium (Bhattacharyya & Patra 1968) due to a point source.

Theoretical Aspects

If a line source of finite length (2b) is used (Fig. 1) to recharge the homogeneous anisotropic aquifer system, then the analytical solution for hydraulic potentials can be developed by applying appropriate domain transformation techniques. To develop the required analytical expression for the hydraulic potential, we start from an expression for the case of a homogeneous isotropic medium. Subsequently, by application of suitable domain transformation techniques the equivalent analytical solution for the anisotropic medium can be formulated.

In case of an infinite homogeneous and isotropic porous medium recharged by a point source of strength Q, the expression for the steady state hydraulic potential (φ) in the porous medium at a distance, r from the source in the YZ-plane can be given as (Bhattacharyya & Patra 1968; Parasnis 1965)

$$\phi = \frac{Q}{2\pi K\sqrt{y^2+z^2}} \tag{1}$$

where K (m/s) is the isotropic hydraulic conductivity. Now, let us consider the hydraulic potential in a homogeneous porous medium due to a line source of finite length, 2b and strength, Q_L (m^2/s) per unit length (Fig.1). The finite-length line source can be assumed to be an equivalent of many infinitesimal point sources. The line source is located along the Y-axis symmetrically with the origin of the coordinate system. It is required to find the hydraulic potential at an arbitrary point, P in the YZ-plane.

Let dλ be an infinitesimally small element of the finite-length line source at a distance λ from the centre. Without loss of generality, dλ may be deemed to be a point source of strength Q_L dλ (m^3/s). If the porous medium is homogeneous and isotropic, then from eqn. (1), it follows that the steady state hydraulic potential at any point P in the YZ -plane would be (Parasnis 1965):

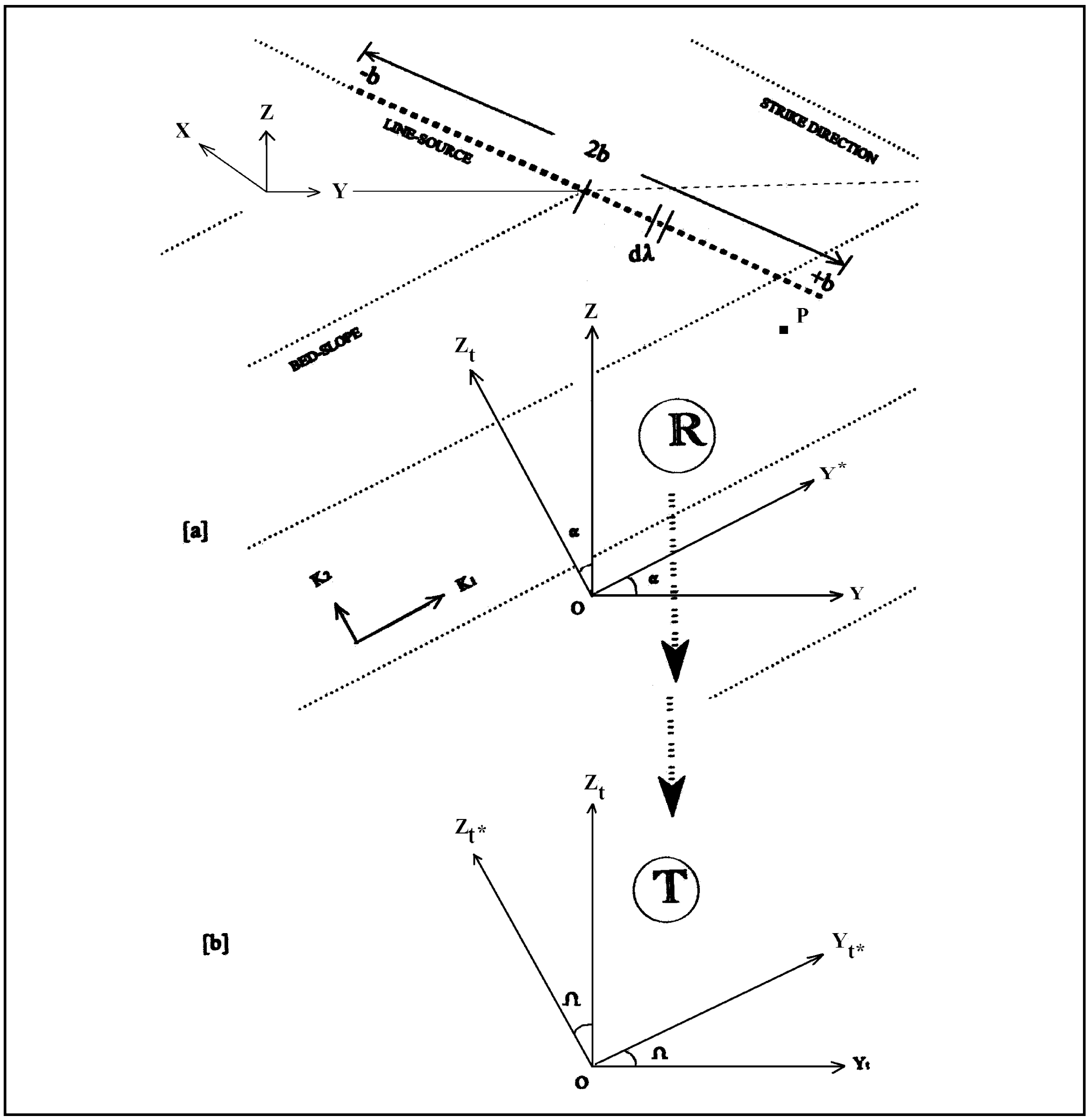

Figure 1. A finite line-source of strength Q_L (m^2/s) is centrally placed at air-earth interface along the strike of the bedding plane in: **[a]** the reference coordinate plane (R); **[b]** the transformed domain (T).

$$d\phi = \frac{Q_L\, d\lambda}{2\pi K \sqrt{z^2 + (\lambda - y)^2}} \quad (2)$$

Integrating Eqn. 2 between the limits of the line source, (-b, +b) yields the following expression of hydraulic potential in the homogeneous, isotropic medium (Parasnis 1965) due to the line source of length, 2b and strength, Q_L (m^2/s).

$$\phi = \frac{Q_L}{2\pi K}\left[\sinh^{-1}\left(\frac{b-y}{z}\right) + \sinh^{-1}\left(\frac{b+y}{z}\right) \right] \quad (3)$$

An existing methodology suggested by Strack 1989 has been adapted with some modifications to develop an analytical solution for the hydraulic potential in the vertical section of the homogeneous anisotropic aquifer system. The procedure involves three steps viz., (i) a transformation of the anisotropic medium into an isotropic domain, (ii) computation of the hydraulic potentials in the fictitious (isotropic) domain and (iii) finally, transformation of the fictitious hydraulic potentials back to the actual physical domain, yielding the required anisotropic hydraulic potentials.

Referring to Fig.1a, let us consider a stratified anisotropic porous medium with principal directions of anisotropy oriented along the Y^* and Z^* axes with hydraulic conductivities K_1 (parallel to the plane of stratification) and K_2 (normal to the plane of stratification). Let the Cartesian coordinates in the physical plane be (Y, Z). The major principal direction of the hydraulic conductivity tensor makes an angle α with the Y-axis. Also, let the Cartesian coordinates of a rotated plane be (Y^*, Z^*), such that the Y^* -axis is inclined at an angle α with the Y-axis. Now, the Cartesian coordinates (Y_t^*, Z_t^*) in a transformed domain [Fig. 1b] are chosen such that they correspond to the coordinates (Y^*, Z^*) in the physical plane. Finally, the Cartesian coordinate system (Y_t, Z_t) is introduced in the transformed domain such that the Y_t -axis corresponds to the Y -axis. Let K_m, as given in the following eqn. 4,

$$K_m = \sqrt{K_1 K_2} \text{ and } \beta = \sqrt{\frac{K_1}{K_2}}, \text{ such that } K_m = \frac{K_1}{\beta} = \beta K_2 \quad (4)$$

denotes the equivalent isotropic hydraulic conductivity of the medium in the transformed domain, (Y_t, Z_t). Further, in the fictitious domain, let the angle between the Y_t^* and Y_t -axes be Ω.

It can be shown that the angle of dip, α in the physical domain and that in the transformed domain, Ω are related by [eqn. (A.7), Annexure I]

$$\tan\Omega = \beta \tan\alpha \quad (5)$$

The hydraulic potential due to the finite-length line source would be isotropic with reference to the transformed domain, (Y_t, Z_t) and can be expressed by virtue of eqn. 3 as:

$$\phi(y_t, z_t) = \frac{Q_L}{2\pi K_m}\left[\sinh^{-1}\left(\frac{b-y_t}{z_t}\right) + \sinh^{-1}\left(\frac{b+y_t}{z_t}\right) \right] \quad (6)$$

Based on the theoretical aspects already presented in the case of hydraulic potentials due to a point source (Jose & Sastry 2005), the following transformation relations have been applied between the two domains:

$$Y_t^* = Y^*$$

$$Z_t^* = \beta Z^*, \text{ where } \beta = \sqrt{K_1/K_2} \quad (7)$$

Now, from trigonometric considerations, the relationship between the physical domain (Y,Z) and the transformed domain (Y_t, Z_t) can be established as (Jose 2001):

$$y_z = y\left[\sqrt{\cos^2\alpha + \beta^2\sin^2\alpha}\right] + z\left[\frac{(1-\beta^2)\sin\alpha\cos\alpha}{\sqrt{\cos^2\alpha + \beta^2\sin^2\alpha}}\right] \quad (8)$$

and

$$z_t = z\left[\frac{\beta}{\sqrt{\cos^2\alpha + \beta^2\sin^2\alpha}}\right] \quad (9)$$

For details see eqn. A.12 and A.13 of Annexure I.

Substituting eqns. 8 and 9 for y_t and z_t, respectively in eqn. 6 gives the following required analytical expression of the hydraulic potential, due to the line source of finite length, 2b and strength Q_L, in the YZ -plane of the homogeneous anisotropic aquifer system:

$$\phi(y, z) = \frac{Q_L}{2\pi K_m}\cdot\left[\sinh^{-1}\left(\frac{B_s - G_s}{\beta z}\right) + \sinh^{-1}\left(\frac{B_s + G_s}{\beta z}\right) \right] \quad (10)$$

where,

$$B_s = b\sqrt{\cos^2\alpha + \beta^2\sin^2\alpha}$$

$$G_s = y\left[\cos^2\alpha + \beta^2\sin^2\alpha\right] + z\,(1-\beta^2)\sin\alpha\cos\alpha \quad (11)$$

It can easily be verified that for a homogeneous and isotropic case eqn. (10) reduces to eqn. (3) by substituting $\beta = 1$ and $K_m = K$ ($K_1 = K_2 = K$) in Eqns. (10) and (11).

Two algorithms, HANI-P and HANI-L have been devised based on the analytical expressions developed in the preceding sections and coded in FORTRAN77 in order to simulate the hydraulic potentials due to a point source as well as a finite-length line source, respectively in hypothetical homogeneous anisotropic porous media with inclined strata. The input information for the computation of hydraulic potentials includes source strength, hydraulic conductivity values in the principal directions, angle of dip of the strata, and grid-node details.

The bedding planes of the rock layers in the hypothetical aquifer system make an angle α with the horizontal (XY) surface. As indicated earlier, the major principal direction of anisotropy, K_1 is uniform in the bedding plane of the strata and the minor principal direction of anisotropy, K_2 is perpendicular to the plane of the strata. The finite-length line recharging source, Q_L [L^2T^{-1}] is located centrally on the earth surface. The boundaries of the hypothetical aquifer system are assumed to be at large distances from the source (i.e., a homogeneous anisotropic aquifer system of infinite extent). Simulations of hydraulic potentials have been carried out with several levels of anisotropy of the aquifer system and for different orientations (α), varied between zero and π/2, of the strata.

The values of aquifer parameter used for the simulation of hydraulic potentials due to a finite-length line source and the various sets of simulations performed using the algorithm HANI-L are given in Table 1 and Table 2, respectively.

RESULTS AND DISCUSSION

The variation in the pattern of the hydraulic potentials due to the finite-length line source with different orientations of the strata is illustrated in Fig. 2. The equipotentials (dotted lines) are plotted for different angles of dip of the strata viz., $\alpha=0$, $\alpha=\pi/12$, $\alpha=\pi/4$ and $\alpha=\pi/2$, when the coefficient of anisotropy, $\beta=4$.

Table 1. Aquifer Parameter values used for the numerical simulations of hydraulic potentials due to a finite-length line source.

Model Parameters	**Numerical Values**
Source strength, Q_L	0.01 m^2/s
Length of the line source, 2b	400 m
Hydraulic conductivity in the major direction, K_1	0.001 m/s
Dip angle of the strata, α	0, π/12, π/4, π/2
Coefficient of anisotropy, β	2, 4 7, 10
Ratio of hydraulic conductivity values, K_1/K_2	4, 16, 49, 100
Grid dimensions	Y= (-500 m, +500 m), Z= (0, -250 m) ΔY= 10 m, ΔZ= 10 m

Table 2. Set of cases where hydraulic potentials due to a finite-length line source have been simulated with different combinations of the angle of dip, α and the coefficients of anisotropy, β.

Hydraulic Potentials due to Line Source in Homogeneous Anisotropic Aquifer System				
Parameters	α=0	α=π/12	α=π/4	α=π/2
β=2	✓	-	✓	-
β=4	✓	✓	✓	✓
β=7	✓	-	✓	-
β=10	✓	-	✓	-

As Fig.2 indicates that the shape of equipotential contours are controlled totally by dip of strata for a fixed coefficient of anisotropy, $\beta = 4$. As streamlines are orthogonal to potential plot along the principal axes of symmetry, one can readily infer the flow direction in each case of Fig.2.

When the stratification is perfectly horizontal, the pattern of recharge due to a finite-length line source in the homogeneous anisotropic aquifer system with varying anisotropy levels can be inferred from Fig.3. Figure 3 illustrates the influence of anisotropy on potential field distribution and it clearly outlines the

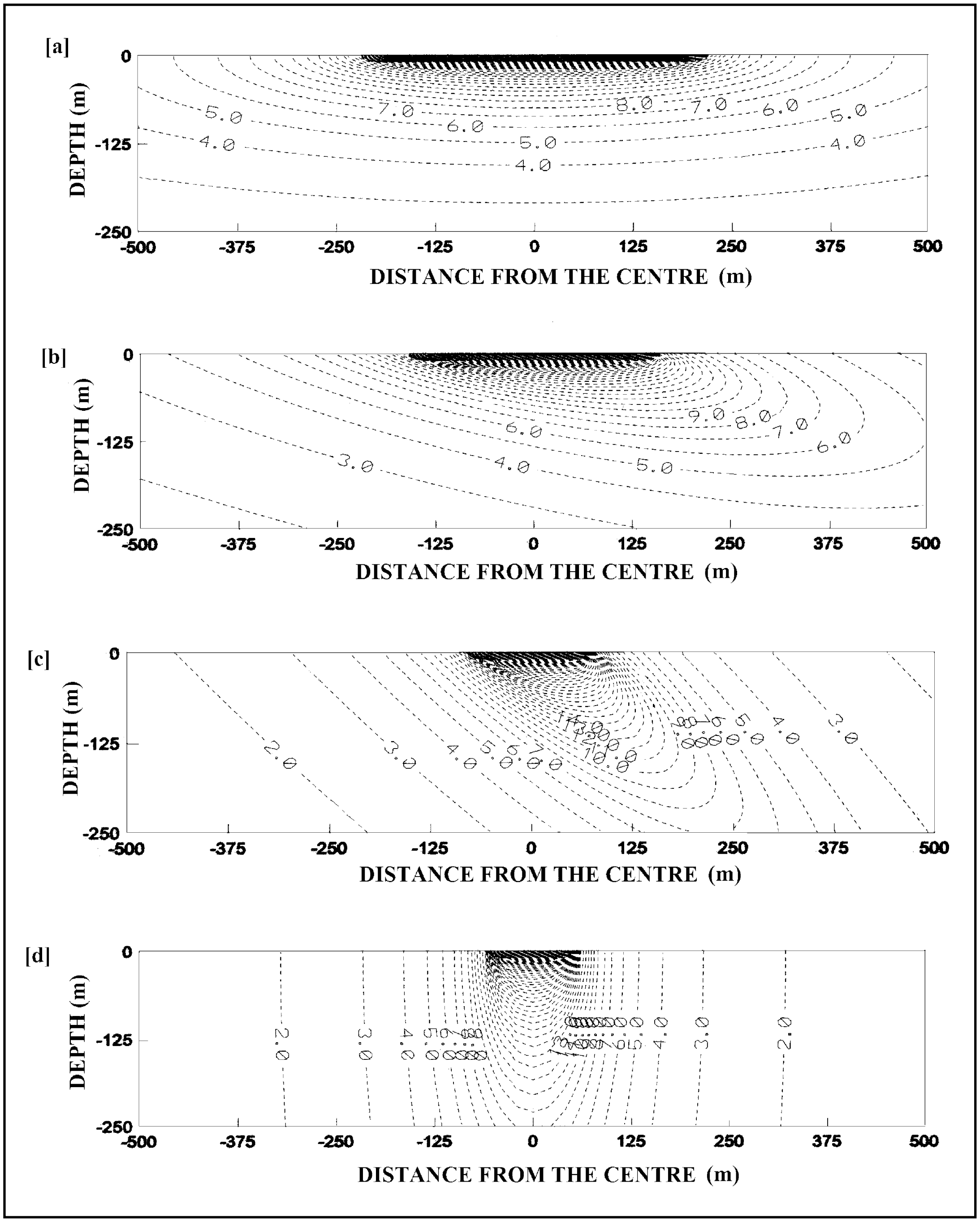

Figure 2. Equipotential distribution (dotted lines) in a homogeneous anisotropic aquifer system due to a finite-length line source for different inclinations (α) of the beds when the coefficient of anisotropy, β=4. For [a] α=0; [b] α=π/12; [c] α=π/4; [d] α=π/2.

importance of this study. With increasing anisotropic coefficient, the potential field contours are getting flattened, thereby indicating an increasingly vertical pattern of stream lines that govern the along principal axes. For all these simulations, the finite-length source had been oriented along the strike of the bedding planes. Further, the plots have been depicted in a vertical plane through the length of the line source. The plots across the finite-length line source have not been shown as the pattern of those are

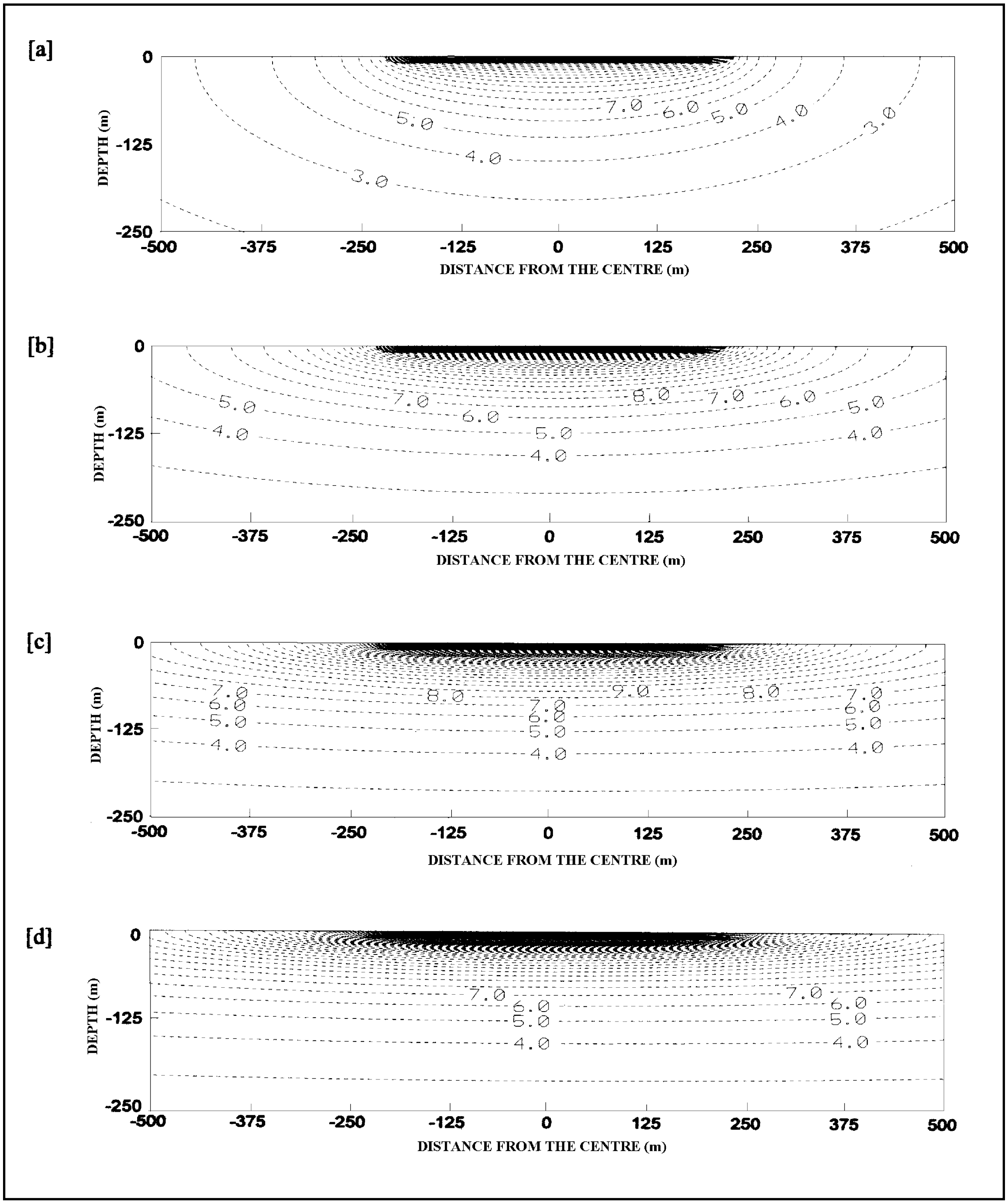

Figure 3. Hydraulic potentials in an anisotropic aquifer system due to a finite-length line source of strength, Q_L=0.01 m^2/s for different coefficients of anisotropy (β) when the angle of dip, α=0. For [a] β=2; [b] β=4; [c] β=7; [d] β=10.

similar to that of the point source (Jose and Sastry, 2005), except for the magnitude of the potential.

A similar set of plots are also given in Fig. 4 for the case when the aquifer strata is sloping at an angle of 45°E with the earth surface. The coefficients of anisotropy applicable to the cases presented in Figs 3 and. 4 are β=2, β=4, β=7 and β=10, respectively.

Both anisotropy and dips of beds influence the hydraulic potential distribution and thereby stream line fields.

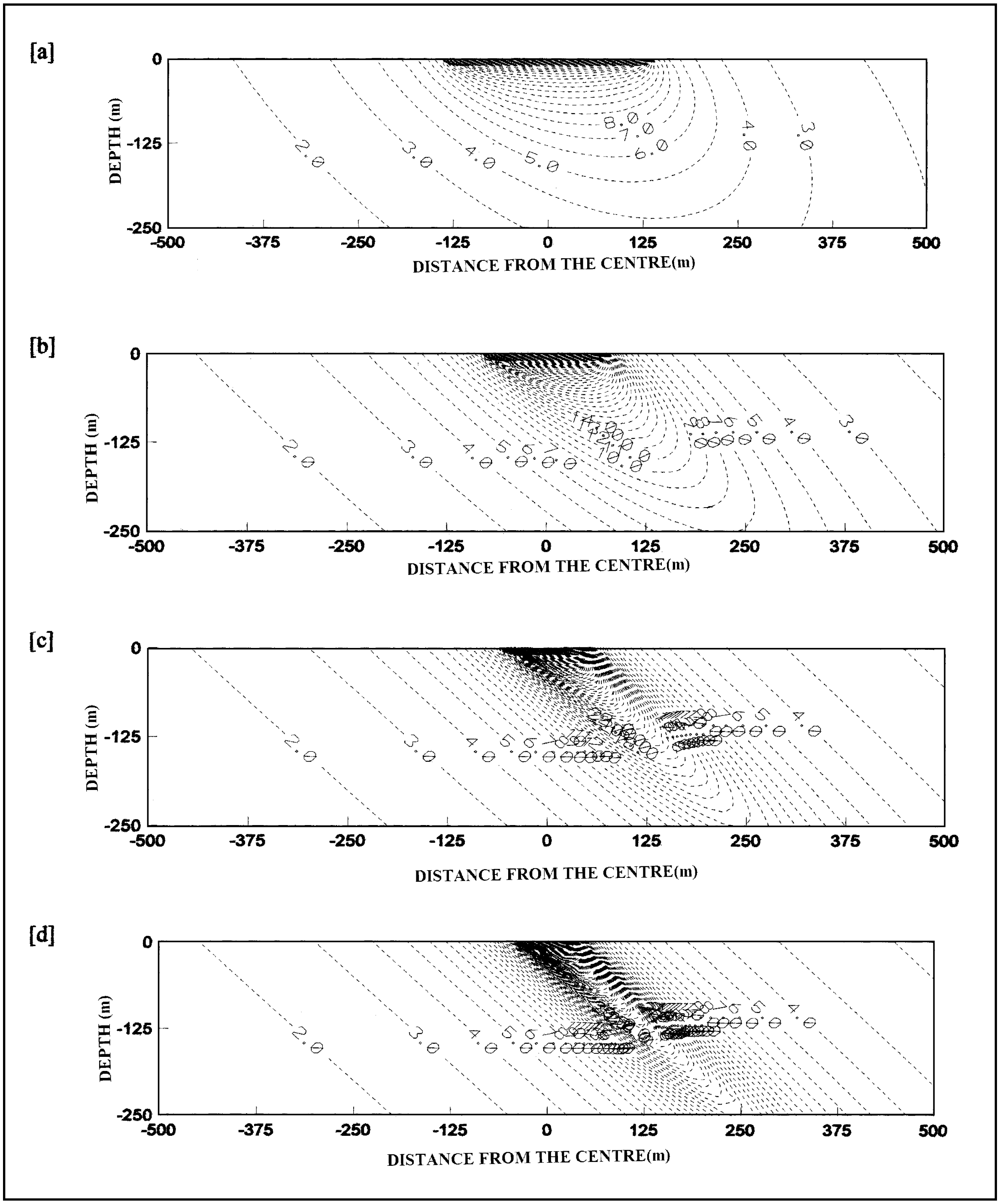

Figure 4. Hydraulic potentials in an anisotropic aquifer system due to a finite-length line source of strength, Q_L=0.01 m^2/s for different coefficients of anisotropy (β) when the angle of dip, $\alpha=\pi/4$. For [a] β=2; [b] β=4; [c] β=7; [d] β=10.

SUMMARY AND CONCLUSIONS

Analytical procedure for computing hydraulic heads in a homogeneous anisotropic aquifer system due to a finite-length line source of recharge has been developed and simulation has been carried out for different coefficients of anisotropy, and orientations of aquifer strata. The results obtained demonstrate the usefulness of analytical solution in simulating hydraulic heads in a homogeneous anisotropic aquifer.

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ANNEXURE I

By considering Fig.1, the coordinates (y^*, z^*) can be expressed in terms of (y, z) as

$$y = y^* \cos\alpha - z^* \sin\alpha$$
$$z = y^* \sin\alpha + z^* \cos\alpha \qquad (A.1)$$

and

$$y^* = y \cos\alpha + z \sin\alpha$$
$$z^* = - y \sin\alpha + z \cos\alpha \qquad (A.2)$$

Using the following transformation

$$y_{t^*} = y^*$$
$$z_{t^*} = \beta z^* \qquad (A.3)$$
$$\textit{where } \beta = \sqrt{K_1 / K_2}$$

we can express y_t and z_t in terms of y_{t^*} and z_{t^*} as

$$y_t = y_{t^*} \cos\Omega - z_{t^*} \sin\Omega$$
$$z_t = y_{t^*} \sin\Omega + z_{t^*} \cos\Omega \qquad (A.4)$$

The y_t- axis the transform domain corresponds to y-axis in the physical plane. Since $z_t = 0$ along y_t-axis we have from eqn. (4) the following expression:

$$\frac{z_{t^*}}{y_{t^*}} = -Tan\,\Omega \quad \textit{for} \quad z_t = 0 \qquad (A.5)$$

Also z = 0 along the y-axis so that from eqn. (A.1) we get

$$y^* \sin\alpha = - z^* \cos\alpha$$
$$\frac{z^*}{y^*} = - \mathrm{Tan}\,\alpha \quad \textit{for } z = 0 \qquad (A.6)$$

From eqns.(A.3) and (A.6) we get

$$\frac{z_{t^*}}{y_{t^*}} = \beta \frac{z^*}{y^*}$$
$$\textit{i.e.,} - \tan\Omega = - \beta \tan\alpha$$
$$\textit{Hence } \tan\Omega = \beta \tan\alpha \qquad (A.7)$$

Equations (A.4), (A.3) and (A.2) allow us to express y_t and z_t in terms of y and z as

$$y_t = (y\cos\alpha + z\sin\alpha)\cos\Omega - \beta(-y\sin\alpha + z\cos\alpha)\sin\Omega$$
$$z_t = (y\cos\alpha + z\sin\alpha)\sin\Omega + \beta(-y\sin\alpha + z\cos\alpha)\cos\Omega \qquad (A.8)$$

This may be written as

$$y_t = \cos\alpha\cos\Omega(y(1+\beta\tan\alpha\tan\Omega) + z(\tan\alpha - \beta\tan\Omega))$$
$$z_t = \cos\alpha\cos\Omega(y(\tan\Omega - \beta\tan\alpha) + z(\tan\alpha\tan\Omega + \beta)) \qquad (A.9)$$

It is possible to select the coordinate transformations in such a way that $-\pi/2 \le \alpha \le \pi/2$. Further, from equation (7) it is evident that $-\pi/2 \le \Omega \le \pi/2$. Then without loss of generality the range of analysis can be restricted same ranges of α and Ω.
Therefore,

$$\cos\Omega \ge 0, \forall\Omega \qquad (A.10)$$

Then we have

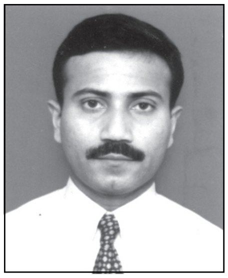
$$\cos\Omega = \frac{1}{\sqrt{1+\tan^2\Omega}} \qquad (A.11)$$

Now, using eqns.(10) and (7) eqn.(9) can be expressed as

$$\begin{aligned}
y_t &= \cos\alpha\cos\Omega[y(1+\beta\tan\alpha\tan\Omega) + z(\tan\alpha - \beta\tan\Omega)] \\
&= y\cos\alpha\frac{1}{\sqrt{1+\tan^2\Omega}}(1+\beta\tan\alpha\beta\tan\alpha) + z\cos\alpha\frac{1}{\sqrt{1+\tan^2\Omega}}(\tan\alpha - \beta^2\tan\alpha) \\
&= y\cos\alpha\frac{(1+\beta^2\tan^2\alpha)}{\sqrt{(1+\beta^2\tan^2\alpha)}} + z\cos\alpha\frac{\tan\alpha(1-\beta^2)}{\sqrt{1+\beta^2\tan^2\alpha)}} \\
&= y\sqrt{\cos^2\alpha + \beta^2\sin^2\alpha} + z\sin\alpha\cos\alpha\frac{(1-\beta^2)}{\sqrt{\cos^2\alpha + \beta^2\sin^2\alpha}} \qquad (A.12)
\end{aligned}$$

Similarly it can be proved

$$\begin{aligned}
z_t &= \cos\alpha\cos\Omega[y(\tan\Omega - z(\tan\alpha\tan\Omega + \beta)] \\
&= \frac{z\beta}{\sqrt{\cos^2\alpha + \beta^2\sin^2\alpha}} \qquad (A.13)
\end{aligned}$$



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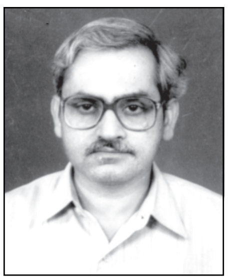

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