J. Ind. Geophys. Union (April 2006)
Vol.10, No.2, pp.131-140

A semi-analytical procedure for the computation of hydraulic heads and streamlines in multi-layered aquifer systems

Rambhatla G.Sastry and Mathew K.Jose[1]
Department of Earth Sciences, IIT, Roorkee-247 667
E-mail: rgss1fes@iitr.ernet.in / rgssastry@yahoo.com
[1]National Institute of Hydrology, Jalvigyan Bhawan, Roorkee-247 667
E-mail: mjose@.nih.ernet.in

ABSTRACT
Analytical solutions are presented for the computation of steady-state hydraulic heads and streamlines in multi-layered aquifer systems. Considering the analogy between direct-current electrical flow and groundwater flow, the proposed methodology, point source over n-layer (NLPNT) invokes the geoelectric sounding principles. Numerical examples for the cases of a 3, 4, and 5-layered aquifer system are presented. Comparison of NLPNT solutions with the corresponding solutions obtained from popular MODFLOW indicate that the NLPNT is quite cost-effective in computing hydraulic heads and streamlines .

INTRODUCTION

Analytical methods are handy compared to numerical techniques when a steady state solution of hydraulic head or stream lines is sought for an aquifer system. The objective here is to develop easy-to-compute analytical solutions for steady-state hydraulic head and streamlines in a stratified aquifer system with a point-source of recharge at the centre. The methodology invokes the geoelectric sounding principles (Bhattacaryya & Patra 1968; Koefoed 1979; Niwas & Israil 1986) as the analogy between electrical flow and groundwater flow is well established (Hubbert 1940; Freeze & Cherry 1979). Further, groundwater flow models have been used in simulating electrical flow through porous media (Osiensky &Williams, 1996) and discussions of direct current electrical resistivity methods and electrical analog models in groundwater applications are available in the literature(Walton 1970; Zohdy Eaton & Mabey 1974; Prickett 1975). In an earlier study, Yates (1988) has developed an analytical solution for flow of water in a saturated three layered aquitard- aquifer- aquitard system of finite length. The solution obtained was for one-dimensional flow in the aquifer. Besides, the solution was limited by the condition that ratios of hydraulic conductivity between layers should approach zero. However, in the present study analytical expressions for steady state hydraulic head and streamlines in the presence of a point-source have been derived for a multi-layered aquifer system without any restrictions on the number of layers and layer conductivities. Here, the hydraulic conductivity can vary from layer to layer in any fashion and the flow is three dimensional with a cylindrical symmetry around the point-source of recharge. By making use of the theoretical developments presented in the paper, a semi-analytical solution procedure (NLPNT) has been devised to compute the required hydraulic heads and streamlines.

THEORY

In geoelectrical prospecting, the depths and electrical resistivities of horizontal layers of earth are determined by using the electrical potential and electrical field at any point on the surface of the earth. As such, analytical solution for electrical potentials on the surface of a stratified earth medium exists in the geophysical literature (Bhattacharyya & Patra 1968; Koefoed 1979). However, for geohydrological applications the hydraulic head and flow need to be obtained in the various layers below the earth-surface, and for which analytical solutions are non-existent. Therefore, the present study is aimed at deriving a set of expressions for hydraulic head and streamlines in a multi-layered aquifer system.

Consider a horizontally stratified aquifer system with n layers in a cylindrical coordinate frame, (R,Z,θ) with its origin at the point source, A (see Fig. 1). Let K_1, K_2,....., K_n be the hydraulic conductivities, and h_1, h_2,....., h_n be the depths to the bottom of respective layers from the surface. It is assumed that the last layer extends to infinite depth (half-space). Then, the Laplace equation satisfied at any point in Layer i (for i=1,2,. . . ,n) by the steady state hydraulic head, φ_i (r,z) (written as φ_i) is given by

$$\frac{\partial^2 \phi_i}{\partial r^2} + \frac{1}{r}\frac{\partial \phi_i}{\partial r} + \frac{\partial^2 \phi_i}{\partial z^2} = 0 \qquad (1)$$

The general solution of eqn. (1) can be written as

$$\phi_i(r, z) = \frac{q}{2\pi K_1}\left\{ \int_0^\infty \left[e^{-\lambda z} + A_i(\lambda) e^{-\lambda z} + B_i(\lambda) e^{\lambda z} \right] J_0(\lambda r)\, d\lambda \right\} \qquad (2)$$

where q [m^3/s] is the recharge by the point source; $A_i(\lambda)$ and $B_i(\lambda)$ are rational functions in which numerator and denominator are polynomials in $e^{-2\lambda}$; $J_0(\lambda r)$ is the Bessel function of zeroth order. The variable of integration λ [L^{-1}] ranges from 0 to ∞.

Now, the unknown functions $A_i(\lambda)$ and $B_i(\lambda)$ in eqn. (2) need to be evaluated subject to certain boundary conditions (Koefoed, 1979) given below:

(i) At the air-earth interface the vertical component of flow must be zero; i.e.,

$$K_1 \frac{\partial \phi_1}{\partial z}\Big|_{z=0} = 0 \qquad (3a)$$

This implies that, in eqn. (2) the functions $A_1(\lambda)$ and $B_1(\lambda)$ must be identical so that,

$$A_1(\lambda) = B_1(\lambda) \qquad (3b)$$

The surface potential can be obtained by using eqns. (2) and (3b) as:

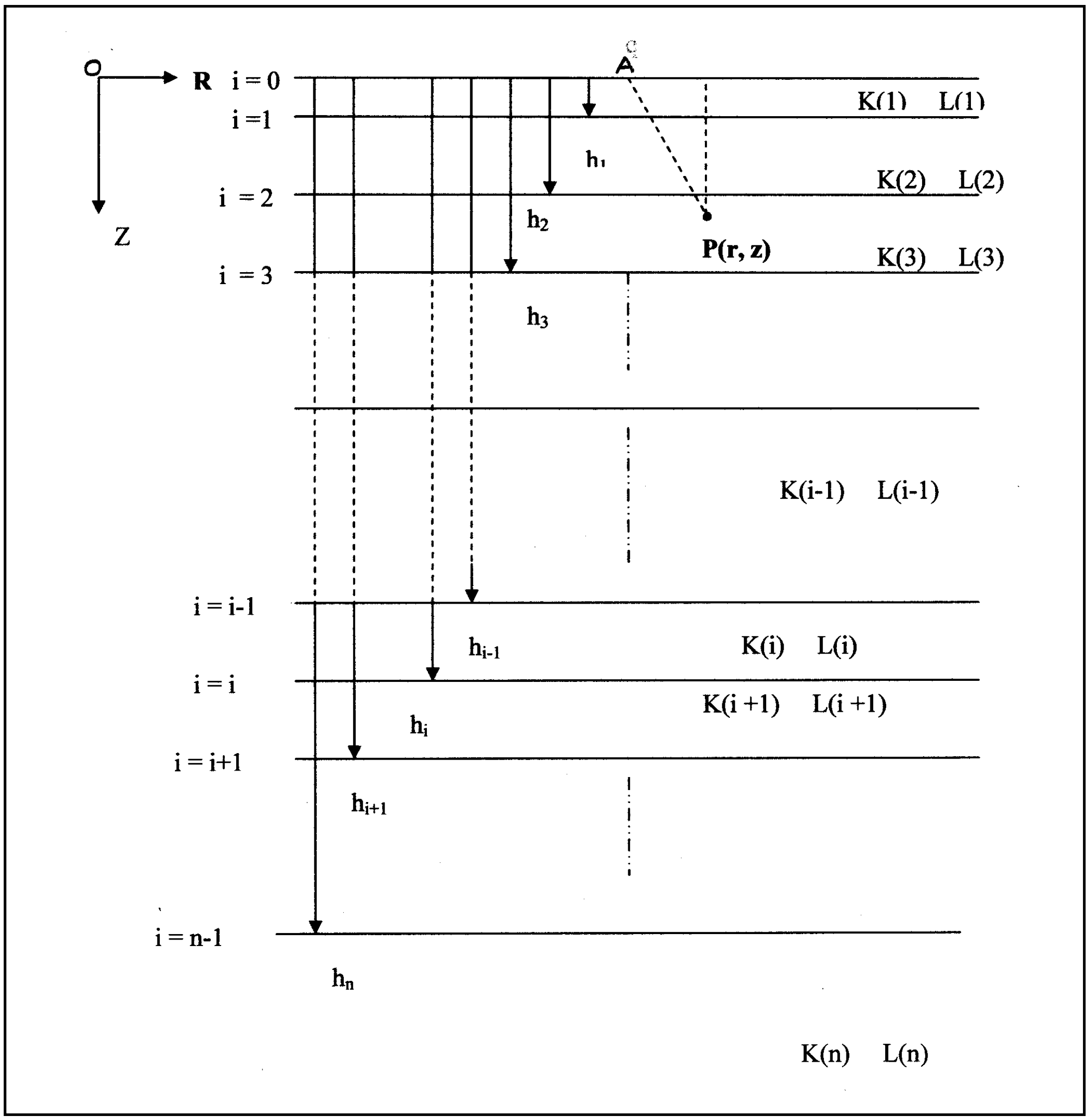


Figure 1. Schematic diagram for the multi-layered aquifer system with a point source at A; P(r,z) is any arbitrary point in the cylindrical coordinate system.

$$\phi_1(r) = \frac{q}{2\pi K_1} \int_0^{\infty} P_1(\lambda)\, J_0(\lambda r)\, d\lambda \quad (4)$$

where

$$P_1(\lambda) = 1 + 2A_1(\lambda) \quad (5)$$

$P_1(\lambda)$ is the kernel function determined by thicknesses and hydraulic conductivity of the layers. The analytical solution for eqn. (4) is available (Koefoed 1979) in the geophysical context.

(ii) At each of the boundary planes in the subsurface, the hydraulic head as well as flow must be continuous. So, at any i[th] interface within the medium,

$$\phi_i = \phi_{i+1} \quad and \quad K_i \frac{\partial \phi_i}{\partial z} = K_{i+1} \frac{\partial \phi_{i+1}}{\partial z} \quad (6a)$$

Applying these conditions to eqn. (2), we obtain the following identities:

$$A_i(\lambda)\, e^{-\lambda h_i} + B_i(\lambda)\, e^{\lambda h_i} = A_{i+1}(\lambda)\, e^{-\lambda h_i} + B_{i+1}(\lambda)\, e^{\lambda h_i} \quad (6b)$$

$$K_i[1 + A_i(\lambda)\, e^{-\lambda h_i} - B_i(\lambda)\, e^{\lambda h_i}] = K_{i+1}[\{1 + A_{i+1}(\lambda)\}\, e^{-\lambda h_i} - B_{i+1}(\lambda)\, e^{\lambda h_i}] \quad (6c)$$

(iii) At infinite depth/ distance, the hydraulic head must approximate to zero, i.e.,

$$\lim_{z \to \infty} \phi(r, z) = 0 \quad and \quad \lim_{r \to \infty} \phi(r, z) = 0 \quad (7a)$$

Eqn. (7a) necessitates $B_n(\lambda)\, e^{\lambda z}$ to vanish in eqn. (2). Therefore,

$$B_n(\lambda) = 0 \quad (7b)$$

ANALYTICAL PROCEDURES

Based on the theoretical considerations in the preceding section, the following formulations have been developed to compute hydraulic head and streamlines in a multi-layered aquifer system.

Recurrence Relation for $A_i(\lambda)$ and $B_i(\lambda)$

In order to compute the hydraulic potentials at various points below the surface of the layered medium, the first requirement is to evaluate the rational functions $A_i(\lambda)$ and $B_i(\lambda)$ in eqn. (2) for various layers. The following procedure has been developed to evaluate $A_i(\lambda)$ and $B_i(\lambda)$ in a recursive manner:

Using eqns. 6(b) and 6(c), a recurrence formula has been established to relate $A_i(\lambda)$ with $A_{i+1}(\lambda)$, and $B_i(\lambda)$ with $B_{i+1}(\lambda)$; and it is expressed as the following matrix equation:

$$\begin{bmatrix} A_{i+1} \\ B_{i+1} \end{bmatrix} = \begin{bmatrix} a_i & b_i e^{2\lambda h_i} \\ b_i e^{-2\lambda h_i} & a_i \end{bmatrix} \begin{bmatrix} A_i \\ B_i \end{bmatrix} + \begin{bmatrix} -b_i \\ b_i e^{-2\lambda h_i} \end{bmatrix} \quad (8)$$

where

$$a_i = \frac{K_{i+1} + K_i}{2K_{i+1}}$$

$$b_i = \frac{K_{i+1} - K_i}{2K_{i+1}} \quad (9)$$

Eqn. (8) can be used to evaluate $A_i(\lambda)$ and $B_i(\lambda)$ progressively for i=1,2,...,n provided $A_1(\lambda)$ and $B_1(\lambda)$ are already known. In order to obtain $A_1(\lambda)$ [$=B_1(\lambda)$], the Pekeris recurrence relation (Koefoed 1979) given by the following equation is employed.

$$P_i(\lambda) = \frac{P_{i+1}(\lambda) + (K_{i+1}/K_i)\tanh(\lambda d_i)}{(K_{i+1}/K_i) + P_{i+1}(\lambda)\tanh(\lambda d_i)} \quad (10)$$

where $P_i(\lambda)$ is defined as

$$P_i(\lambda) = \frac{1 + A_i(\lambda) + B_i(\lambda)\, e^{2\lambda h_{i-1}}}{1 + A_i(\lambda) - B_i(\lambda)\, e^{2\lambda h_{i-1}}} \quad (11)$$

Applying the boundary condition for the n[th] layer given by eqn. 7b in eqn. (11) we get

$$P_n(\lambda) = 1 \quad (12)$$

Also, for the top layer $P_1(\lambda)$ and $A_1(\lambda)$ are related by eqn. (5). It can also be deduced from eqn. (11) by application of boundary conditions (eqn. 3b) for the top layer . Therefore, eqn. (10) in conjunction with eqn. (12) can be used to evaluate $P_1(\lambda)$ and, thereby, $A_1(\lambda)$ or $B_1(\lambda)$ in a recursive manner.

Hydraulic Head

Since $A_i(\lambda)$ and $B_i(\lambda)$ are rational functions in which the numerator and the denominator are polynomials in $e^{-2\lambda}$, these functions can be expressed as approximate exponential series (Hamming 1962) with a finite number of terms, *m* as

$$A_i(\lambda) = \sum_{j=1}^{m} f_j\, e^{-\varepsilon_j \lambda} \quad (13)$$

$$B_i(\lambda) = \sum_{j=1}^{m} g_j\, e^{-\varepsilon_j \lambda} \quad (14)$$

where f_j and g_j are coefficients in the respective expansions and ε_j determines the position of approximating functions along the abscissa.

Now, substitution of $A_i(\lambda)$ and $B_i(\lambda)$ from equations (13) and (14) into eqn. (2) followed by application of the Integral of Lipschitz (Watson, 1944) will facilitate solution of the integral equation for hydraulic head. Thus, we get the expression for hydraulic head in any layer as:

$$\phi_i(r, z) = \frac{q}{2\pi K_1}\left[\frac{1}{(r^2+z^2)^{1/2}}\right] + \sum_{j=1}^{m}\frac{f_j}{[r^2+(z+\varepsilon_j)^2]^{1/2}} + \sum_{j=1}^{m}\frac{g_j}{[r^2+(z-\varepsilon_j)^2]^{1/2}} \quad (15)$$

Eqn. (15) may be used to compute the hydraulic heads in the respective layers of the multi-layered aquifer system provided, that the coefficients f_j and g_j are ascertained.

When the hydraulic conductivities of various layers in a multi-layered aquifer system are the same, then it becomes a homogeneous aquifer. Then, the coefficients f_j and g_j in eqn. (15) become zeroes. As a result, the solution for hydraulic head (eqn. 15) reduces to $\varphi_i = q/2\pi K_1(r^2 + z^2)^{1/2}$, which is nothing but the exact solution for steady state potential in a homogeneous aquifer.

Determination of Coefficients f_j and g_j

The following procedure has been used to determine the coefficients f_i and g_i:

If the rational functions $A_i(\lambda)$ and $B_i(\lambda)$ are known at discrete values λ_t (t = 1, 2, ..., s for s>m or s<m), then eqns. (13) and (14) can be rewritten as:

$$A_i(\lambda_t) = \sum_{j=1}^{m} f_j\, e^{-\varepsilon_j \lambda_t} \quad (16)$$

and

$$B_i(\lambda_t) = \sum_{j=1}^{m} g_j\, e^{-\varepsilon_j \lambda_t} \quad (17)$$

Now, equations (16) and (17) form *s* equations each with $A_i(\lambda_t)$ and $B_i(\lambda_t)$, which can be used to evaluate *2m* unknowns in each set, (f_j, ε_j) and (g_j, ε_j). But, eqns. (16) and (17) are obviously nonlinear system of equations in ε_j, and solution is rather difficult. However, *m* values of ε_j can be assigned as per some criterion to convert those equations to over-determined linear system (s>m) of equations in f_j and g_j and to get a solution.

Therefore, let us consider eqn. (16) where (f_j, ε_j) are to be evaluated. Replacing the functions $A_i(\lambda_t)$ and f_j by vector quantities (column matrices) and the exponential term by vector quantities (column matrices) and the exponential term by an (s x m) matrix yields

$$[A_i] = [A_i(\lambda_1), A_i(\lambda_2), \ldots, A_i(\lambda_s)]^T \quad (18)$$

$$[E] = [e^{-\varepsilon_j \lambda_r}] \quad for \quad r = 1, s \,\&\, j = 1, m \;;\; s > m \quad (19)$$

$$[F] = [f_1, f_2, \ldots, f_m]^T \quad (20$$

Therefore, eqn. (16) can be rewritten as a matrix equation

$$[A_i]_{(sx1)} = [E]_{(sxm)}[F]_{(mx1)} \quad (21)$$

The least-squares solution of eqn. (21) is given by Morrison (1969) where $[E_{ls}]^{-1}$ is the least-square inverse of matrix [*E*]. It is established that

$$[F] = [E_{ls}]^{-1}[A_i] \quad (22)$$

$$[E_{ls}]^{-1} = [E^T E]^{-1}[E^T] \quad (23)$$

Therefore, combining eqns. (22) and (23), we get

$$[F] = [E^T E]^{-1}[E^T]. \quad [A_i] \quad (24)$$

Now, eqn. (24), consisting of a set of linear equations, can be solved by appropriate methods. An arithmetic progression has been chosen to assign values of ε_j in the solution procedure. The ε_j values are thus assigned by the expression

$$\varepsilon_j = j\varepsilon_0 \quad (25)$$

where ε_0 is a certain base value chosen which is approximately equal to the total thickness of the first *(n-1)* layers. The best set of coefficient for f_j can then be evaluated for a given set of corresponding ε_j values in order that the equations are satisfied within an accepted range of error.

Streamlines

The stream function can be obtained from the hydraulic potential function (Harr 1962) by using the Cauchy-Riemann equations. Let ψ_i(r, z) be the stream function (written as ψ_i) for the i[th] layer. Then,

$$-K_i\frac{\partial \phi_i}{\partial r} = \frac{\partial \psi_i}{\partial z} \quad (26)$$

$$-K_i\frac{\partial \phi_i}{\partial z} = -\frac{\partial \psi_i}{\partial r} \quad (27)$$

Integration of either eqn. (26) or eqn. (27) will yield the stream function for the i[th] layer as

$$\psi_i = -\int K_i\frac{\partial \phi_i}{\partial r}\, dz + C_1 \quad (28)$$

$$\psi_i = -\int K_i\frac{\partial \phi_i}{\partial z}\, dr + C_2 \quad (29)$$

In eqns. (28) and (29), the constants of integration C_1 and C_2 can be taken to be zero using boundary conditions (7a).

Thus, the stream function for the i[th] layer may be evaluated using,

$$\psi_i = -K_i\int\left(\frac{\partial \phi_i}{\partial r}\right) dz \quad (30)$$

The final expression (Jose 2001) for stream function in the i[th] layer is obtained in two steps: (i) evaluate the derivative $\partial\phi_i/\partial r$ using eqn. (15) and (ii) substitute the derivative into equation (30), followed by integration. Thus, we have

$$\psi_i(r, z) = \frac{q K_i}{2\pi K_1}\left[\frac{z}{r\sqrt{r^2+z^2}} + \sum_{j=1}^{m}\frac{(z+\varepsilon_j) f_j}{r\sqrt{r^2+(z+\varepsilon_j)^2}} + \sum_{j=1}^{m}\frac{(z-\varepsilon_j) g_j}{r\sqrt{r^2+(z-\varepsilon_j)^2}}\right] \quad (31)$$

Eqn. (31) is the expression for streamlines in a multi-layered aquifer system.

NUMERICAL EXPERIMENTS

An algorithm, NLPNT has been devised using the above analytical solutions (15) & (31) to obtain steady state hydraulic heads and streamlines respectively in a multi-layered aquifer system. Multi-layered aquifer models with 3, 4, and 5 layers respectively have been considered for the numerical experiments. The hydraulic heads and streamlines computed using NLPNT have been reproduced in the form of contour plots, in vertical sections. The corresponding results, for the considered aquifer models, obtained by using the three-dimensional groundwater flow model MODFLOW Mc Donald & Harbaugh, 1984) have been used to compare the NLPNT results.

Description of Models

A point source of strength, q = 0.01 m^3/s is placed at the centre of the ground surface to recharge the aquifer. The hydraulic heads and streamlines have been computed in the central section of the aquifer model at the nodes of a rectangular grid with a lateral extent of 400 m each towards either side of the point source. The lateral boundaries extend to infinite distance where the hydraulic head tends to be zero. The thickness of a layer in any of the models has been taken as 100 m. Hence, the depth of an aquifer model depends on the number of layers in the model. The various layers in an aquifer model have been assigned different values for hydraulic conductivity.

To enable comparison, the steady state hydraulic heads have also been simulated using MODFLOW with identical grid set-up and boundaries. Near-zero values for the hydraulic head have been assigned as boundary conditions at large distances in the MODFLOW domain to conform with the assumptions in the NLPNT solution procedure. A recharge well (with q = 0.01m^3/s) has been introduced at the central node for the point source. All the required parameters are assigned cell-wise in the model. The model employs an iterative Strongly Implicit Procedure (SIP) to compute hydraulic heads.

The depth of the 3-layered aquifer model is 300m, consisting of three layers of 100 m each. The hydraulic conductivity values for the layers, from the top, are: $K_1 = 10^{-3}$ m/s, $K_2 = 10^{-5}$ m/s, and $K_3 = 10^{-4}$ m/s respectively. Thus, the middle layer emulates an aquitard with $K_1 > K_2 < K_3$.

RESULTS AND DISCUSSION

The hydraulic heads obtained from NLPNT (continuous curves) as well as MODFLOW (dashed contours) are plotted as equi-potential lines in Fig. 2. The equi-potentials are spaced uniformly with 0.005m for all the plots. The dashed-horizontal lines in the plots represent the interfaces between aquifer layers. An inspection of Fig. 2 reveals that the analytical (NLPNT) and numerical (MODFLOW) solutions are in good agreement.

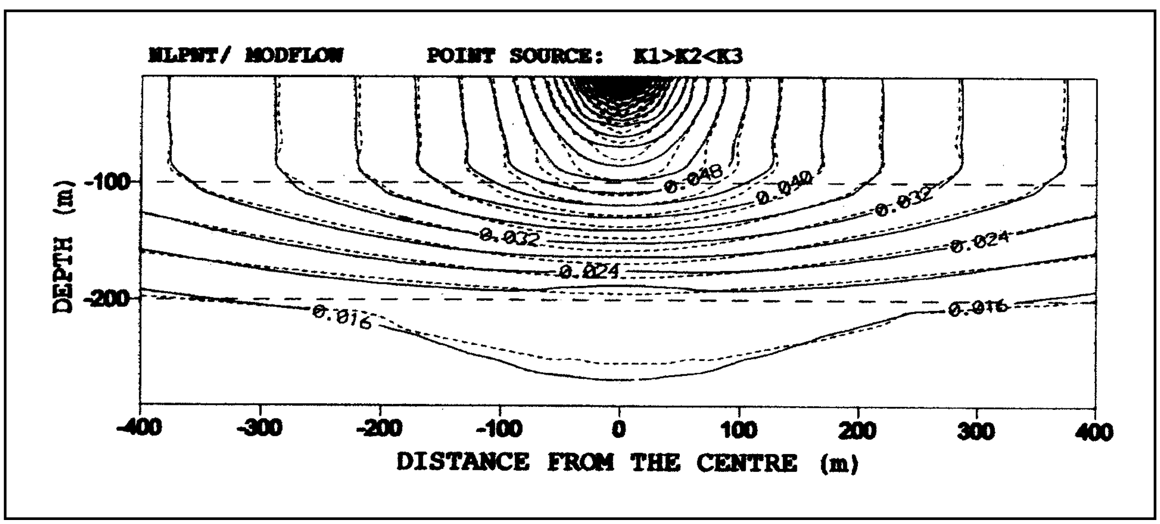


Figure 2. Equi-potential plot in the vertical section of a 3-layered aquifer system ($K_1 > K_2 < K_3$) computed by NLPNT (solid contours) and MODFLOW (dashed contours).

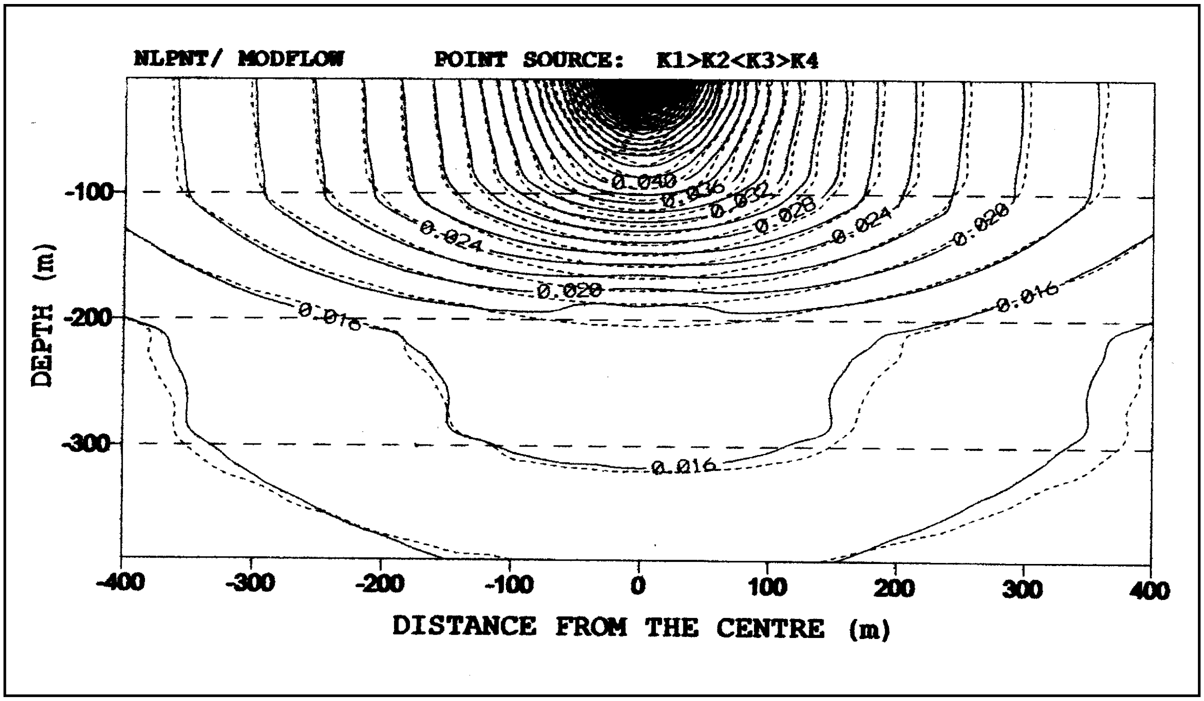


Figure 3. Equi-potential plot in the vertical section of a 4-layered aquifer system ($K_1>K_2<K_3>K_4$) computed by NLPNT (solid contours) and MODFLOW (dashed contours).

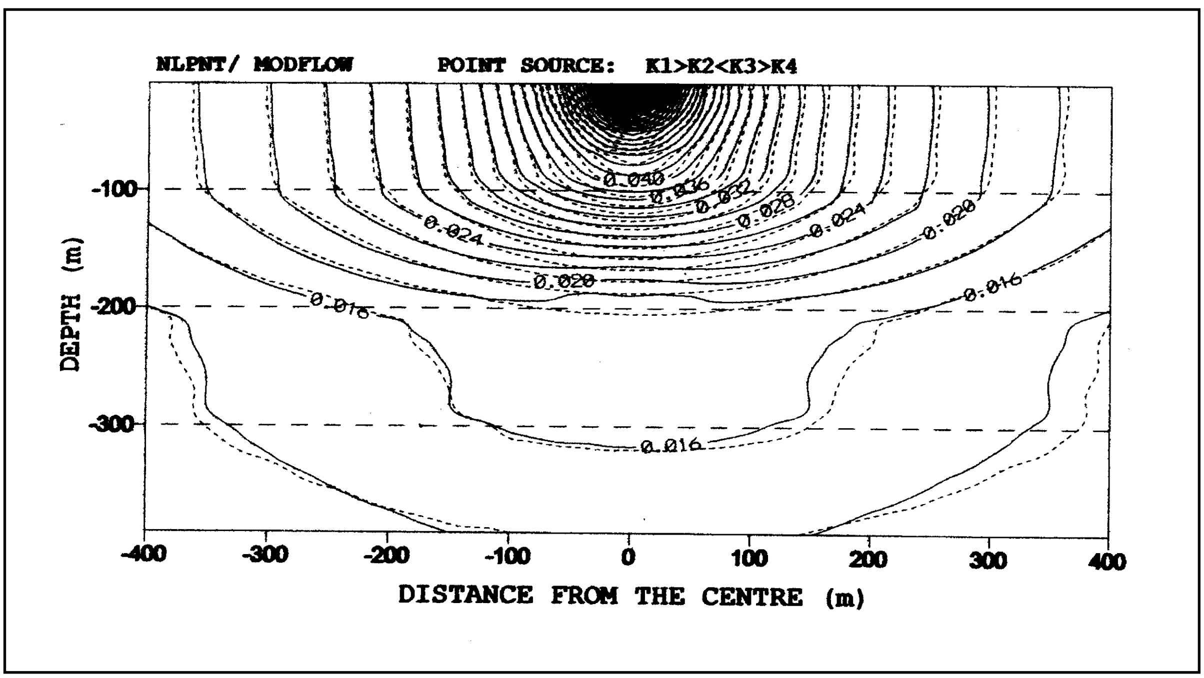


Figure 4. Equi-potential plot in the vertical section of a 5-layered aquifer system ($K_1>K_2>K_3>K_4>K_5$) computed by NLPNT (solid contours) and MODFLOW (dashed contours).

The 4-layered aquifer model is 400 m deep with two sets of hydraulic conductivity values for its layers. It has a repetitive layer design such that $K_1>K_2<K_3>K_4$, where $K_1= 10^{-3}$ m/s, $K_2= 10^{-5}$ m/s, $K_3= 10^{-3}$ m/s, and $K_4= 10^{-5}$ m/s respectively. The equi-potentials shown by continuous curves for this aquifer model are depicted in Fig. 3. Super-imposed is the corresponding equi-potentials (dashed-contours) obtained from MODFLOW. On comparison, it can be seen that the two sets of equi-potentials match very closely. The streamlines have also been computed by NLPNT.

Finally, hydraulic heads and streamlines have been computed in a 5-layered aquifer system. The aquifer depth is 500 m with each layer of 100 m thickness. The hydraulic conductivity values for the layers have been assigned in a diminishing pattern starting with the top layer (i.e. $K_1>K_2>K_3>K_4>K_5$). The numerical values for the hydraulic conductivity are: $K_1= 10^{-3}$ m/s, $K_2= 5x10^{-4}$ m/s, $K_3= 10^{-4}$ m/s, $K_4= 5x10^{-5}$ m/s, and $K_5= 10^{-5}$ m/s respectively. The analytically as well as numerically obtained equi-potentials for this aquifer model are plotted in Fig. 4. The 5-layered model results also seem to compare well with the MODFLOW results (Fig. 4). Also, the streamlines computed by NLPNT is shown in Fig. 5. The sharp deflections in the streamlines at the interfaces of the layers are due to the high contrast between conductivity values. It has been observed that the streamlines follow the usual Snell's Law (tangent law) of incidence and refraction (Hubbert, 1940), in all the cases.

The distribution of the two sets of hydraulic heads (analytical and numerical) can be compared at two sections (Fig. 6) in the vertical plane, one at the centre (point source) and the other at a distance of 200 m away from the point source. It can be seen that the difference between the two sets of computed heads is negligible.

Towards a quantitative comparison of analytical and numerical results, a *mean relative difference* and a *maximum relative difference* between the corresponding values of hydraulic heads have been estimated for the three earlier described aquifer models and presented by Fig. 7 and Fig. 8 respectively. The mean relative difference and the maximum relative difference, respectively are expressed as a percentage of an arbitrary base value, which happens to be the hydraulic head at the central node of the respective MODFLOW solution. Figure 7 indicates that the mean relative difference is less than 2% at all the points within the aquifer. Similarly the maximum extent of relative difference between NLPNT and MODFLOW solutions can be discerned from Fig. 8. Evidently, even the maximum relative difference is less than 2% in most of the aquifer domain, barring the

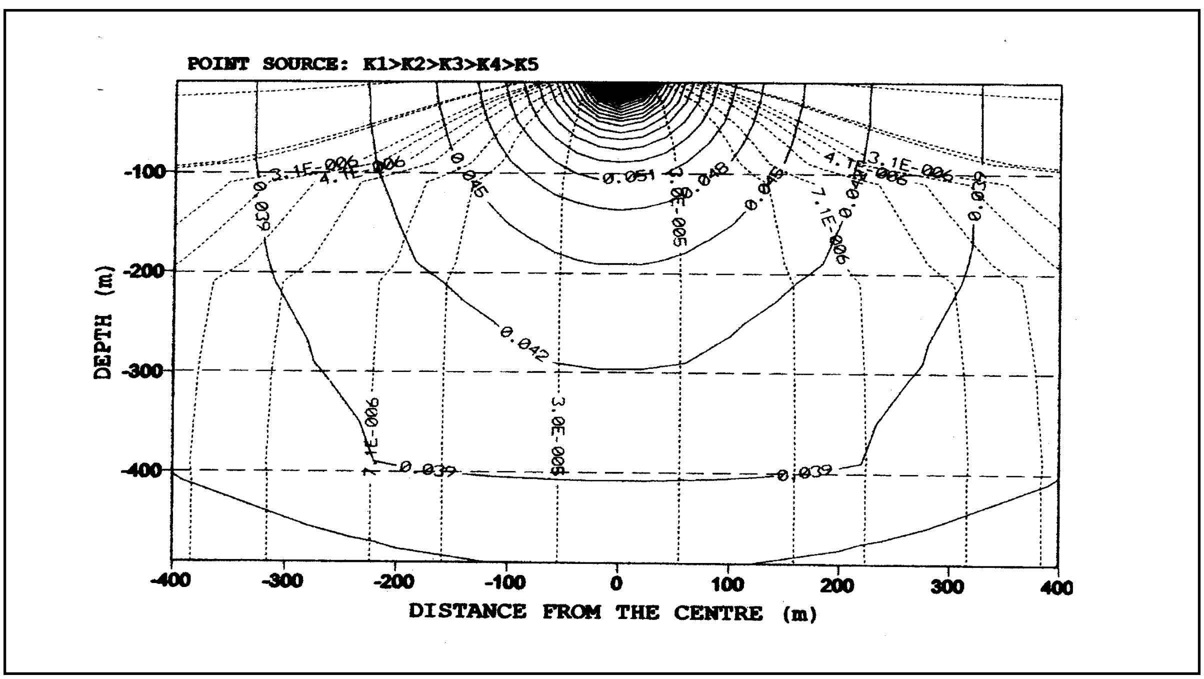


Figure 5. Equi-potential plot (solid contours) and streamlines (dashed contours) in the vertical section of a 5-layered aquifer system ($K_1>K_2>K_3>K_4>K_5$) computed by NLPNT.

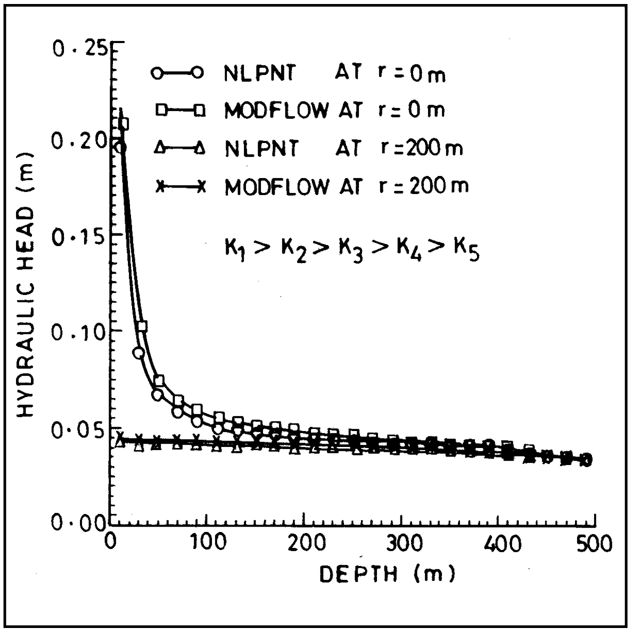


Figure 6. Vertical distribution of hydraulic head obtained from NLPNT (solid line) and MODFLOW (dashed line) at the source and at a distance 200m from the source for the 5-layered aquifer system with $K_1>K_2>K_3>K_4>K_5$.

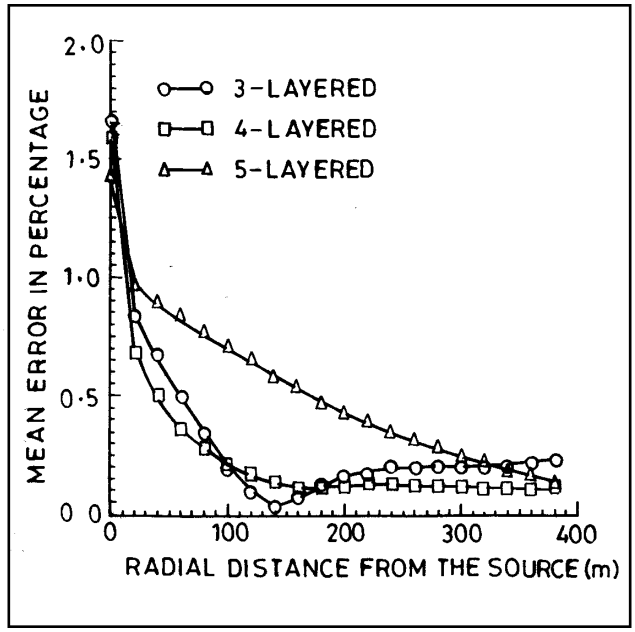


Figure 7. Mean relative difference between the hydraulic heads computed by NLPNT and MODFLOW (for the 3, 4, and 5 layered cases), expressed as a percentage of the MODFLOW-hydraulic head at the central node.

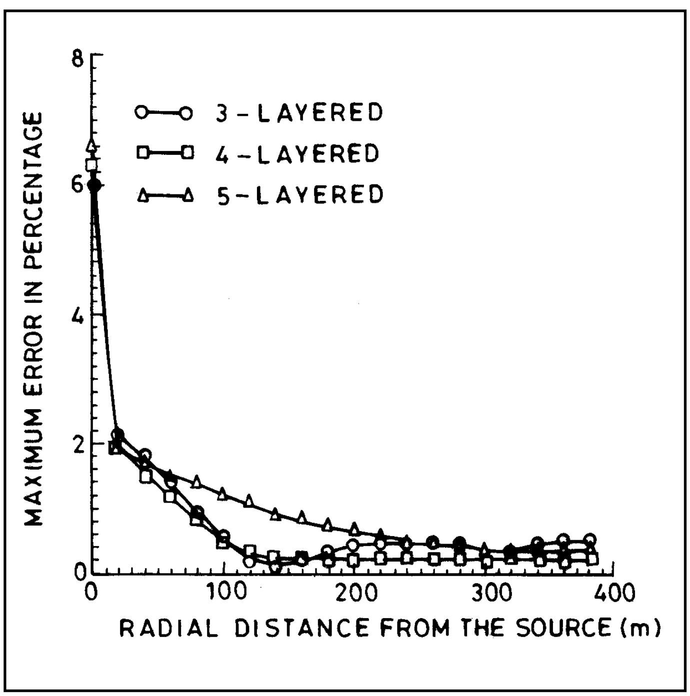


Figure 8. Maximum relative difference between the hydraulic heads computed by NLPNT and MODFLOW (for the 3, 4, and 5 layered cases), expressed as a percentage of the MODFLOW-hydraulic head at the central node.

points near the source where it is around 6%. The slightly larger deviation near the point source can be attributed to singularity effects in the solution. Further refinement of the MODFLOW boundary conditions (presently, near-zero values for hydraulic head had been arbitrarily assigned at large distances) in conjunction with the NLPNT solutions may reduce the relative difference drastically. Nevertheless, the close agreement of NLPNT results with that of MODFLOW establishes the validity of the analytical solution procedure in computing hydraulic head as well as streamlines in a multi-layered aquifer system.

CONCLUSIONS

Based on the geoelectrical sounding theory, analytical expressions for the solution of steady-state hydraulic heads and streamlines have been derived. The analytical solution assumes that the stratified aquifer is of infinite extent, and cylindrical symmetry around the point source kept on the air-earth interface exists. A semi-analytical computational procedure (NLPNT) has been developed and demonstrated for a few multi-layered aquifer models. The hydraulic heads computed by NLPNT have been compared with corresponding results from the numerical groundwater flow model, MODFLOW and they have been found to be in good agreement thereby establishing the validity of the given solution procedure.

Since the proposed procedure requires only a few input parameters, it provides a means for easier and fast computation when steady state hydraulic heads and streamlines are to be determined in a multi-layered aquifer system. An added advantage is the straight computation of streamlines using the analytical expressions, unlike many numerical flow models. Further, it is possible to extend the given solution procedure (which assumes a point source of recharge) to line-source(s) and an area-source(s).

REFERENCES

Bhattacharya, P.K. & Patra, H.P., 1968. Direct current geoelectric sounding- Principles and interpretation, Elsevier Scientific Publishing Co., Amsterdam, pp. 200.

Freeze, R.A. & Cherry, J.A., 1979. Groundwater, Prentice-Hall, Englewood Cliffs, New York, pp. 604.

Hamming, R.W., 1962. Numerical methods for scientists and engineers, McGgraw-Hill Book Company, New York.

Harr, M.E., 1962. Groundwater and Seepage, McGraw-Hill Book Company, New York, pp. 315.

Hubbert, M.K., 1940. The theory of groundwater motion, *J. Geol.*, 48(8), 785-944.

Jose, M.K., 2001. Simulation of flow in multilayered aquifer system with and without discontinuity, Ph.D Thesis, IIT, Roorkee, pp.185.

Koefoed, O., 1979. Geosounding principles- Vol.1, Elsevier Scientific Publishing Co., Amsterdam, pp. 385.

Mc Donald, M.G. & Harbaugh, A.W., 1984. A modular three dimensional finite difference groundwater flow model, USGS National Centre, Reston, Virginia, USA, pp. 528.

Morrison, N., 1969.Introduction to sequential smoothing and prediction, McGraw-Hill Book Company, New York.

Niwas,S. & Israil,M., 1986. Computation of apparent resistivities using an exponential approximation of kernel functions, *Geophysics*, 51(8), 1594-1602.

Osiensky, J.L. & Williams, R.E., 1996. A two dimensional MODFLOW numerical approximation of mise-a-la-masse electrical flow through porous media, *Groundwater*, 34(4), 727-733.

Prickett, T.A., 1975. Modelling techniques for groundwater evaluation, in *Advances in Hydrosciences*,, 10, 1-143.

Walton, W.C., 1970. Groundwater resources evaluation, McGraw-Hill, New York, pp. 664.

Watson, G.N., 1944. A treatise on the theory of Bessel functions (Second Edition), Cambridge University Press, Cambridge, pp. 804.

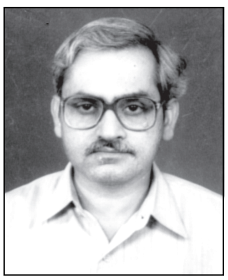
Yates, S.R., 1988. An analytical solution to saturated flow in a finite stratified aquifer, *Groundwater*, 26 (2), 199-206.

Zohdy, A.A.R., Eaton, G.P. & Mabey, D.R., 1974. Application of surface geophysics to groundwater investigations, in *Techniques of water resources investigations*, USGS, Book 2, Chap. D1.

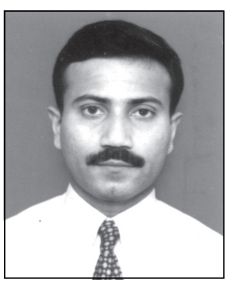
(Accepted August 5th 2005. Received July 18th 2005; in original form July 26th 2004)

NOTATIONS USED

R	Radial axis of the cylindrical coordinate system, [m]
Z	Vertical axis of the cylindrical coordinate system, [m]
(r,z)	Coordinates of any point P in the medium
n	Number of layers
$\mathbf{K_i}$	Hydraulic Conductivity of i^{th} layer, [m/s]
$\mathbf{h_i}$	Depth to i^{th} interface, [m]
q	Source strength, [m^3/s]
$\varphi_i(\mathbf{r},z)$	Hydraulic head at arbitrary point P(r,z) in i^{th} layer, [m]
$\psi_i(\mathbf{r},z)$	Stream function in the i^{th} layer of the aquifer system
$J_0(\lambda \mathbf{r})$	Bessel function of order zero
$A_i(\lambda)$, B_i(»)	Unknown functions for i^{th} layer in the equation for hydraulic head
λ	Variable of integration, [m^{-1}]



Dr. Rambhatla G. Sastry has obtained M.Sc(Tech.) degree in Applied Geophysics from Andhra University in July 1973 and Ph.D. degree in Geophysics from Moscow State University, USSR in April 1980. He has served as CSIR Pool Officer at NGRI during 1980-81 and joined the geophysics faculty of Department of Earth Sciences, University of Roorkee during May 1981. Currently he is serving as Professor in Geophysics at IIT, Roorkee. He was Commonwealth Academic Staff Fellow for the year 1989 at Geophysics Department, University of Edinburgh, Edinburgh (U.K.). His research interests include Exploration Geophysics, Geophysical Inversion and Hydrological Modelling.



Dr. Mathew K. Jose holds Post-Graduate Degrees in Hydrological Engineering from the UNESCO-International Institute for Infrastructural, Hydraulic and Environmental Engineering (UNESCO-IHE), Delft, The Netherland and also in Meteorology from the Cochin University of Science and Technology (CUSAT), Cochin, India. He obtained Doctoral Degree in Earth Sciences (groundwater modelling) from the Indian Institute of Technology (IIT) Roorkee, India. He has been a scientist at the National Institute of Hydrology (NIH), Roorkee for more than a decade, wherein he participated in a number of research studies, international projects of UNDP, World Banke etc. as well as nationally sponsored projects in the area of hydrology and water resources. He has visited a number of water resources/ hydrological institutions abroad and has many published research papers to his credit. His specialisation includes groundwater hydrology, flow and transport modelling, hydrological data processing and watershed management. Currently he is deputed to the North Eastern Regional Institute of Water and Land Management (NERIWALM), Tezpur as Associate Professor of Water Resources Engineering.