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Fractal Analysis for Geomagnetic Secular Variations

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ABSTRACT
Fractal Analysis deals with the science of complexity. Fractal analysis is more economical and most powerful method for analyzing the time series data. Fractal geometry allows the description of natural patterns by simple numbers, to facilitate their comparison and to establish and test models of pattern formation (Kruhl, 1994) The purpose of this paper is to study the geometrical complexity of geomagnetic secular variations estimated by fractal dimension, the disorder of the unpredictable secular variations estimated by Lyapunov exponent and the non linear system represented at each instant of time by a point which traces out a trajectory during this time defined by the state of variables- phase space analysis, for the available data of geomagnetic secular variations at Indian observatories. The result of the analysis confirms the previous results of secular variation anomalies at the Indian observatories. Regional inconsistencies for the declination (D) and vertical (Z) components for the Hyderabad and Sabhawala observatories are brought out in this paper. Analytical technique and the results of the analysis are discussed

INTRODUCTION

A fractal is a recently discovered kind of geometrical object and it is furthermore one which describes nature much better that Euclidean objects, which are based on regular geometrical shapes like straight line, circles, etc. As Benoit Mandelbrot said:"Clouds are not spheres, mountains are not cones, coastlines are not circles and bark is not smooth... nor does lightning travel in a straight line." Several aspects of these natural phenomena can be described much well as fractals. Many aspects of geology and geophysics are complex just as various problems in biology, economics and human behavior are complex. There are also links between important problems in such diverse areas engaging the attention of a number of scientists motivating them to propose a new science of complexity. The new paradigm of science includes fractals, chaos and self organized criticality. The studies are extremely active and it is impossible to predict with certainty what the future holds in different real life situations. Fractals provide rational means for the extrapolation and interpolation of seemingly disconnected observations. It is recognized that complex fractal dimensions lead to log-periodic behavior. It has been suggested that log-periodic behavior may lead to a viable earthquake prediction strategy (Donald Turcotte 1997). Fractal analysis is having its own advantage over the power spectrum analysis. Fractal analysis is free of the assumptions of data continuity; it can also access the asymptotic of the power spectrum directly.

Kabin & Papitashvili (1998) preferred the fractal analysis than the conventional Fourier transforms method to study the properties of the IMF and the Earth's magneto tail field. Geomagnetic Secular variations at Indian Observatories have been widely studied by many scientists. Changes in the main field are not linear in time, it is smooth, and that it often keeps the same sign over several decades (Parkinson 1983). The secular trend in the vertical component and the direction of the migration of dip equator in the Indian zone are consistent (Rangarajan & Deka 1991). A strong correlation between the southern observatories and sudden change in characteristics for the low-to mid latitude observatories of Alibag and Sabhawala of secular variations have been found (Nandini Nagarajan 1992). The secular changes in the dip equator are determined by the eccentric dipole approximation of the earth's main field (Rangarajan 1994). A comparison between the observed annual means and IGRF models reveals very low secular variations anomaly in the Indian region (Bhardwaj & Rangarajan 1997). Field disturbance measured by observatories plays an important role in modeling the main field. The secular changes and non-dipole pattern movements are used by the paleo magneticians to help to define the process within the liquid outer core and core-mantle boundary where the Earth's main field characteristics are generated (Campbell 1997). As confirmed by IGRF, a chaotic geomagnetic field implies the impossibility of predicting its behavior over an interval longer than a few years. Prediction of observatory data for global models in a phase space with at least E=3(embedding dimension) instead of simple extrapolation in time would greatly improve the global model. Instead of producing the smoothest possible model, the fractal property which is present for a longer period of time should be taken into account. The data of the observatories, Alibag, Hyderabad, Kodaikanal, Sabhawala and Trivandrum are used for this analysis. Fractal dimensions are calculated by the method followed by Kabin & Papitashvilli (1998). The result of the analysis confirms the low dimensional chaotic (Donald Turcotte 1997) behavior of geomagnetic secular variations at the Indian region.

DATA ANALYSIS

Annual mean values of the geomagnetic field components horizontal (H), vertical (Z) and declination (D) of the Earth's magnetic field published in the Indian Magnetic Data for the period from 1960 to 1999 have been used for this analysis. For Trivandrum Observatory, the recoding of data stopped from October 1999 as the station was wound up and hence no data afterwards. For Kodaikanal Observatory, there was no data after 1997 due to nonfunctioning of Z variometer. Though the effect of geomagnetic storm time variations (Sridharan & Ramasamy 2002) is likely to be present in this data, the secular trend for quiet and disturbed days are not different (Bharadwaj & Rangarajan 1997) For the Hyderabad observatory, the data is available from 1966 and for Sabhawala, the data is available from 1964 onwards. The locations of the observatories are given in Table 1.

Table 1. Locations of Observatories

Station	Geographic		Dipole
1. Alibag	Latitude Longitude	18° 37′N 72° 52′E	9.7° N 145.6°
2. Hyderabad	Latitude Longitude	17° 25′N 78° 33′E	7.9°N 148.9°
3. Kodaikanal	Latitude Longitude	10° 14′N 77° 28′E	0.9°N 149.1°
4. Sabhawala	Latitude Longitude	30° 22′N 77° 48′E	20.9°N 151.5°
5. Trivandrum	Latitude Longitude	08° 29′N 76° 58′E	0.8°S 148.5°

Fractal Dimension

A mathematical fractal is defined as any series for which the Hausdorff dimension exceeds the discrete topological dimension (Drazin 1992) To each subset of the Euclidean space $\Re^m$ there is assigned a topological dimension d which is an integer satisfying the property $0 \leq d \leq m$. Mandelbrot & Van Ness (1968) defined a fractal as a set with dimension D and topological dimension d satisfying $d<D$. In fractal geometry a point is defined as a dimension of '0', a straight line is defined as a dimension of '1' and a plane is defined as a dimension of '2', etc. For H component the dimension is almost '1' for all the stations and that is not the case for D and Z components.

We have implemented the technique developed by Higuchi (1988) and followed by Kabin & Papitashvilli (1998).

If we have the observational time series of values $X(t_i)$ where time intervals are supposed to be equal, then the increments of X can be defined as

Table 2. Fractal dimensions for secular variations

Station	D-Component	H-Component	Z-Component
Alibag	1.20058	1.01014	1.05207
Hydereabad	0.77211	1.06765	1.08524
Kodaikanal	0.93063	1.0352	1.06363
Trivandrum	1.09439	1.01412	1.07855
Sabhawala	1.16774	1.05312	0.54655

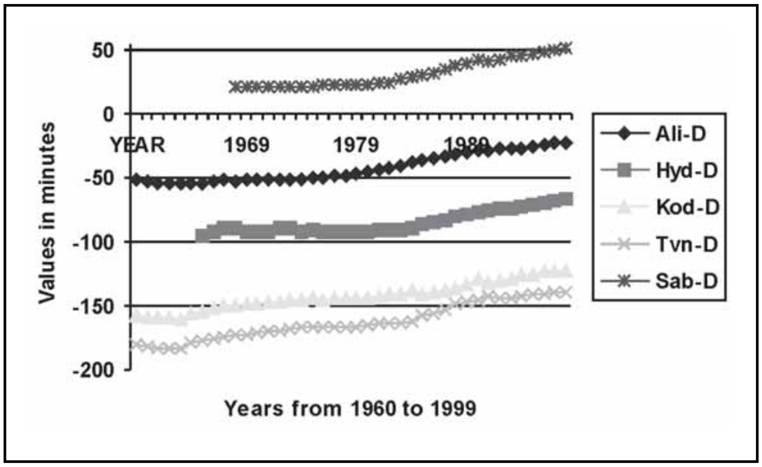

Figure 1. Secular variations of D from 1960 to 1999

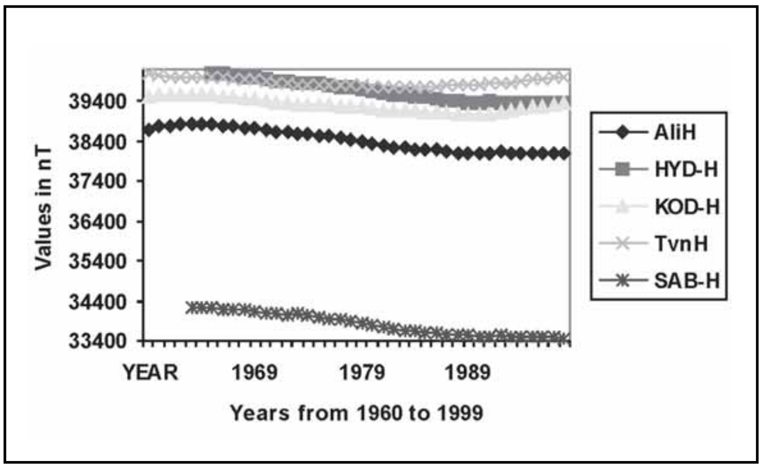

Figure 2. Secular variations of H from 1960 to 1999

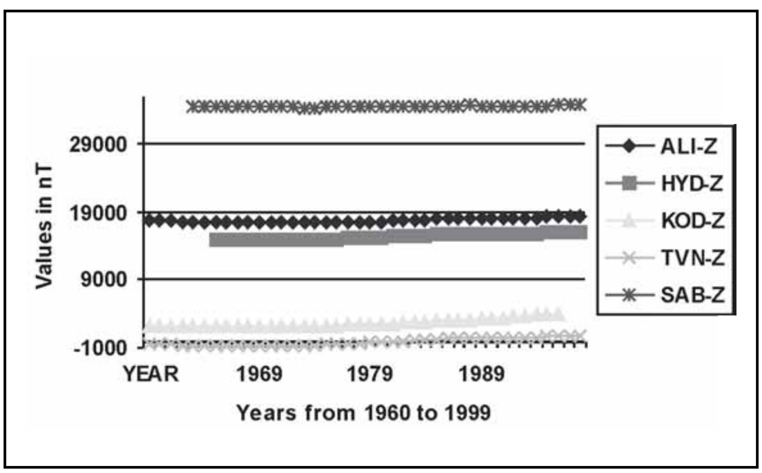

Figure 3. Secular variations of Z from 1960 to 1999

$\Delta(t_j - t_i) = X(t_j) - X(t_i)$, $j>i$. The non-normalized apparent length of the time series curve is defined as: $L_k = \Sigma i |\Delta(t_{i+k} - t_i)|$.

If X (t) is a fractal function, then the graph ln (L_k) versus ln (k) should be a straight line with a slope 1-d ('d' is the fractal dimension) for small enough $\Delta_k t$. For large values of $\Delta_k t$, the graph ln (L_k) versus ln (k) can deviate from the straight line approximation because in this case it is derived for just a few data points chosen from the entire data set. In this range there is not enough statistical information to approximate the fractal properties of the curve.

Accordingly the fractal dimensions of the geomagnetic secular variations of D, H and Z for the five stations are calculated and their values are given in the Table 2 and Fig. 4. Fractal analysis does not suffer from discontinuity of data. If one or two data points are deviating or missing, that will not affect the result, as the slope of the straight line will not change much, when the logarithm values of the total variations are considered for the graph. Consequently, the indication of somewhat unnatural deviations from the smooth secular trends will not cause significant difference from the expected values.

Lyapunov Exponent

Lyapunov exponent may be interpreted in terms of information theory as giving the rate of loss of information about the location of the initial point x_0 for measuring the disorder of the system. The usual test for chaos is the calculation of the largest Lyapunov exponent. A positive largest Lyapunov exponent indicates the presence of chaos. The Lyaponuv exponent is defined (Drazin 1992) as follows:

Consider a continuously differential map F: R→R and suppose that there exists λ such that

$$|F^n(x_o+\varepsilon) - F^n(x_o)| \sim \varepsilon e^{n\lambda} \text{ as } \varepsilon\to 0, n\to\infty \text{ provided } \varepsilon e^{n\lambda}\to 0$$

$$\text{i.e., } \quad \varepsilon \left| \frac{dF^n}{dx_o}(x_o) \right| \sim \varepsilon e^{n\lambda} \quad \text{as } n\to\infty$$

to express the average exponential separation of the orbit starting at $x_o+\varepsilon$ from the orbit starting at x_o.

Therefore,

$$\lambda = \lim_{N\to\infty} \left\{ \frac{1}{N} \ln \left| \frac{dF^n}{dx_o}(x_o) \right| \right\}$$

$$\lambda = \lim_{N\to\infty} \left\{ N^{-1} \ln | F'(X_{N-1}) F'(X_{N-2}) \ldots\ldots F'(X_0) | \right\}$$

$$\lambda = \lim_{N\to\infty} \left\{ \frac{1}{N} \sum_{n=0}^{N-1} \ln | F'(Xn) | \right\} \text{ where } x_n = F^n(x_0)$$

This shows that λ is a measure of the exponential separation of the neighboring orbits averaged over all points of an orbit around an attractor.

Accordingly, the Lyapunov exponent for the variations of the first order difference for the elements D, H and Z have been determined and the values are furnished Table 3.

Table 3. Lyapunov Exponent for Secular variations

Station	D-Component	H-Component	Z-Component
Alibag	-0.13358	1.25599	1.342067
Hyderabad	0.079871	1.276273	1.488319
Kodaikanal	0.009777	1.411562	1.544632
Trivandrum	0.010177	1.240287	1.473539
Sabhawala	0.105016	1.164021	0.105016

Phase space Analysis

Phase space is conceived as a coordinate space defined by the state of variables of dynamical system (Donald Turcotte 1997). In experiments, we usually do not have the luxury of working with the actual vectors of phase space variables. Normally only the time series of a single variable is available to characterize the behavior of each system. Therefore to be able to analyze experimental as well as numerical data one has to rely upon the phase space reconstruction method (Abarbanel 1996)

A dynamical system is one whose state changes with time (t). Two main types of dynamical system are: (i) the one in which the time variable is discrete (t∈ Z or N) and (ii) one with a continuous time variable (t∈R). A discrete dynamical system can be represented as the iteration of a function, i.e.,

$$X_{t+1} = f(X_t), \; t\in Z \text{ or } N$$

When t is continuous, the dynamics is usually described by a differential equation, dx/dt = x′ = X(x), where x represents the state of the system (Arrowsmith & Place 1991) and the values assumed by x give a geometrical description of the solutions in the phase space. If the scattered final states exhibit sensitive dependence on the incident condition, then the process is called chaotic (Kruhl 1994). Accordingly, the time delay embedding technique is used here to carry out the phase space analysis. The idea of time delay embedding technique is as follows:

Let us suppose that the given time series pertains to the measurement of a variable x (t) among the describing system. The first step in the embedding technique is to construct an m-component delay or state vector X_i at time $t=t_i$ as

Table 4. Correlation coefficient of D, H and Z components for phase space reconstructions

Station	D-Component			H-Component			Z-Component		
	t_1	t_2	t_3	t_1	t_2	t_3	t_1	t_2	t_3
Alibag	0.9974	0.9930	0.9877	0.9962	0.9883	0.9772	0.9919	0.9775	0.9592
Hyderabad	0.9868	0.9698	0.9589	0.9983	0.9950	0.9897	0.9963	0.9866	0.9728
Kodaikanal	0.9910	0.9821	0.9735	0.9906	0.9666	0.9259	0.9969	0.9929	0.9839
Trivandrum	0.9941	0.9850	0.9752	0.9769	0.9285	0.8535	0.9967	0.9885	0.9768
Sabhawala	0.9944	0.9879	0.9789	0.9973	0.9928	0.9870	0.9069	0.7939	0.6949

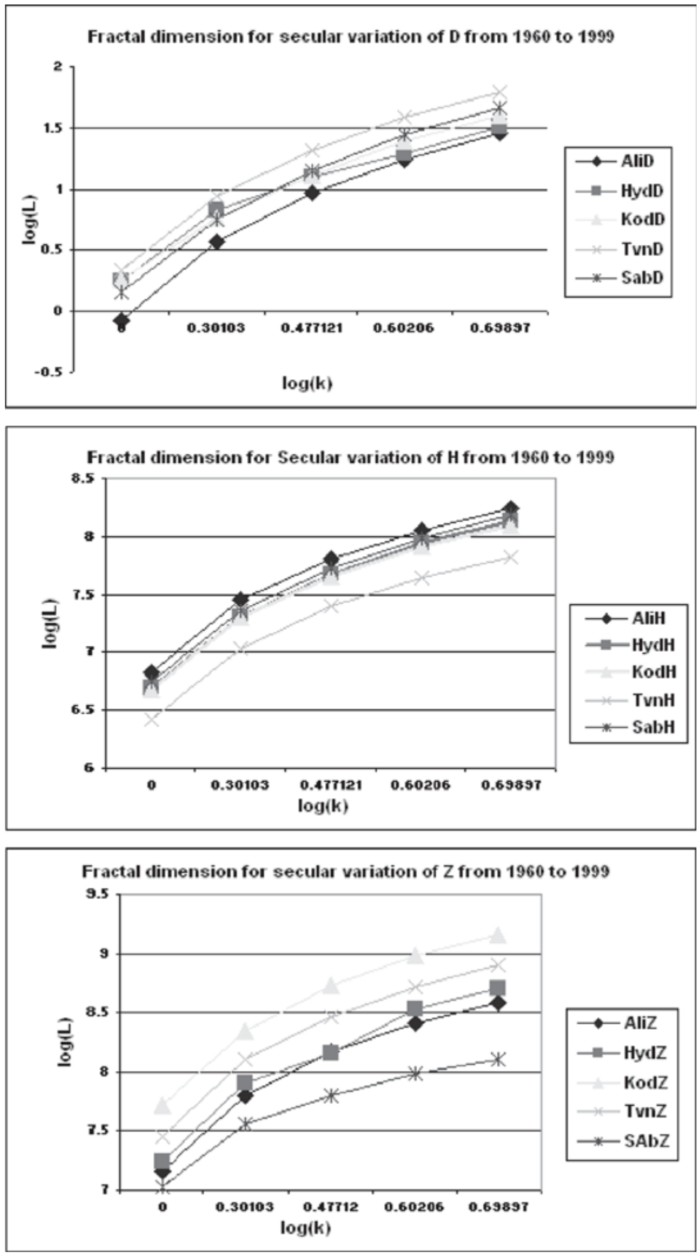

Figure 4. Fractal dimension of D, H and Z elements of secular variations from 1960 to 1999

$X_i = [\ x_1(t_i),\ _{X2}(t_i),\ \ldots\ldots\ldots\ldots\ x_m(t_i)]$, with $x_k(\ t_i\ ) = x\ (\ t_i + (k\text{-}1)\tau\ )$ where τ is an appropriate time delay. The time delay t should be so chosen that it is small enough to resolve the physical process of interest. Now the m-dimensional reconstructed phase portrait will have the same properties of Lyapunov exponent and fractal dimension as one constructed from the measurement of N independent variables. The points in the phase space are chaotic in the sense that two nearby points diverge rapidly with time before escaping to infinity (Lakhina 1994) The figures 5, 6 and 7 show the phase space reconstructions of the geomagnetic secular variations for the values of t =1, t =2 and t =3 (years). The corresponding correlation coefficients are furnished in Table 4.

RESULTS AND DISCUSSION

Fractal dimensions, Lyapunov exponents and correlation coefficients of phase space reconstructions are provided in Tables 2, 3 and 4 respectively. Secular variations of D, H and Z during this period are plotted and presented in the figs 1, 2 and 3. The graphs for the fractal dimensions and the correlation of phase space reconstructions are given in the figs 4, 5, 6 and 7. Comparing the values of fractal dimensions, Lyapunov exponent and phase space reconstructions, we infer the following:

The fractal dimension of the declination D component for the stations Alibag (d=1.20058) and Hyderabad (d=0.77211) shows great difference though geographically both the observatories are very close to each other. Secular changes can result from: (a) change in the magnitude of the principal (dipole source) current within the earth (b) motion of that current, causing a shift in the alignment of the dipole axis (the dipole north pole is now moving about 18 km northward and 5 km westward each year) and (c) change in the westward drifting, non dipole parts of the main field representation. The various harmonic components in the field model seem to be moving at different rates. The results that (i) there is a general agreement between the change in the Earth's spin (excess length of a day) and the secular change in declination from 1860 to about 1975 and (ii) one millisecond per day change in time matches with about one minute angular change in the D component per year have been established (Campbell 1997). The difference between the Alibag and Hyderabad observatories in respect of the declination component is attributed to the combination of fossil magnetization in the Decean traps beneath Alibag and deep-seated volcano-tectonic process nearby as evidenced by seismic activity and hot springs in the vicinity (Srivastava & Abbas 1977). The positive value of Lyapunov exponent indicates the presence of chaos. Negative value of Lyapunov exponent indicates that the Declination is linear in nature at the Alibag observatory. The present analysis with extended data indicates the persistence of the difference in D between Alibag and Hyderabad found by Srivastava & Abbas but the secular trends for H and Z are very similar.

The fractal dimension of the horizontal H component for all the stations shows almost thesame value which indicates very low secular variations anomaly in the Indian regions. This again confirms the earlier results of Bhardwaj & Rangarajan (1997)

The low fractal dimension (d=0.54655) for the vertical Z component of the Sabhawala observatory which is far away from the sea and not influenced by geomagnetic coastal effects (Sridharan, Gururajan & Ramasamy 2005) is attributed to the influence of a deep-seated two-dimensional east-west current flow south of Sabhawala, the tectonic history of the region, especially the evolution of the Himalayas and the character of the subsurface geology. Also this anomaly is due to a sedimentary trough running parallel to the Himalayas studied by Nityananda, Agarwal & Singh (1981)

Lyapunov exponents of the first order difference for D, H and Z components listed in Table 3 shows the positive values for all the stations except for the D component in Alibag observatory where the declination component shows a negative value of 0.13358. Positive value for Lypunov exponent denotes the divergence of the neighboring trajectories (Lakhina 1994), and thus confirms the chaotic behavior of the geomagnetic secular variations. The positive value of Lyapunov exponent enables one to identify chaotic nature of the data. Negative value of Lyapunov exponent at Alibag observatory indicates that the Declinations are not varying much compared to the variations of declination with the other observatories. For phase space analysis, the increase in values of t is an indication of divergence. The scattering positions of two nearby orbits indicate the chaotic behavior of a dynamical system (Lakhina 1994). As seen in the figs 5, 6 and 7, the divergence in the scattering plot reveals the low dimensional chaotic behavior of geomagnetic secular variations. Figures 5, 6 and 7 are graphs of the time delay variations for the individual observatories and hence the labels are according to the annual mean values of the observatories. The correlation coefficients are found for the consecutive time delay variations. The high values of correlation coefficients show that the secular variations are not varying rapidly during small intervals of 1 to 3 years. Also the increase in the values of t corresponds to the

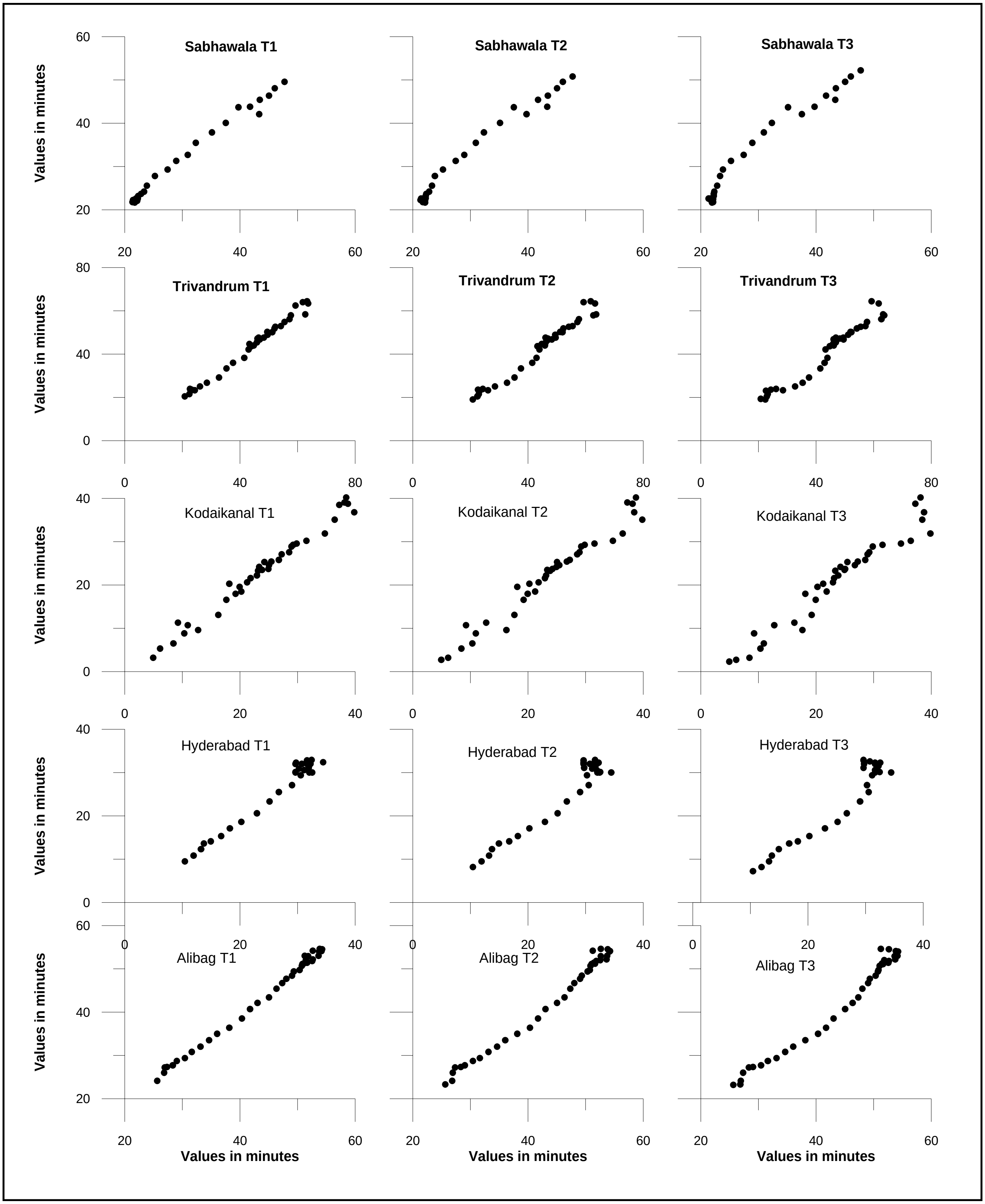

Figure 5. Phase space for Secular variations of D from 1960 to 1999

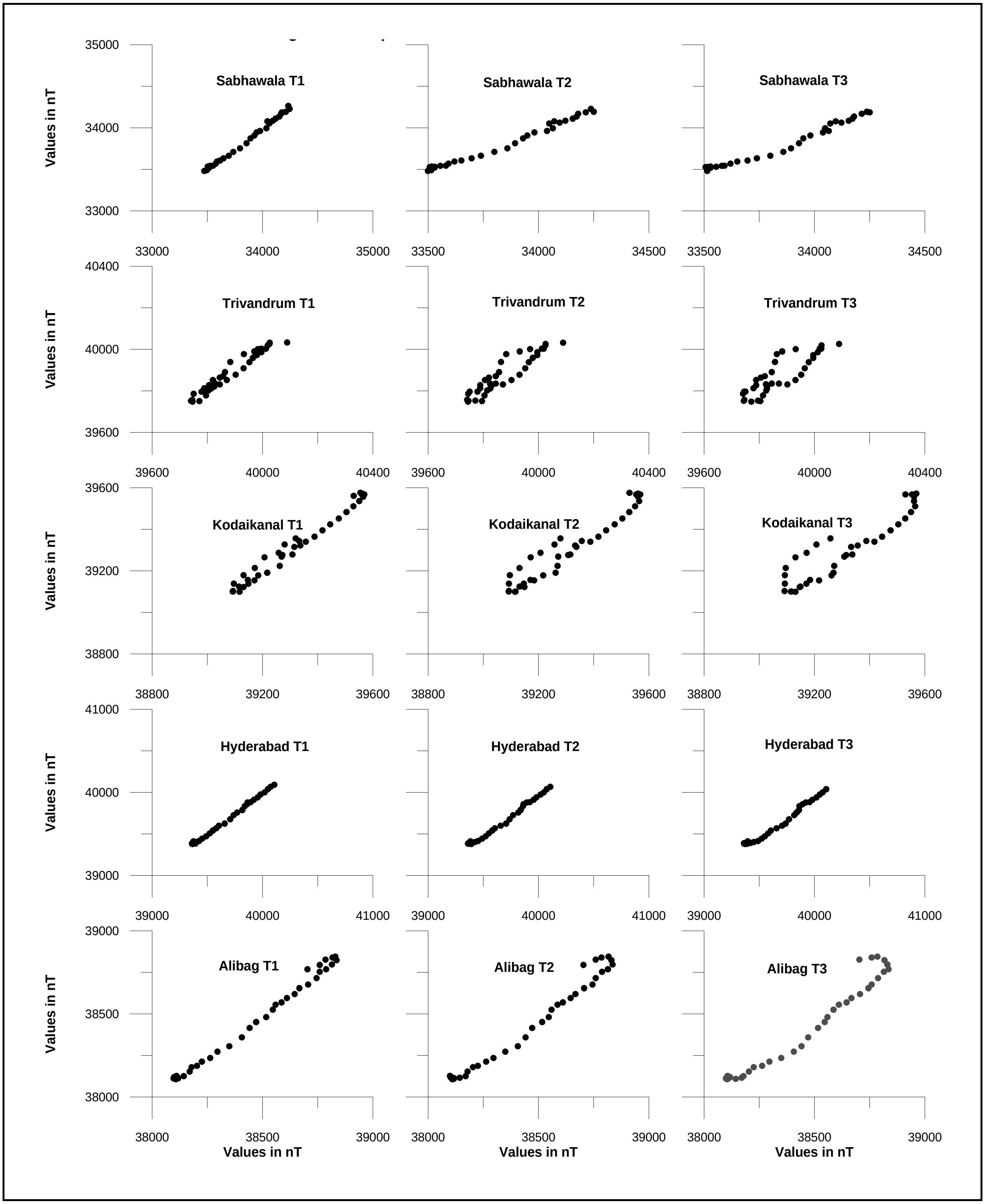

Figure 6. Phase space for Secular variations of H from 1960 to 1999

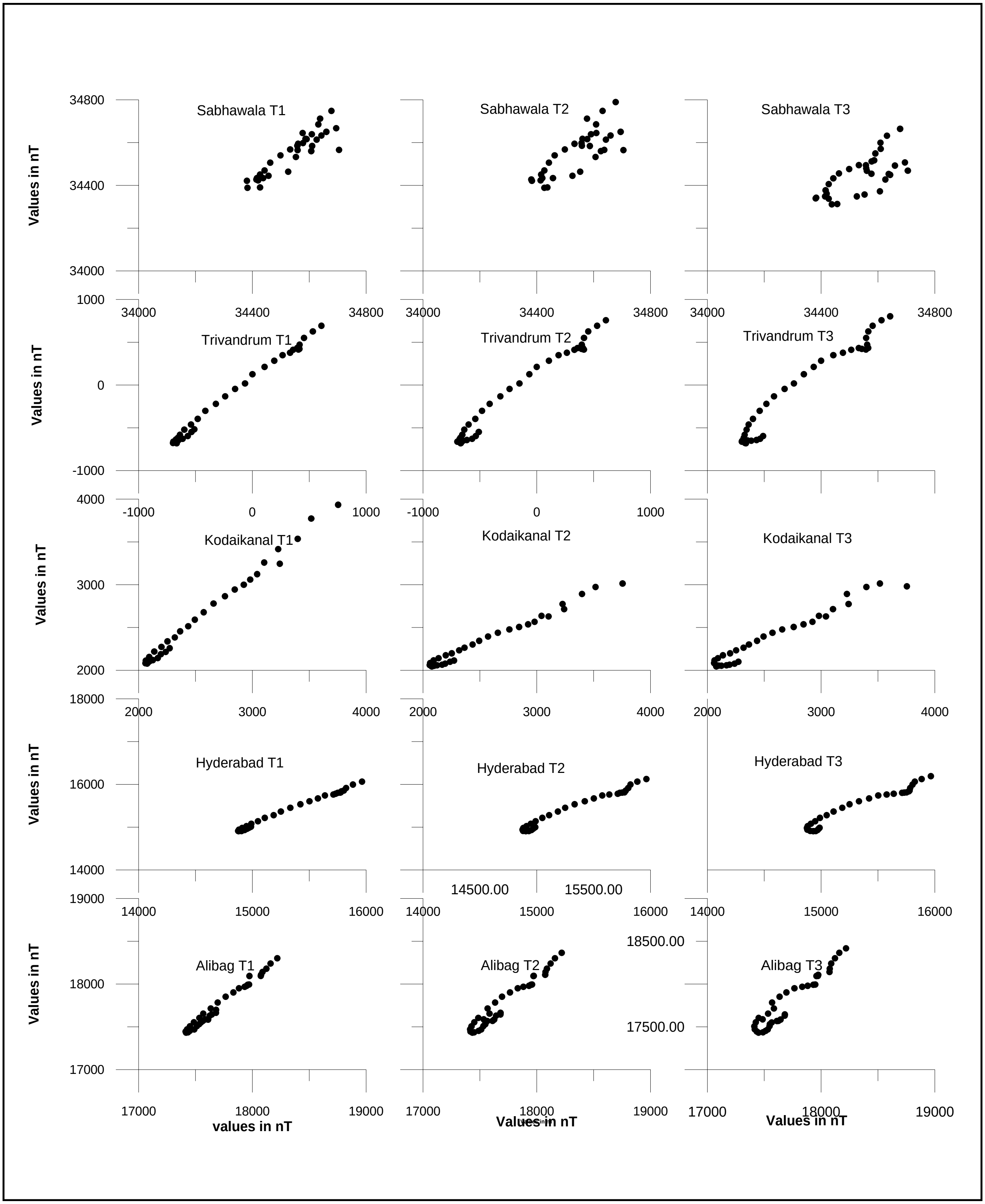

Figure 7. Phase space for Secular variations of Z from 1960 to 1999

decrease in the correlation coefficient. This agrees with the previous results of Nandini Nagarajan (1992)

A feature common to the chaotic systems is that as some external parameter is varied, the dynamical behavior of the system changes. One aim of chaos theory is to describe transitions from simple to complicated motion from a universal point of view (Warden 1993). Though there is very low secular variations anomaly at the Indian geomagnetic observatories, the existence of regional inconsistencies of Hyderabad and Sabhawala observatories for the D and Z components are confirmed in the present study.

CONCLUSIONS

The studies on fractal properties of the time series data of geomagnetic variations at different observatories will provide information on the topological behavior, which may lead to significant theories as well as applications. The recognition that the dynamics of a system can be measured by fractal analysis provides a new exciting and rigorous framework to understand and to predict its pattern. So far there is no clearly developed theory to testify what pre requirements and conditions would be necessary for fractal calculation. Hence we avoid further comments on this topic leaving room for further discussion as there is a lack of scientific demonstration as to how many statistical samples are enough for fractal analysis in respect of geomagnetic secular variations. As a result of this study it is expected that the ideas and methodologies explained in this paper may be very useful for the observatory data analysis towards exploring some new results in Geomagnetism.

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