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Analytical computation of Hydraulic Potentials due to Point, Line and Areal Sources over three - Layered Aquifer System

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ABSTRACT

Steady state analytical expressions for hydraulic potentials and streamlines of a three-layered aquifer system due to a point source recharging have been derived. By applying appropriate convolution techniques, respective expressions for a finite-length line and areal sources are arrived at. Then, the computational algorithms (i) 3LPNT- for a point source, (ii) 3LLIN- for a finite length line source and (iii) 3LARL- for an areal sourc have been designed.

Our simulation results for these three different sources, when compared with those by standard numerical groundwater flow model, MODFLOW indicate the cost-effectiveness of our approach both in terms of accuracy and computational speed.

INTRODUCTION

Analytical methods are handy compared to numerical techniques when a steady state solution of hydraulic potential or stream function is sought for an aquifer system.

The analogy between electrical flow and groundwater flow is well established (Hubbert, 1940; Freeze & Cherry, 1979). Discussions of direct current electrical resistivity methods and electrical analogue models in groundwater applications are available in the literature (Walton, 1970; Zohdy, Eaton & Mabey, 1974; Prickett, 1975). Further, groundwater flow models have also been used in simulating d.c electrical current flow through porous media (Osiensky & Williams, 1996; Pujari, 1998). Analytical description of flow to a point sink in a perfectly layered subsurface is reported in Nieuwenhuizen et al.(1995).

Analytical solution for electrical potentials on the surface of a stratified earth medium exists in the geophysical literature (Bhattacharyya & Patra, 1968;Koefoed, 1979; Patra & Nath, 1999). However, for geohydrological applications the hydraulic potential and flow need to be determined in the various layers below the earth-surface, and for which analytical solutions are non-existent.

In the present study, analytical expressions for steady state hydraulic potential and streamlines due to a point-source have been derived for a three-layered aquifer system. Since Laplace's equation, the governing differential equation for steady groundwater flow (Fitts, 1991), is a linear partial differential equation, it supports the law of superposition for its solutions. So, the analytical expressions developed for a point source are utilised to generate appropriate semi-analytical expressions involving convolutions in computing the hydraulic potential and streamline distributions within a 3-layered aquifer system for finite -length line source (river, canal etc.) and areal sources (lake, reservoir etc.) respectively. The developed algorithms are tested in a series of numerical experiments, which validate our approach.

THEORY

Consider a horizontally stratified aquifer system with n layers in a cylindrical coordinate frame, (R, Z) with its origin at the point source, A [Fig. 1]. Let K_1, K_2, . . . , K_n be the hydraulic conductivities, and h_1, h_2, . . . , h_n be the depths to the bottom of respective layers [Fig. 1] from the surface. It is assumed that the last layer extends to infinite depth (half-space). Then, the Laplace's equation to be satisfied at any point in layer

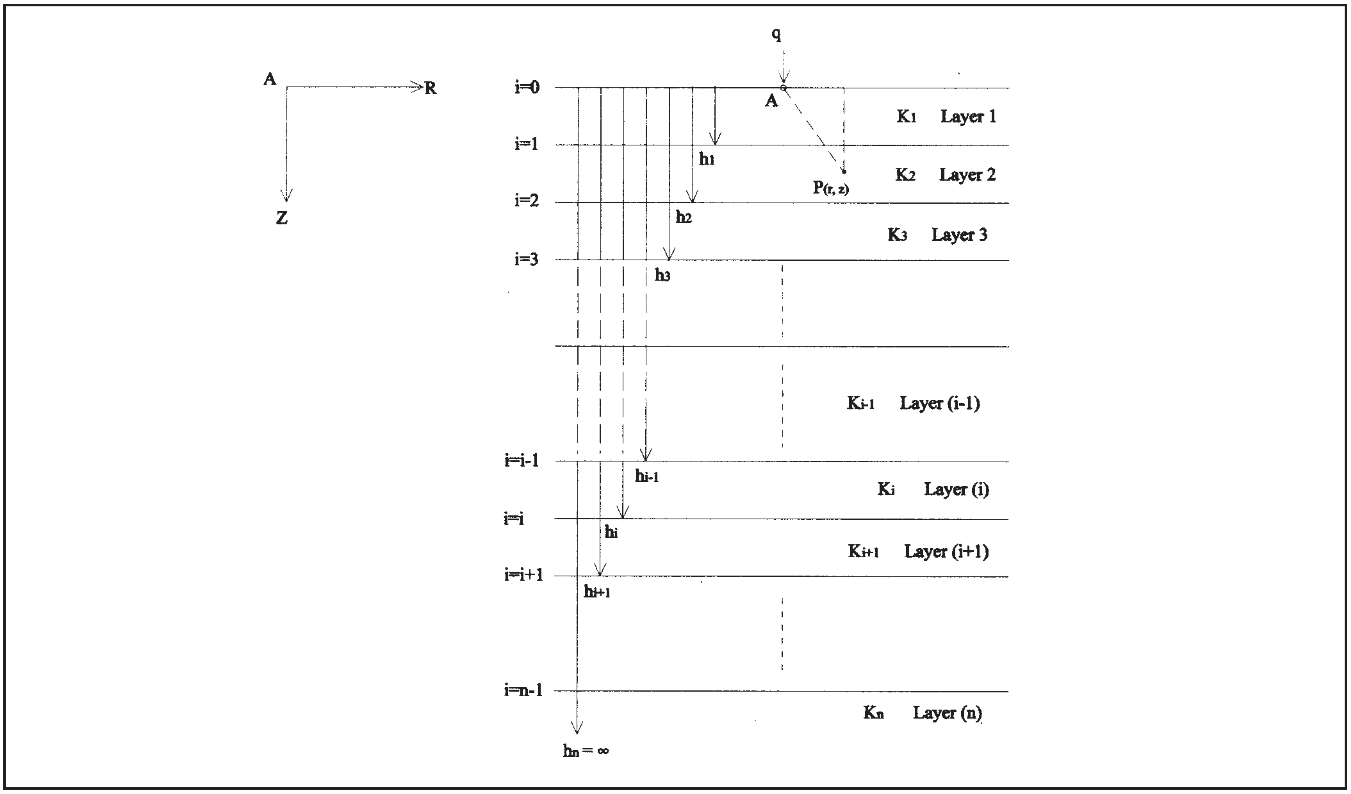

Figure 1. Schematic diagram of the multilayered aquifer system with reference to the cylindrical coordinate system. A point-source is located at A; Hydraulic potential (ϕ_i) and stream function (ψ_i) are computed at any arbitrary point P(r,z) in the aquifer system.

i (for i = 1, 2, . . . , n) by the steady state hydraulic potential ϕ_i (r, z), notated as ϕ_i for brevity, is:

$$\frac{\partial^2 \phi_i}{\partial r^2} + \frac{1}{r}\frac{\partial \phi_i}{\partial r} + \frac{\partial^2 \phi_i}{\partial z^2} = 0 \tag{1}$$

The general solution (Bhattacharyya & Patra, 1968) of eqn. (1) can be written as:

$$\phi_i(r,z) = \frac{q}{2\pi K_1}\left\{\frac{1}{(r^2+z^2)^{1/2}} + \int_0^\infty \left[A_i(\lambda)e^{-\lambda z} + B_i(\lambda)e^{\lambda z}\right]J_0(\lambda r)\,d\lambda\right\} \tag{2}$$

where q (m^3/s) is the recharge by the point source; $A_i(\lambda)$ and $B_i(\lambda)$ are rational functions in which numerator and denominator are polynomials in $e^{-2\lambda}$; and $J_0(\lambda r)$ is the Bessel function of zeroeth order. The variable of integration λ [L^{-1}] ranges from 0 to ∞.

Invoking the Integral of Lipschitz from the theory of Bessel functions (Watson, 1980), viz.,

$$\int_0^\infty e^{-\lambda z}\, J_0(\lambda r)\,d\lambda = \frac{1}{(r^2+z^2)^{1/2}} \tag{3}$$

eqn. (2) may be re-written as:

$$\phi_i(r,z) = \frac{q}{2\pi K_1}\left\{\int_0^\infty \left[e^{-\lambda z} + A_i(\lambda)e^{-\lambda z} + B_i(\lambda)e^{\lambda z}\right] J_0(\lambda r)\,d\lambda\right\} \tag{4}$$

Now, the unknown functions $A_i(\lambda)$ and $B_i(\lambda)$ in eqn. (4) need to be evaluated subject to certain boundary conditions (Koefoed, 1979) given below:

(i) At the air-earth interface (earth-surface) the vertical component of flow must be zero; i.e.,

$$K_1 \frac{\partial \phi_1}{\partial z}\Big|_{z=0} = 0 \tag{5a}$$

This implies that, in eqn. (4) the functions $A_1(\lambda)$ and $B_1(\lambda)$ must be identical so that,

$$A_1(\lambda) = B_1(\lambda) \tag{5b}$$

It can be noted, as a corollary, that the surface potential, ϕ_1 (r, z = 0) can be obtained by substitution of eqn. (5b) into (4) as:

$$\phi_1(r) = \frac{q}{2\pi K_1}\int_0^\infty P_1(\lambda)\, J_0(\lambda r)\,d\lambda \tag{6}$$

where,

$$P_1(\lambda) = 1 + 2A_1(\lambda) \tag{7}$$

$P_1(\lambda)$ is the kernel function determined by thicknesses and hydraulic conductivity of the layers. A solution for the surface potential due to eqn. (6) is often employed in geophysical literature (Bhattacharyya & Patra, 1968;Koefoed, 1979; Patra & Nath, 1999).

(ii) At each of the boundary planes in the subsurface, the hydraulic potential as well as flow must be continuous. So, at any i[th] interface within the medium,

$$\phi_i = \phi_{i+1} \quad and \quad K_i \frac{\partial \phi_i}{\partial z} = K_{i+1} \frac{\partial \phi_{i+1}}{\partial z} \tag{8a}$$

Applying these conditions to eqn. (4), we obtain the following identities:

$$A_i(\lambda)\, e^{-\lambda h_i} + B_i(\lambda)\, e^{\lambda h_i} = A_{i+1}(\lambda)\, e^{-\lambda h_i} + B_{i+1}(\lambda)\, e^{\lambda h_i} \tag{8b}$$

$$K_i[1 + A_i(\lambda)\, e^{-\lambda h_i} - B_i(\lambda)\, e^{\lambda h_i}] = K_{i+1}[\{1 + A_{i+1}(\lambda)\}\, e^{-\lambda h_i} - B_{i+1}(\lambda)\, e^{\lambda h_i}] \tag{8c}$$

(iii) At infinite depth/ distance, the hydraulic potential must approximate to zero, i.e.,

$$\lim_{z \to \infty} \phi(r,z) = 0 \quad and \quad \lim_{r \to \infty} \phi(r,z) = 0 \tag{9a}$$

Equation (3.9a) requires $B_n(\lambda)\, e^{\lambda z}$ to vanish in eqn. (4). Therefore,

$$B_n(\lambda) = 0 \tag{9b}$$

HYDRAULIC POTENTIAL AND STREAM FUNCTION: THREE-LAYERED AQUIFER SYSTEMS

As mentioned earlier, analytical solution for surface (electrical) potential for a stratified earth medium (Bhattacharyya & Patra, 1968) is available in the literature. However, for groundwater analyses the analytical expression of hydraulic potentials within the porous medium needs to be derived as it is non-existent. Hydrogeological investigations indicate that aquifer systems with single, two or three layers are most common. Therefore, the theoretical analysis developed here is restricted to three-layered media [Fig. 2].

An alternate solution technique has been presented to compute the hydraulic potentials and streamlines in a three-layered aquifer system. Suitable convolution algorithms have also been developed to extend the analytical solutions for the point source to the case of a line-source and an areal-source.

By using the equations (5b), (8b), (8c) and (9b), a system of *2n* equations in *2n* unknown functions $A_i(\lambda)$ and $B_i(\lambda)$ can be formed. By introducing the following notations,

$$u_i = e^{-\lambda h_i}$$

$$v_i = e^{+\lambda h_i} \tag{10}$$

$$t_i = K_{i+1}/K_i$$

the corresponding system of equations in $A_i(\lambda)$ and $B_i(\lambda)$ can be written as (Koefoed, 1979):

$$\begin{aligned}
&(u_1+v_1)A_1(\lambda) - u_1A_2(\lambda) - v_1B_2(\lambda) &&= 0\\
&(v_1-u_1)A_1(\lambda) + t_1u_1A_2(\lambda) - t_1v_1B_2(\lambda) &&= (1-t_1)u_1\\
&u_2A_2(\lambda) + v_2B_2(\lambda) - u_2A_3(\lambda) - v_2B_3(\lambda) &&= 0\\
&-u_2A_2(\lambda) + v_2B_2(\lambda) + t_2u_2A_3(\lambda) - t_2v_2B_3(\lambda) &&= (1-t_2)u_2\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots && (11)\\
&u_{n-1}A_{n-1}(\lambda) + v_{n-1}B_{n-1}(\lambda) - u_{n-1}A_n(\lambda) &&= 0\\
&-u_{n-1}A_{n-1}(\lambda) + v_{n-1}B_{n-1}(\lambda) + t_{n-1}u_{n-1}A_n(\lambda) &&= (1-t_{n-1})u_{n-1}
\end{aligned}$$

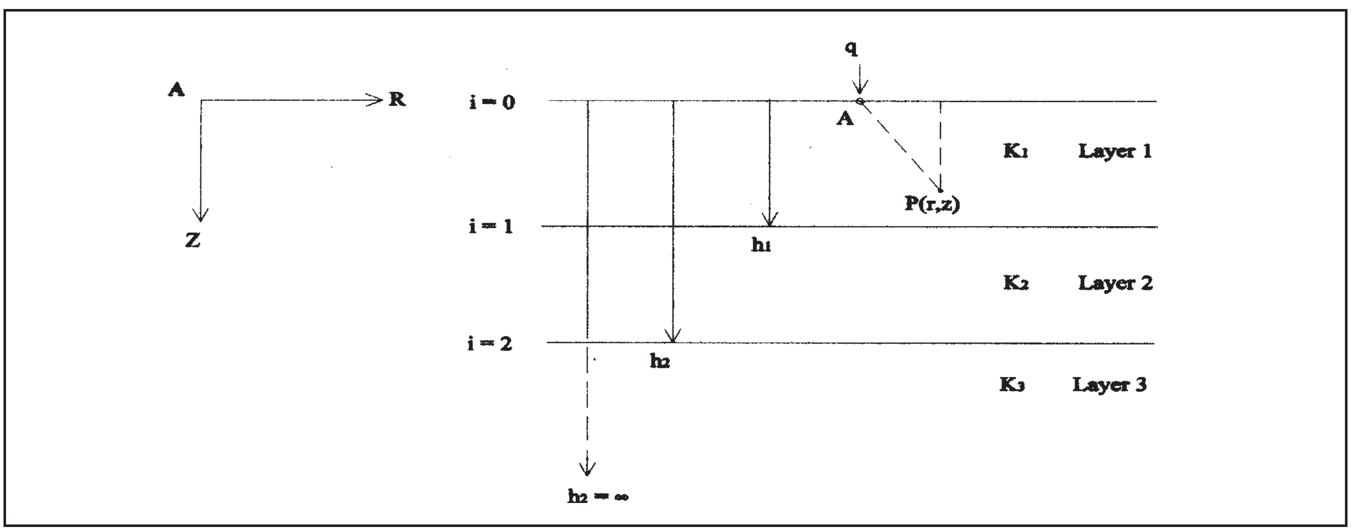

Figure 2. Schematic diagram of the three-layered aquifer system with a point-source located at A, on air-earth interface; P(r,z) is an arbitrary point in the cylindrical coordinate system.

After necessary algebraic manipulations, the arrived hydraulic potentials in layer 1, layer 2, and layer 3, are respectively given by:

$$\phi_1(r,z) = \frac{q}{2\pi K_1}\left[\frac{1}{\sqrt{r^2+z^2}} + \sum_{m=0}^{\infty}\frac{a_m}{\sqrt{r^2+(2mh_0-z)^2}} + \sum_{m=0}^{\infty}\frac{a_m}{\sqrt{r^2+(2mh_0+z)^2}}\right] \quad (12)$$

$$\phi_2(r,z) = \frac{q}{2\pi K_1}\left[\frac{1}{\sqrt{r^2+z^2}} + \sum_{m=0}^{\infty}\frac{d_m}{\sqrt{r^2+(2mh_0-z)^2}} + \sum_{m=0}^{\infty}\frac{b_m}{\sqrt{r^2+(2mh_0+z)^2}}\right] \quad (13)$$

$$\phi_3(r,z) = \frac{q}{2\pi K_1}\left[\frac{1}{\sqrt{r^2+z^2}} + \sum_{m=0}^{\infty}\frac{f_m}{\sqrt{r^2+(2mh_0+z)^2}}\right] \quad (14)$$

where h_0 is a chosen fixed thickness value, and a_m, b_m, d_m, and f_m are the coefficients in polynomial expansions which involve hydraulic conductivities and layer thicknesses.

For practical applications, river sections and canals can be represented as line sources of finite length. Analytically, the integration of a point source along a finite length results in a finite-line source (Parasnis, 1967). A linear convolution of hydraulic potentials due to point sources along a given profile can therefore be considered as the hydraulic potentials due to a finite-line source. As such, the algorithm for point source, 3LPNT has been modified by incorporating appropriate linear convolution techniques. The evolved algorithm, 3LLIN simulates the hydraulic potential in the porous medium due to a line source of finite length kept on the earth surface.

Following a methodology similar to that for the line source, integration of a point source over a finite area results in an areal source. Lakes, ponds, recharge tanks etc. may be viewed as areal sources of recharge. The recharge intensity (indicated by the strength of the source) may be either uniform over the spread area of the source or varying from point to point. The algorithm for point source, 3LPNT can be modified by the introduction of a two-dimensional convolution technique to compute the hydraulic potentials due to such areal sources. The resulting algorithm 3LARL simulates the hydraulic potentials in the porous medium due to an areal source of given strength.

STREAM FUNCTION

Generating streamlines in the stratified aquifer system enables visualising the groundwater flow regime. The stream function can be obtained from the hydraulic potential function (Harr, 1962) by virtue of the Cauchy-Riemann equations. Let $\psi_i(r,z)$ be the stream function (written as ψ_i) for the i^{th} layer. Then,

$$-K_i\frac{\partial\phi_i}{\partial r} = \frac{\partial\psi_i}{\partial z} \quad (15)$$

$$-K_i\frac{\partial\phi_i}{\partial z} = -\frac{\partial\psi_i}{\partial r} \quad (16)$$

Integration of either eqn. (15) or eqn. (16) will yield the stream function for the i^{th} layer as:

$$\psi_i = -\int K_i\frac{\partial\phi_i}{\partial r}dz + C_1 \quad (17)$$

or

$$\psi_i = \int K_i\frac{\partial\phi_i}{\partial z}dr + C_2 \quad (18)$$

In eqn. (17) and eqn. (18), the constants of integration C_1 and C_2 can be taken to be zero by virtue of the boundary conditions given by eqn. (9a).

Thus, the stream function for the i^{th} layer may be evaluated using,

$$\psi_i = -K_i\int\frac{\partial\phi_i}{\partial r}dz \quad (19)$$

The final expression for stream function in the i^{th} layer is obtained in two steps:

(i) To evaluate the derivatives $\partial\phi_i/\partial r$ for i=1,2, and 3 using eqn. (12), eqn. (13), and eqn. (14) and (ii) To substitute those derivatives into equation (19), followed by integration. The evaluation of the resulting integral has been performed by using the identity (Dwight, 1967) given by eqn. (20):

$$\int\frac{dx}{X^{3/2}} = \frac{4ax+2b}{(4ac-b^2)X^{1/2}}, \quad \text{where } X^{1/2} = (ax^2+bx+c)^{1/2} \quad (20)$$

Thus, we have, the required expressions for the stream functions as:

$$\psi_1(r,z) = \frac{q}{2\pi}\left[\frac{z}{r\sqrt{r^2+z^2}} + \sum_{m=0}^{\infty}\frac{a_m(z-2mh_0)}{r\sqrt{r^2+(2mh_0-z)^2}} + \sum_{m=0}^{\infty}\frac{a_m(z+2mh_0)}{r\sqrt{r^2+(2mh_0+z)^2}}\right] \quad (21)$$

$$\psi_2(r,z) = \frac{qK_2}{2\pi K_1}\left[\frac{z}{r\sqrt{r^2+z^2}} + \sum_{m=0}^{\infty}\frac{d_m(z-2mh_0)}{r\sqrt{r^2+(2mh_0-z)^2}} + \sum_{m=0}^{\infty}\frac{b_m(z+2mh_0)}{r\sqrt{r^2+(2mh_0+z)^2}}\right] \quad (22)$$

$$\psi_3(r,z) = \frac{qK_3}{2\pi K_1}\left[\frac{z}{r\sqrt{r^2+z^2}} + \sum_{m=0}^{\infty}\frac{f_m(z+2mh_0)}{r\sqrt{r^2+(2mh_0+z)^2}}\right] \quad (23)$$

Using the derived analytical solutions [set of equations (12), (13), & (14)] for hydraulic potentials and for streamlines [set of equations (21), (22), & (23)], an algorithm, 3LPNT has been devised. It is used for computing the steady state hydraulic potentials and streamlines, respectively in a single, two layered or three layered aquifer systems.

MODEL DESCRIPTION

The hydraulic potentials and streamlines have been computed in the case of three layered aquifer systems using 3LPNT, at the nodes of a rectangular grid of dimensions 1180 m by 580 m in the RZ-plane [vide Fig. 2] in order to illustrate the solution procedure. The vertical plane has been discretised into 59 columns and 29 layers (vertical discretisation) of dimension 20m each. The model parameters of the system are: h_1 = 100m, h_2 = 200m and h_3 = ∞ by letting n = 3, s_1 = 1, s_2 = 2, and h_0 = 100 m. A point source of strength, q = 0.01 m^3/s has been assigned to recharge the aquifer system. Hydraulic conductivities for the layers (K_1, K_2 and K_3) have been so chosen as to form several types of layered aquifer systems viz., , Type-I: $K_1>K_2>K_3$, Type-II: $K_1<K_2>K_3$ and Type-III: $K_1>K_2<K_3$.

VALIDATION TEST

For the validation purpose, steady state simulation of hydraulic potential has been performed by using a three dimensional finite difference groundwater flow model, MODFLOW (Mc Donald & Harbaugh, 1984) with identical grid setup and boundary conditions as that of the semi-analytical solution procedure. Thus, the three dimensional model grid has been discretised into 59 rows, 59 columns and 29 layers (vertical discretisation) of dimension 20m each. A recharge well (with q = 0.01 m^3/s) has been introduced at the central node (L1, R30, C30) of the top layer to act as the source. Constant head boundary condition has been assigned to the boundaries (at sufficiently large distance from the source) of the model grid with near-zero head values. Model parameters have been assigned cell-wise in the model. While MODFLOW uses an iterative procedure to compute hydraulic potential, computation by 3LPNT is direct.

NUMERICAL EXPERIMENTS

Based on the analytical solutions presented for the hydraulic potential and streamlines, the computational algorithms 3LPNT, 3LLIN and 3LARL for the cases of a point source, a finite-length line source and an areal

Table 1. Types of aquifer systems (based on hydraulic conductivities) for which simulations are performed with 3LPNT.

AQUIFER TYPE	K_1 (m/s)	K_2 (m/s)	K_3 (m/s)
$K_1 > K_2 > K_3$	1e-03	1e-04	1e-05
$K_1 < K_2 > K_3$	1e-04	1e-03	1e-05
$K_1 > K_2 < K_3$	1e-03	1e-05	1e-04

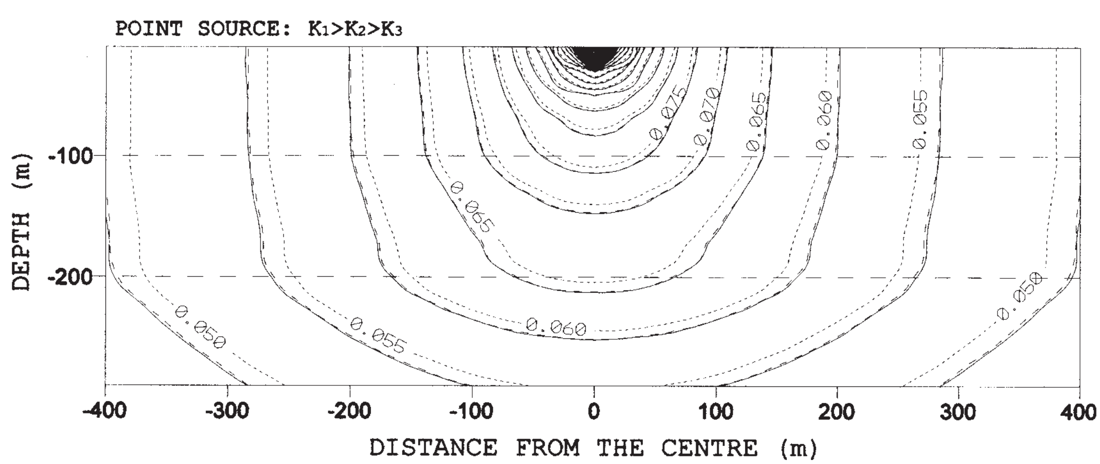

Figure 3. Convergence of the hydraulic potential due to a point source computed by 3LPNT with 100 terms (dotted lines), 200 terms (dashed lines) and 300 terms (solid lines) of the series expression in the analytical solution. The hydraulic conductivities of 3-layered aquifer satisfy the ineqality, $K_1>K_2>K_3$ (Table 1).

source, respectively have been coded in FORTRAN77 for demonstrating the solution procedure with several numerical examples. Comparison of respective results with that corresponding to MODFLOW simulations has also been given. Descriptions of these numerical experiments are given in the following sections:

Hydraulic Potentials Due to a Point Source

The hydraulic potential and streamline distribution have been computed in several types of aquifer systems [see Table 1] using 3LPNT and MODFLOW. Figure 3 demonstrates the convergence

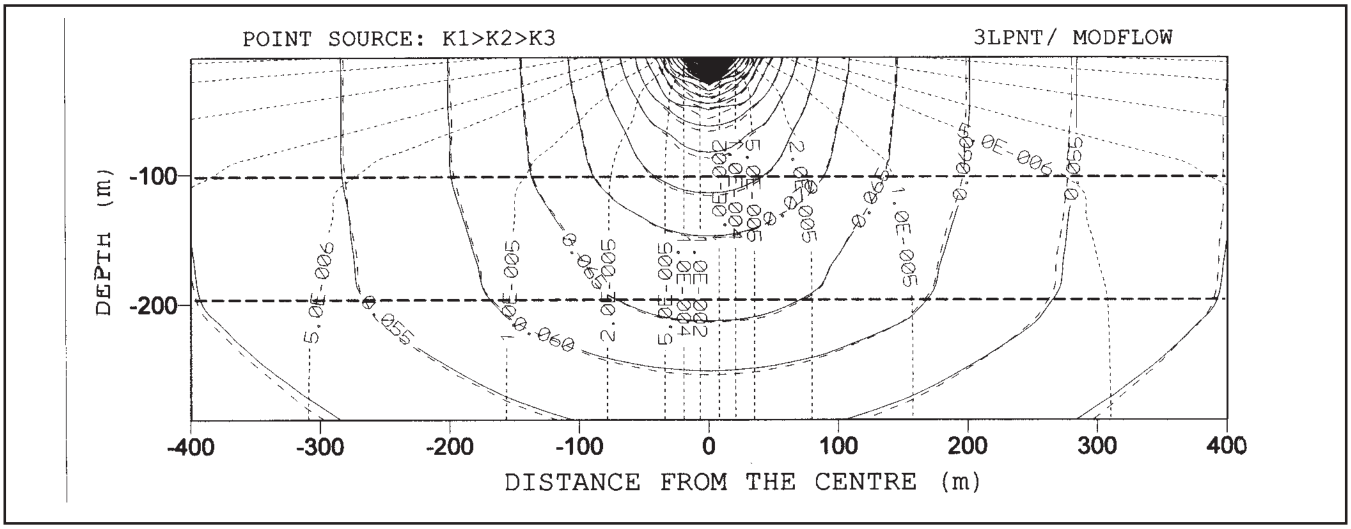

Figure 4a. Comparison of contour plots of equipotentials in the vertical section of the layered aquifer system ($K_1>K_2>K_3$) by our 3LPNT (solid contours) and MODFLOW (dashed contours). Dotted contours are the streamlines computed by 3LPNT.

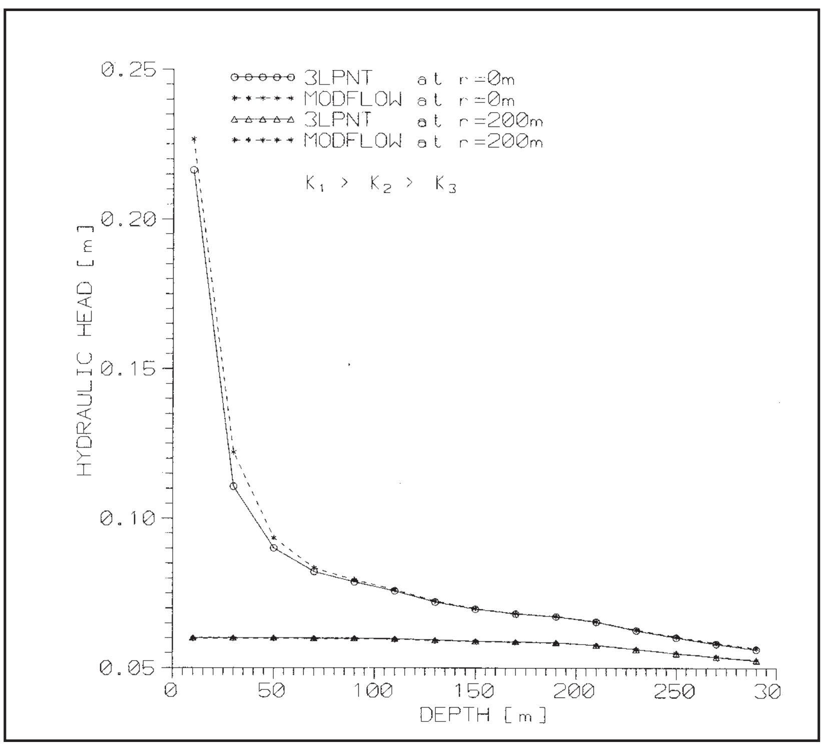

Figure 4b. Vertical distribution of hydraulic potentials obtained from 3LPNT (solid line) and MODFLOW (dashed line) at the centre and at a distance 200m from the source for the case $K_1>K_2>K_3$.

aspects of the proposed analytical scheme. Several contour plots of hydraulic potential and streamlines have been presented for the different 3-layered cases [Figs. 4a, 5a and 6a]. The plots have been depicted in the vertical section, bound by (-400m, 400m) in the horizontal direction and (0, 300m) in the vertical. The dashed horizontal lines in the plots demarcate the interfaces between layers.

The intervals used for the equipotential contours are: 0.005m for Fig. 4a, 0.01m for Fig. 5a and 0.005m for Fig. 6a. The equipotentials corresponding to MODFLOW simulations (dashed contours) have also been provided for comparison. Contour plots of hydraulic potentials computed by 3LPNT and

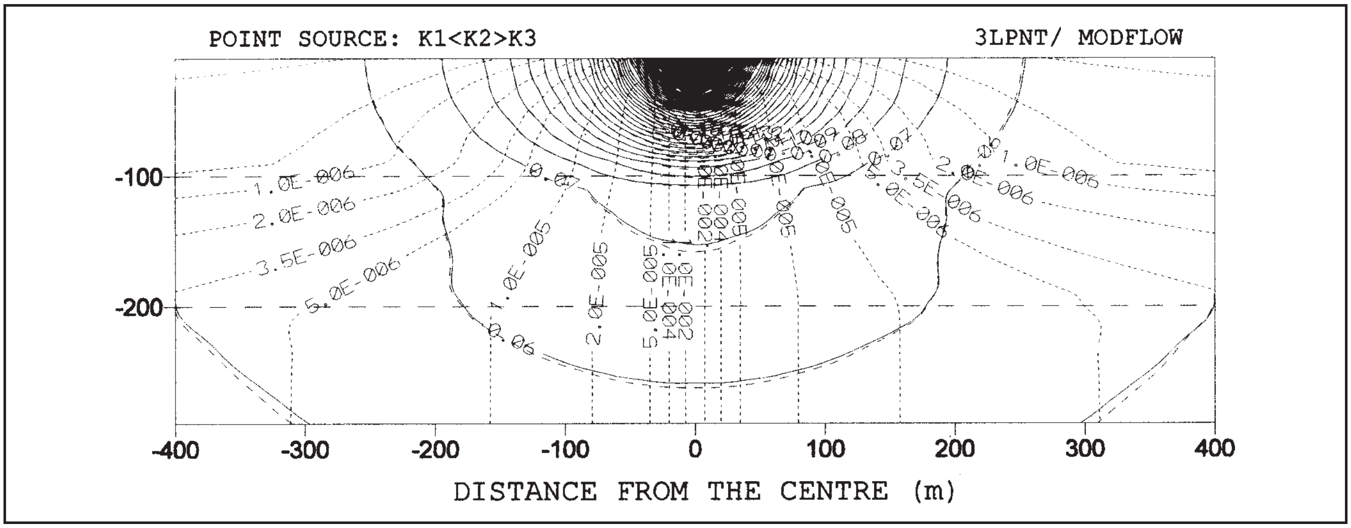

Figure 5a. Comparison of contour plots of equipotentials in the vertical section of the layered aquifer system ($K_1<K_2>K_3$) by our 3LPNT (solid contours) and MODFLOW (dashed contours). Dotted contours are the streamlines computed by 3LPNT.

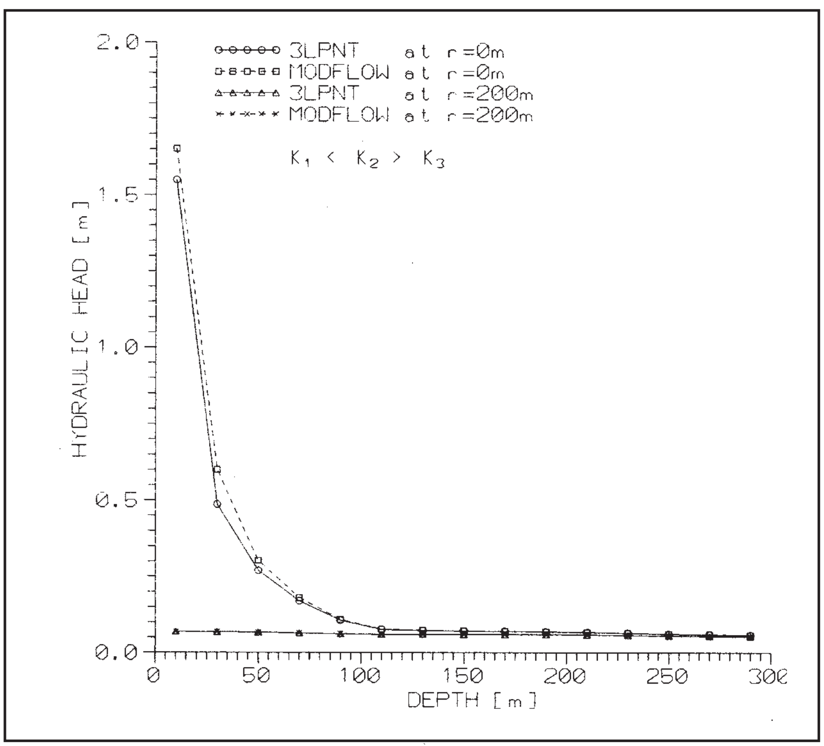

Figure 5b. Vertical distribution of hydraulic potentials obtained from 3LPNT (solid line) and MODFLOW (dashed line) at the centre and at a distance 200m from the source for the case where $K_1<K_2>K_3$.

MODFLOW have been merged into single plots to facilitate visual comparison. To avoid clustering, the streamlines have been plotted selectively.

A performance comparison of computed hydraulic potentials between the analytical and numerical methods can be made by visual inspection of Fig. 4a, Fig.5a and Fig. 6a for the equipotentials, and Fig. 4b, Fig. 5b and Fig.6b for the vertical distribution in the case of various types of aquifer systems. It is noticed that there is a fairly good agreement between the hydraulic potentials computed by 3LPNT and those computed by MODFLOW. In general, the discrepancy between MODFLOW and 3LPNT results is less than 2%.

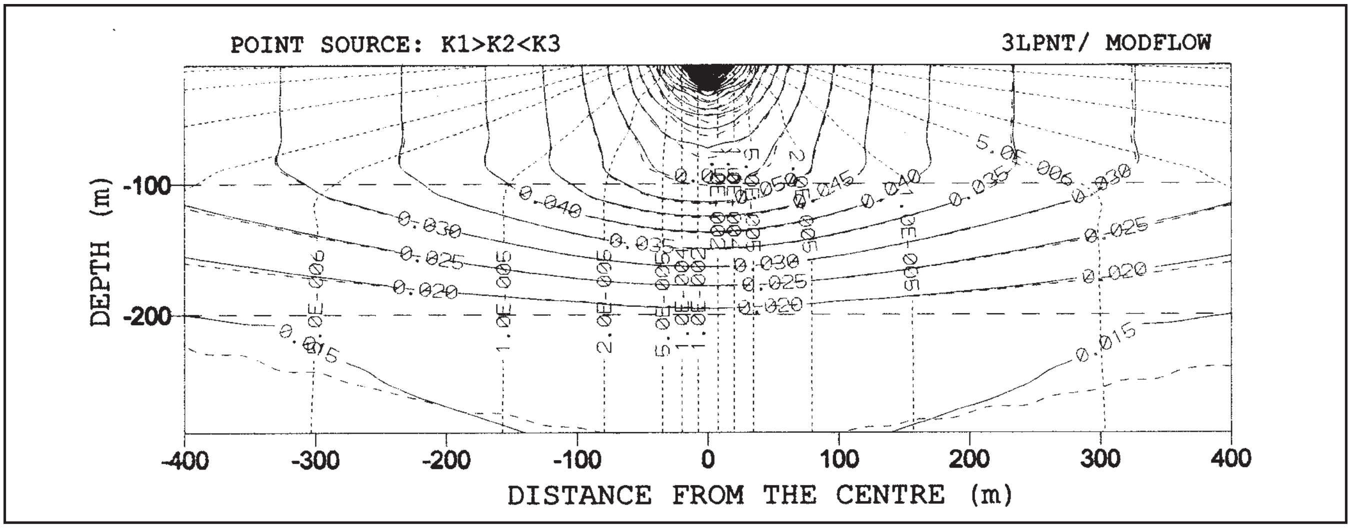

Figure 6a. Comparison of contour plots of equipotentials in the vertical section of the layered aquifer system ($K_1>K_2<K_3$) computed by our 3LPNT (solid contours) and MODFLOW (dashed contours). Dotted contours are the streamlines computed by 3LPNT.

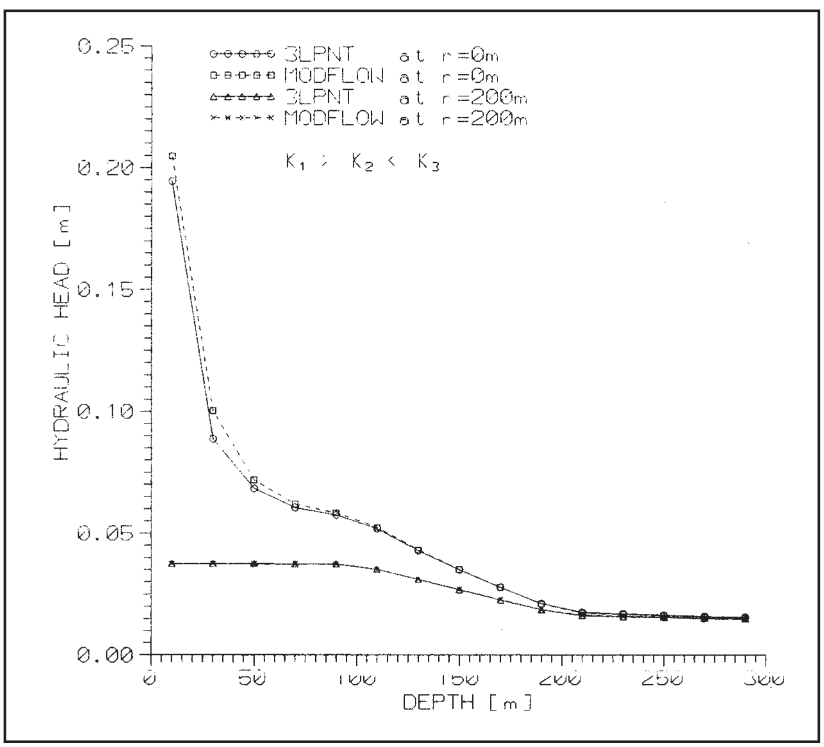

Figure 6b. Vertical distribution of hydraulic potentials obtained from 3LPNT (solid line) and MODFLOW (dashed line) at the centre and at a distance 200m from the source for the case where $K_1>K_2<K_3$.

Hydraulic Potentials Due to a Line Source

A line source of finite length, 2b=200m (discretised in 20 nodes) and strength, $q=0.1 m^2/s$ is used to simulate the hydraulic potentials in three layered aquifer systems using the simulation algorithm, 3LLIN. The hydraulic conductivities of the various layers in different cases have been assigned as indicated in Table 2. The hydraulic potentials in the vertical section of the porous medium along the strike of finite-line source for the various cases are depicted as equipotential contour plots [Fig.7 and Fig.8]. Since such plots across the strike of the finite-line source are quite similar to that of the point source, those have been omitted here.

Table 2. Types of aquifer systems (based on hydraulic conductivities) for which simulations are performed with 3LLIN.

AQUIFER TYPE	K_1 (m/s)	K_2 (m/s)	K_3 (m/s)	Source Length
$K_1 > K_2 > K_3$	1e-03	1e-04	1e-05	200 m
$K_1 > K_2 < K_3$	1e-03	1e-05	1e-04	200 m
$K_1 < K_2 > K_3$	1e-04	1e-03	1e-05	200 m

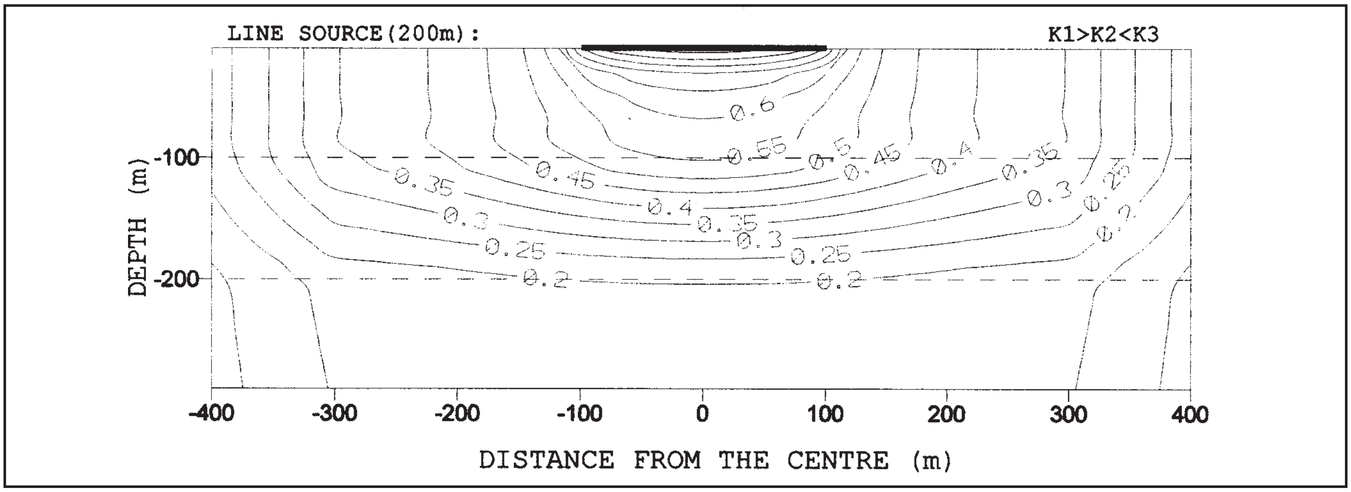

Figure 7. Comparison of contour plots of equipotentials in the vertical section of a layered aquifer system ($K_1 > K_2 < K_3$) due to a finite-length line source along its strike (computed by our 3LLIN); The top panel shows the position of the line source. Dashed horizontal lines represent the layer interfaces.

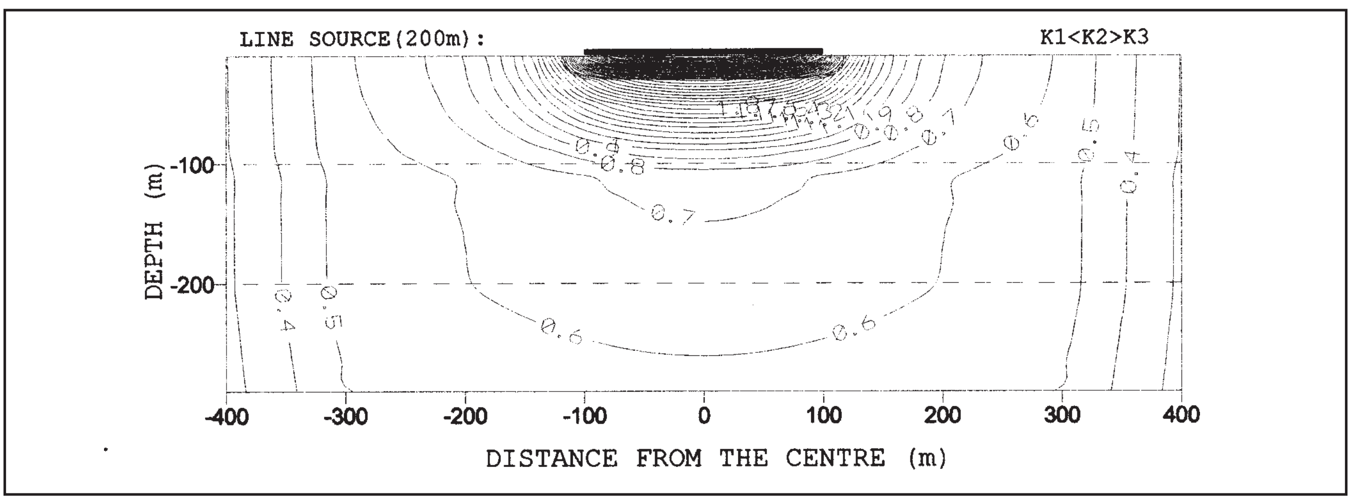

Figure 8. Comparison of equipotential contours in the vertical section of a layered aquifer system ($K_1 < K_2 > K_3$) due to a finite-length line source along its strike (computed by our 3LLIN); The top panel shows the position of the line source. Dashed horizontal lines represent the layer interfaces.

Hydraulic Potentials Due to an Areal Source

The hydraulic potentials due to an areal source (discretised in 20x20 nodes) of strength, q=1.0m/s placed at the air-earth interface of layered aquifer systems have been computed using 3LARL. The type of aquifer systems and hydraulic conductivities of the layers are given in Table 3. Plots of equi-potential lines in the vertical section are shown in Figs. 9, 10 and 11 for different cases.

Table 3 Types of aquifer systems (based on layer hydraulic conductivities) for which simulations are performed with 3LARL.

AQUIFER TYPE	K_1 (m/s)	K_2 (m/s)	K_3 (m/s)	Source Area (m^2)
$K_1 > K_2 > K_3$	1e-03	1e-04	1e-05	40000
$K_1 > K_2 < K_3$	1e-03	1e-05	1e-04	40000
$K_1 < K_2 > K_3$	1e-04	1e-03	1e-05	40000

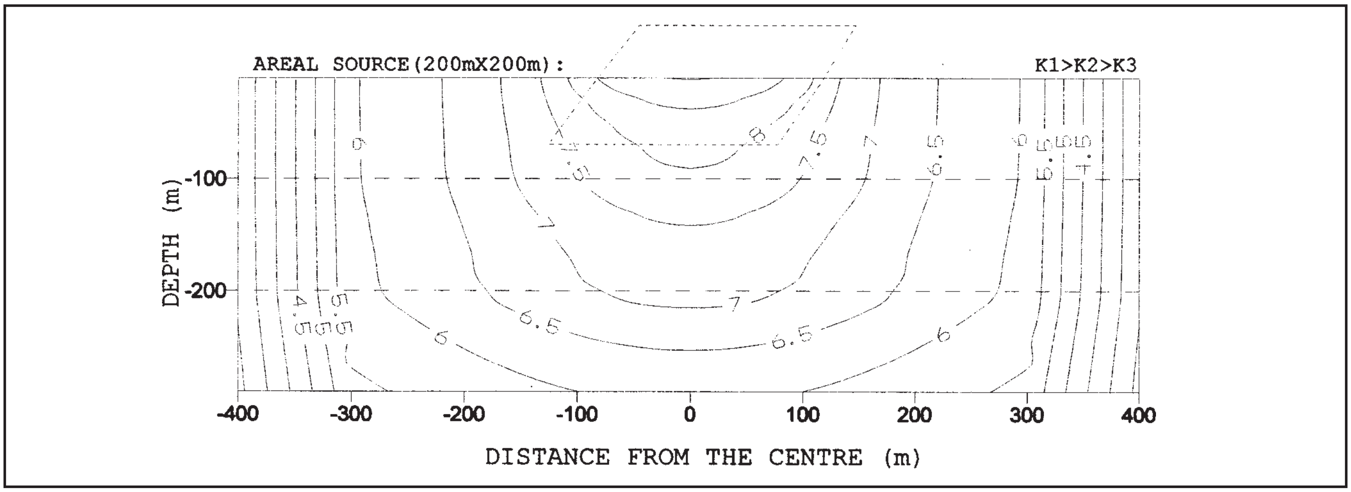

Figure 9. Contour plot of equipotential in the vertical section of a layered aquifer system ($K_1 > K_2 > K_3$) due to an areal source (computed by our 3ARL); In the top panel, a dashed rectangle shows the position of the areal source. Dashed horizontal lines represent the layer interfaces.

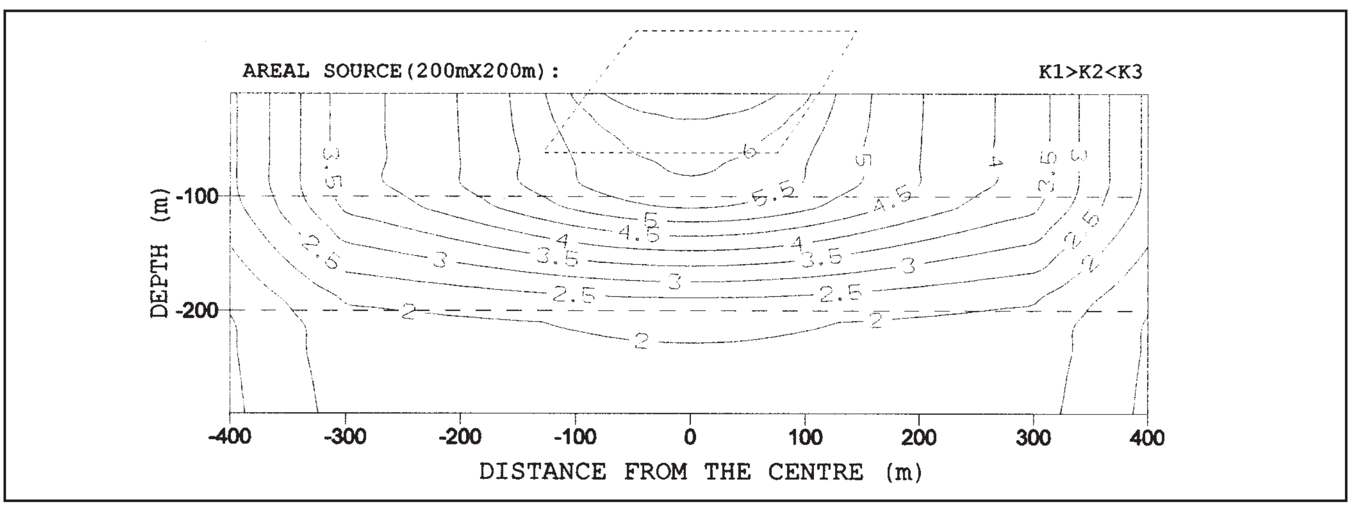

Figure 10. Contour plot of equipotential in the vertical section of a layered aquifer system ($K_1 > K_2 < K_3$) due to an areal source (computed by our 3ARL); In the top panel, a dashed rectangle on top of the aquifer system shows the position of the areal source. Dashed horizontal lines represent the layer interfaces.

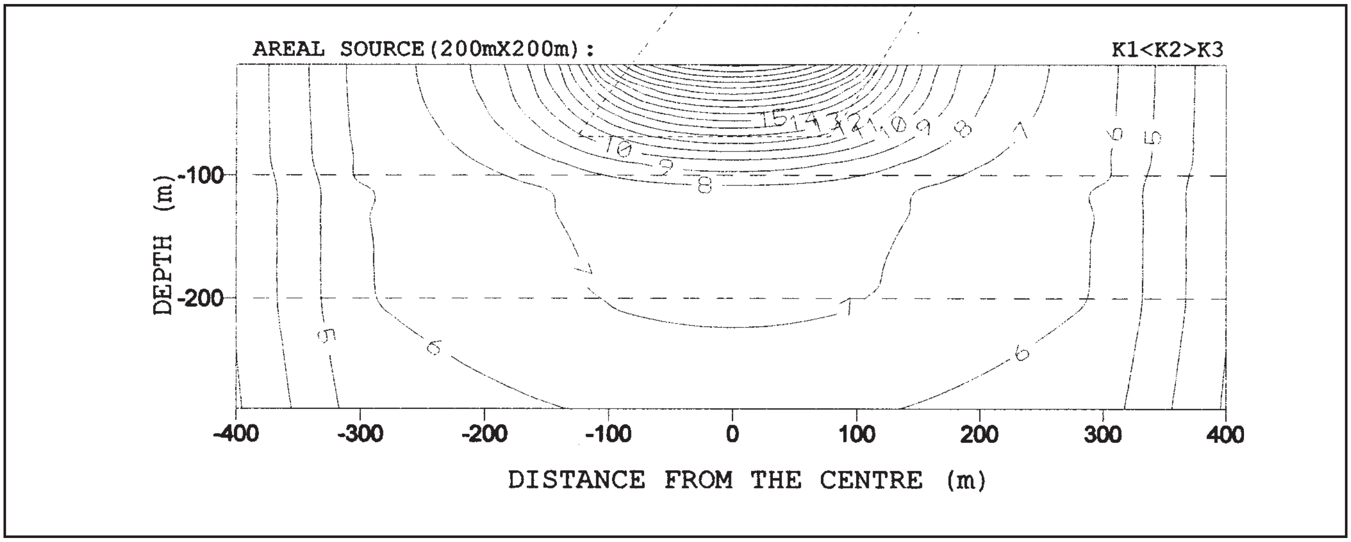

Figure 11. Contour plot of equipotential in the vertical section of a layered aquifer system ($K_1 < K_2 > K_3$) due to an areal source (computed by our 3ARL); In the top panel, a dashed rectangle on top of the aquifer system shows the position of the areal source. Dashed horizontal lines represent the layer interfaces.

DISCUSSION

Based on the geoelectrical sounding theory, analytical expressions for the solution of steady-state hydraulic potentials and streamlines due to a point source have been derived for three-layered aquifer systems.

Comparison was made between the analytical solutions and a finite-difference solution using MODFLOW to check the veracity of the developed solution techniques. The equipotential distributions obtained through 3LPNT and MODFLOW matched well, there by establishing the computational effectiveness of the analytical solution. It has been observed that the streamlines follow the tangent law of incidence and refraction (Hubbert, 1940), in all the cases. However, the small deviations exhibited between the results of the analytical models and that of the MODFLOW can be attributed to inaccuracies arising out of assigning finite boundaries with non-zero values for the hydraulic potentials in the MODFLOW simulations. It may be recalled that the analytical solutions, in the strict sense, are applicable to an infinite aquifer system with hydraulic potentials equal to zero at infinity. Further, there may be some rounding-off errors in MODFLOW simulations because of the iterative solution procedures.

The simulation algorithm, 3LPNT requires only a few input parameters such as source strength, layer conductivities, layer thicknesses and grid information for computation. Therefore, when steady state hydraulic potential/ stream function is to be determined in a layered aquifer like the one described, these analytical solutions provide a means for easier computation. An added advantage lies in the straight computation of streamlines using the analytical expressions, unlike many numerical flow models. Further, the solution procedure has been extended to the cases of a line source and an areal source by incorporating appropriate convolution techniques. For such sources, numerical means require elaborate model preparations followed by intricate computational schemes.

In terms of CPU time, the analytical model is found to be about twenty five times faster than MODFLOW simulations when executed in a 32-bit Pentium Pro personal computer. Besides, the analytical methods require only very few input parameters, whereas for the MODFLOW simulations detailed data preparations are necessary.

CONCLUSIONS

Analytical expressions are derived for hydraulic potential and stream line distribution in a 3-layered aquifer system for a point source over air-earth interface. Based on convolution technique, results have been extended to finite-length line and areal sources over air-earth interface of a 3-layered earth. Our simulations indicate that the proposed analytical/ semi-analytical schemes are cost-effective when compared to finite-difference based MODFLOW method.

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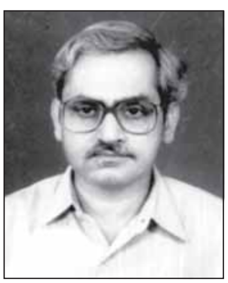

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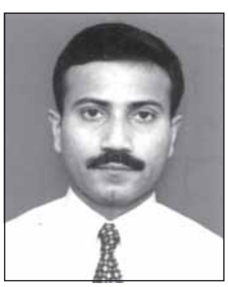

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