

# Empirical relation for estimating shear wave velocity from compressional wave velocity of rocks

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## ABSTRACT

In situ shear wave velocity is one of the most important parameters for evaluating dynamic elastic moduli and needs special energy sources for generation and receivers for detection. Identification of shear wave phase on a seismic record needs skill as it is not the first wave to arrive and is usually embedded in surface waves. To estimate shear wave velocity for site characterization and for predicting the in situ quality of rock, empirical relations were proposed by Carroll (1969) to predict shear wave velocities from compressional wave velocities of rock samples. His empirical relation was used on the actual field data and the estimated shear wave velocities were compared with in situ velocities measured using cross-hole seismic technique. The shear wave velocities measured at four nuclear power plant sites were used to generate the correlation between compressional and shear wave velocities. A power fit model with a regression coefficient of 0.84 was developed. This correlation is valid for rocks having compressional wave velocity ranging from 4000 m/sec to 6000 m/sec and Poisson's ratio between 0.22 and 0.28. The proposed relation is based on 185 data pairs of in situ compressional and shear wave velocity measurements conducted at four nuclear power project sites. The sites with different host rocks are: Kota (Rajasthan), Kaiga (Karnataka), Tarapur (Maharashtra) and Kakrapar (Gujarat). The empirical relationship proposed in this paper is first of its kind in India. The proposed empirical relation can be used to predict shear wave velocities at similar rock sites anywhere in India.

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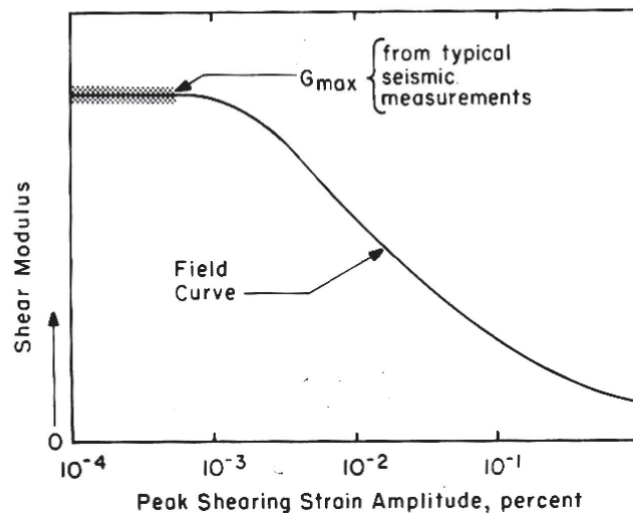
## INTRODUCTION

The operation of nuclear reactor results in the generation of unbalanced dynamic forces and moments which are transmitted to the foundation and the underlying rock. The foundation for nuclear reactors must, therefore, be designed to ensure stability under the combined effect of static and dynamic loads. Even though the magnitude of dynamic load is small, it is applied repetitively over long periods of time. The vibration response of a reactor-foundation-rock system is defined by natural frequency and the amplitude of vibration under normal operating conditions of the reactor. The natural frequency and amplitude of vibration are the two most important parameters to be determined in designing foundation for any reactor. For dynamic loads, the design criterion is that the natural frequency of the reactor-foundation-rock system should not coincide with the operating frequency of reactor. In fact, a zone of resonance is generally defined and the natural frequency of reactor-foundation-rock system must lie outside this zone. For determination of natural frequency of rock-

foundation system, shear wave velocity of rock is one of the important input parameters (Dowrick 2003).

Discontinuity sets in rock mass such as block size and block form, permeability, failure criteria and deformation moduli determine the geotechnical behaviour of rock. Generally, the discontinuity sets for rock mass are characterized manually by measuring each discontinuity. This not only is a tedious work, but also requires access to rock mass. This problem to some extent can be solved by measuring in situ shear wave velocities which are affected by discontinuities.

The determination of acoustic parameters of rock, particularly the elastic moduli, has important application in assessing the response of structures to static and dynamic loads (Barkan 1962; Judd 1965). In general, the dynamic elastic moduli determined by cross-hole seismic technique, because of low strains, tend to yield higher values than those determined by static method (Simmons & Brace 1965). Fig. 1 shows the variation of elastic moduli with strain (Borm 1978; Richart, Anderson & Stokoe 1977). Cross-hole seismic technique involves strains of the order of



**Figure 1.** Variation of in situ shear modulus with strain (After Richart et al., 1977)

$10^{-5}$  to  $10^{-6}$  and therefore, yields higher values of shear moduli termed as  $G_{\max}$ . Dynamic methods being non-invasive and non-destructive are more amenable to in situ testing than static methods and also sample relatively large volumes of material. When statically determined elastic moduli are preferable in the design criteria, the dynamic moduli are also useful in that they may be considered as upper limits for the various moduli. For determining dynamic moduli from seismic measurements, three independent parameters viz., density, shear and compressional wave velocity are required. Methods of obtaining shear wave velocity data suffer from obtaining a recognizable shear phase on the seismic record without resorting to elaborate recording devices and energy sources (Carroll 1969). Another approach which has been used to estimate in situ dynamic moduli is to assume a Poisson's ratio and measure compressional wave velocity and density in the medium (Wantland 1964). Poisson's ratio for rocks vary widely and therefore shear wave velocities from the same can not be estimated correctly (Carroll 1969).

In addition to the above, the shear wave velocities of the subsurface materials are also needed for characterizing the discontinuities in the rock (Boominathan 2004), for predicting amplification of the earthquake wave, for assessment of liquefaction potential (Ravendra Nath et al., 1992) and for advanced finite element programs for dynamic analysis of structures.

The various geophysical techniques available for in situ measurement of shear wave velocities and their advantages and disadvantages are described below.

### Spectral Analysis of Surface Waves (SASW)

Nazarian & Stokoe (1984) first described the SASW method to the earthquake engineering community, sometimes referred to as "C X W" (Boore & Brown 1998). SASW uses an active source of seismic energy. Signal is recorded repeatedly by a pair of 1 Hz seismometers at small (1m) to large (500 m) distances (Nazarian & Desai 1993). The seismometers are vertical particle velocity sensors. SASW technique uses the dispersive characteristics of surface waves (Rayleigh waves) to determine the variation of the shear wave velocity of a layered earth with depth. In this technique the assumption is made that the most energetic arrivals recorded are Rayleigh waves which does not hold when noise overwhelms the power of artificial source as in urban areas or where body wave phases are more energetic than the Rayleigh waves. In such situations, SASW will not yield reliable results (Brown 1998; Sutherland & Logan 1998). Boore & Brown (1998) found that SASW models consistently under predicted shallow shear wave velocities.

### Multi-channel Analysis of Surface Waves (MASW)

MASW technique (Park, Miller & Xia 1999) was developed to overcome the shortcomings of SASW in the presence of noise. The simultaneous recording of 12 or more receivers at short (1-2m) to long (50-100m) distances from an impulsive or vibratory source gives statistical redundancy to the measurements of phase velocities. Miller et al., (2000) were able to obtain excellent MASW results in the noisy environment of an operating oil refinery.



the data are acquired fast and no elaborate processing is needed. Average shear wave velocity of the strata, up to the depth the source is clamped, is measured. The disadvantage of the technique is that only average velocity up to a particular depth is obtained.

### Down-hole Seismic Method

In this technique, the receiver is lowered in the hole and the shear waves are generated at the surface. This technique also provides average shear wave velocity up to a particular depth and volume of the

material contributing to the shear wave velocities is that located very close to the borehole (Redpath 1973).

### Cross-hole Seismic Method

This technique is by far the most reliable method for shear wave velocity determination. The velocity of the subsurface materials is measured at various depths between two closely spaced (5-7 m apart) boreholes without any interference from the nearby horizons (Bruce 1977). However, cross-hole test needs at least two closely spaced boreholes and special shear wave

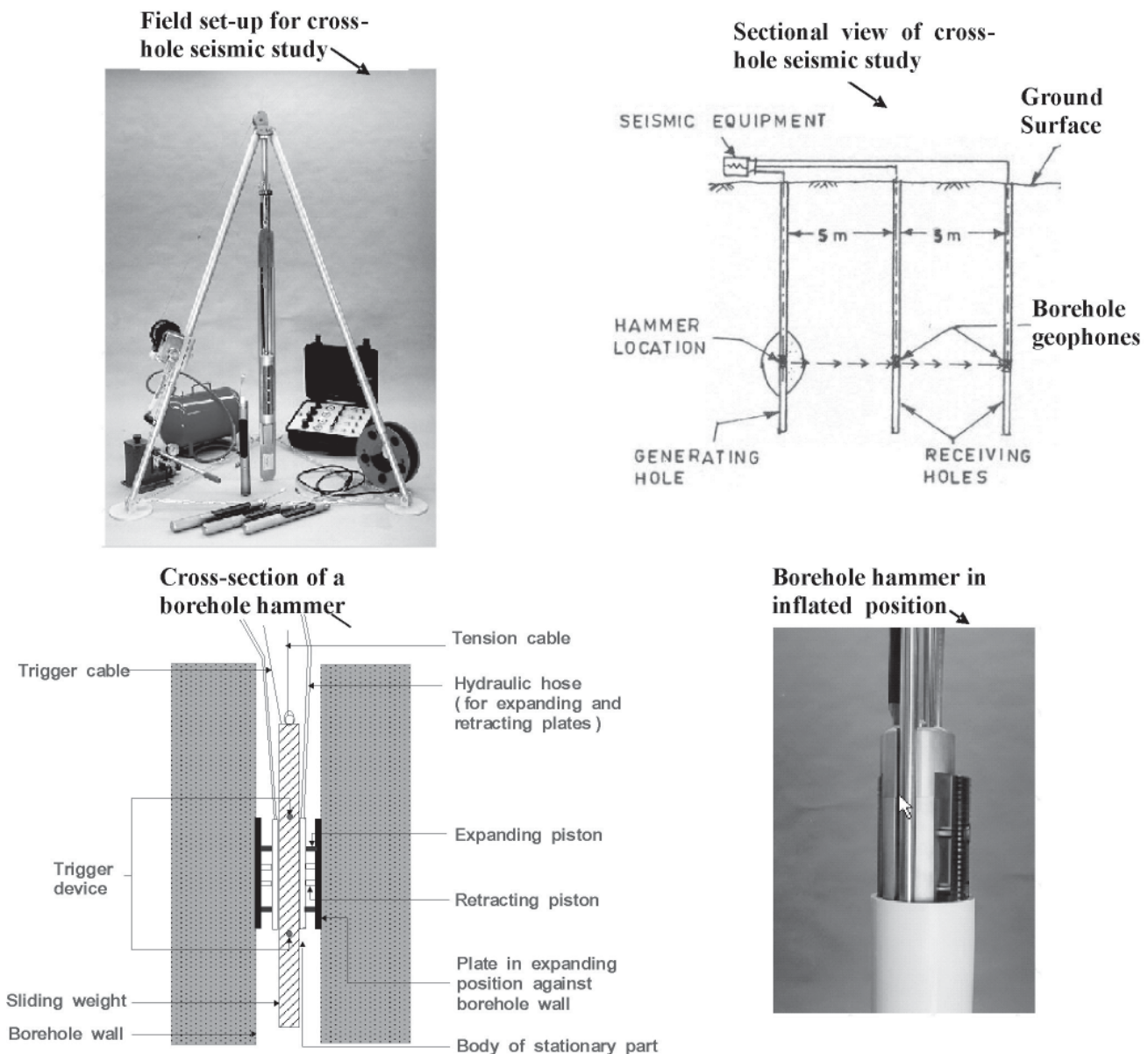


Figure 3. Components of cross-hole seismic survey



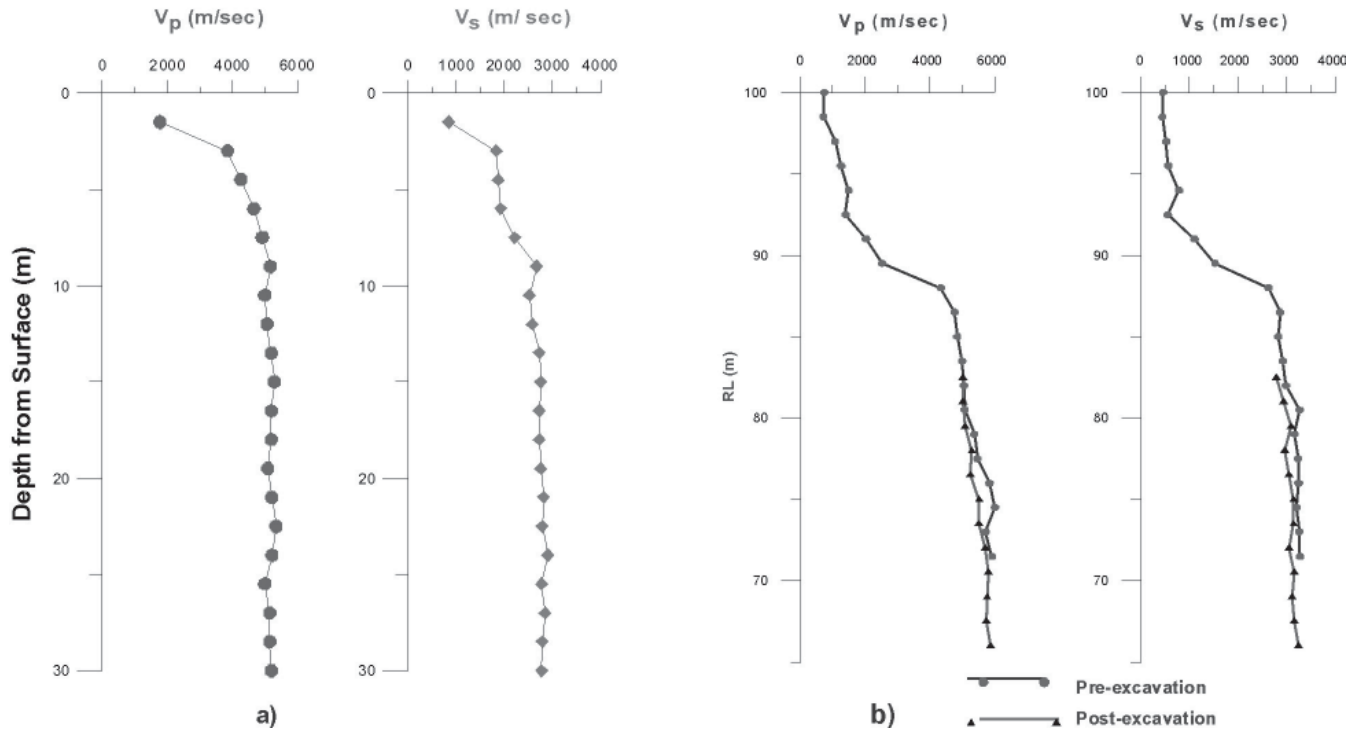
generating and recording equipment (Fig. 2).

The in situ shear wave velocities with depth were measured using cross-hole seismic technique at four nuclear power plant sites. The nuclear power plants are:

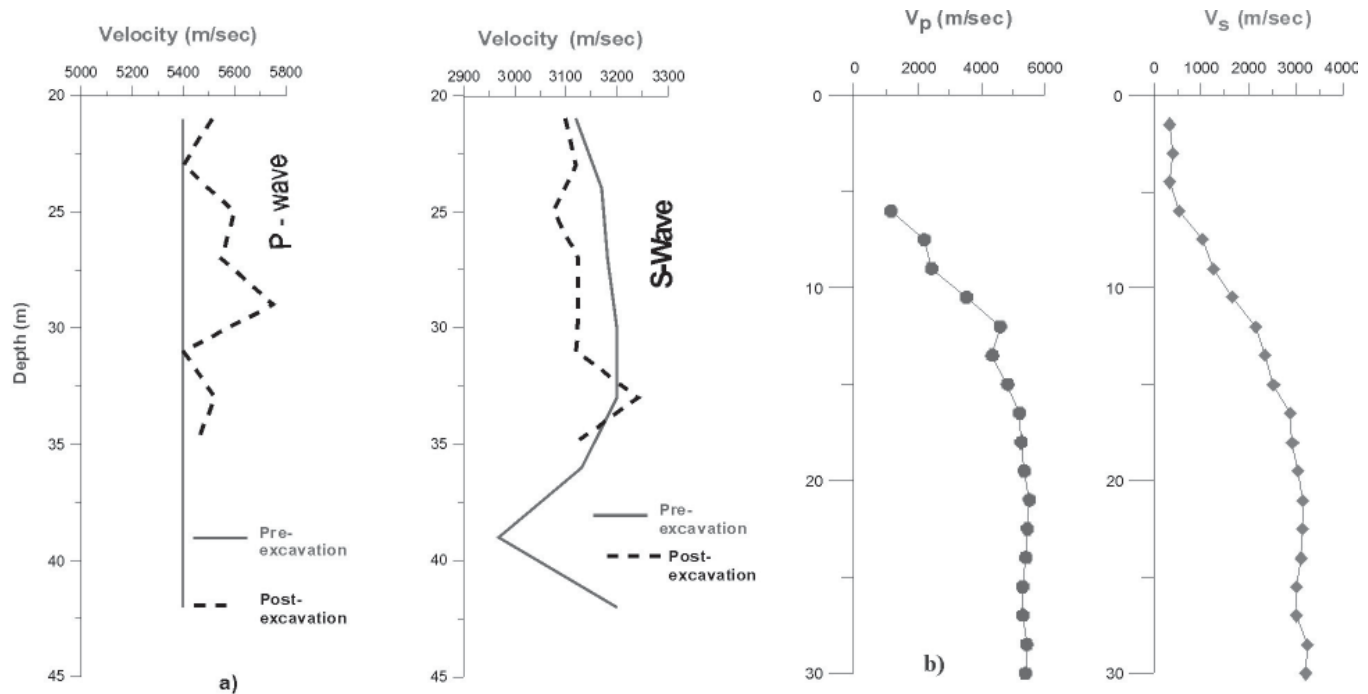
i) Rajasthan Atomic Power Plant site, Rawatbhatta, Kota, Rajasthan, ii) Kaiga Atomic Power Plant site, Kaiga, Uttara Kanada District, Karnataka, iii) Tarapur Atomic Power Plant site, Tarapur, Maharashtra and iv) Kakrapar Atomic Power Plant site, Kakrapar, Gujarat. The locations of these power plant sites marked on geological map of India are shown in Fig.2. Cross-hole seismic studies were carried out in two mutually perpendicular directions in three boreholes of dia NX (inner dia 76 mm) drilled using rotary hydraulic drilling. The borehole at the corner was used for generating seismic waves and the same were picked-up by two three-component geophones lowered in the remaining holes. Seismic waves were generated by a mechanical impulse instrument which consisted of a stationary part and a moving part (Fig.3). The stationary part comprises a hydraulic cylinder block with eight horizontal pistons – four pistons for expansion to grip it with the borehole wall and the other four for contraction. The movable part can be lifted above or lowered below the stationary cylinder (Fig.3). The entire assembly was lowered into the

generating borehole at various depths by a tension cable. Seismic waves from the downward motion of the hammer were produced by raising it manually and allowing it to drop freely on the top of the cylinder. Seismic waves from the upward motion of the hammer were produced by manually raising the hammer so that it hits the bottom of the cylinder. The phases of the shear waves from the two hittings are opposite to each other, thus facilitating their identification (Ravendra Nath et al., 1992).

The seismic waves so produced were picked-up by three-component geophones planted in the remaining holes. Since shear waves can not be transmitted through liquid and are highly attenuated in semi liquid drilling mud, it is necessary to position the geophones in firm contact with the material. To achieve this, geophone must be clamped to the borehole wall. Under these circumstances, the borehole may be either dry or filled with water or drilling mud. This situation will have no effect on shear wave velocity measurement. Yet another advantage of holding geophone against borehole wall is that it does not introduce any errors in distance measurement because of caving etc. particularly in soft rock conditions. The variations of in situ compressional and shear wave velocities with depth at the four atomic power plant sites are depicted in Figs. 4 and 5.



**Figure 4.** Compressional and shear wave velocities with depth (a) Rajasthan Atomimc Power Plant (b) Kaiga Atomimc Power Plant



**Figure 5.** Compressional and shear wave velocities with depth (a) Tarapur Atomic Power Plant (b) Kakrapar Atomic Power Plant

### Empirical Relation

Site specific shear wave velocities measured by various geoscientists show that significant variations in shear wave velocity with depth exist and that these variations qualitatively correlate with material type, their condition (compaction, strength) and structural loading conditions (Phil & Andy 1990).

To overcome the difficulty of determining site specific shear wave velocity or of assuming the shear wave velocity, Carroll (1969) proposed an empirical relation to predict shear wave velocity from compressional wave velocity (which is easier and straightforward to determine) on volcanic and other rock samples. The major disadvantage of determination of velocity on rock samples is that the values are representative of only a small volume of the rock. Unless the in situ conditions of stress, fluid content etc., of the rock from which the sample was taken are reproduced in the laboratory, measurements on samples can differ significantly from those values existing in situ. This is because the acoustic properties of rock exhibit an environmental dependency, particularly with respect to stress. Consequently, it is desirable to determine a method of estimating the dynamic moduli from in situ acoustic measurement and at the same time avoid the difficulty of measuring shear wave velocity.

The relation on rock samples proposed by Carroll (1969) was tested for estimating in situ shear wave velocity from compressional wave velocity for basalt, granite-gneiss, quartzite and other rocks having compressional wave velocities ranging from 4000 m/sec to 6000 m/sec and Poisson's ratio between 0.22 and 0.28. The estimated shear wave velocities from empirical relation were compared with the cross-hole measured in situ shear wave velocities. Using 185 in situ measurements, a new empirical relation based on power fit model between in situ compressional and shear wave velocities was proposed.

### Carroll's Formula

Carroll (1969) proposed the following empirical relation between compressional ( $V_p$ ) and shear ( $V_s$ ) velocities on rock samples :

$$V_s = 0.937562 V_p^{0.81846} \quad \text{..... (1)}$$

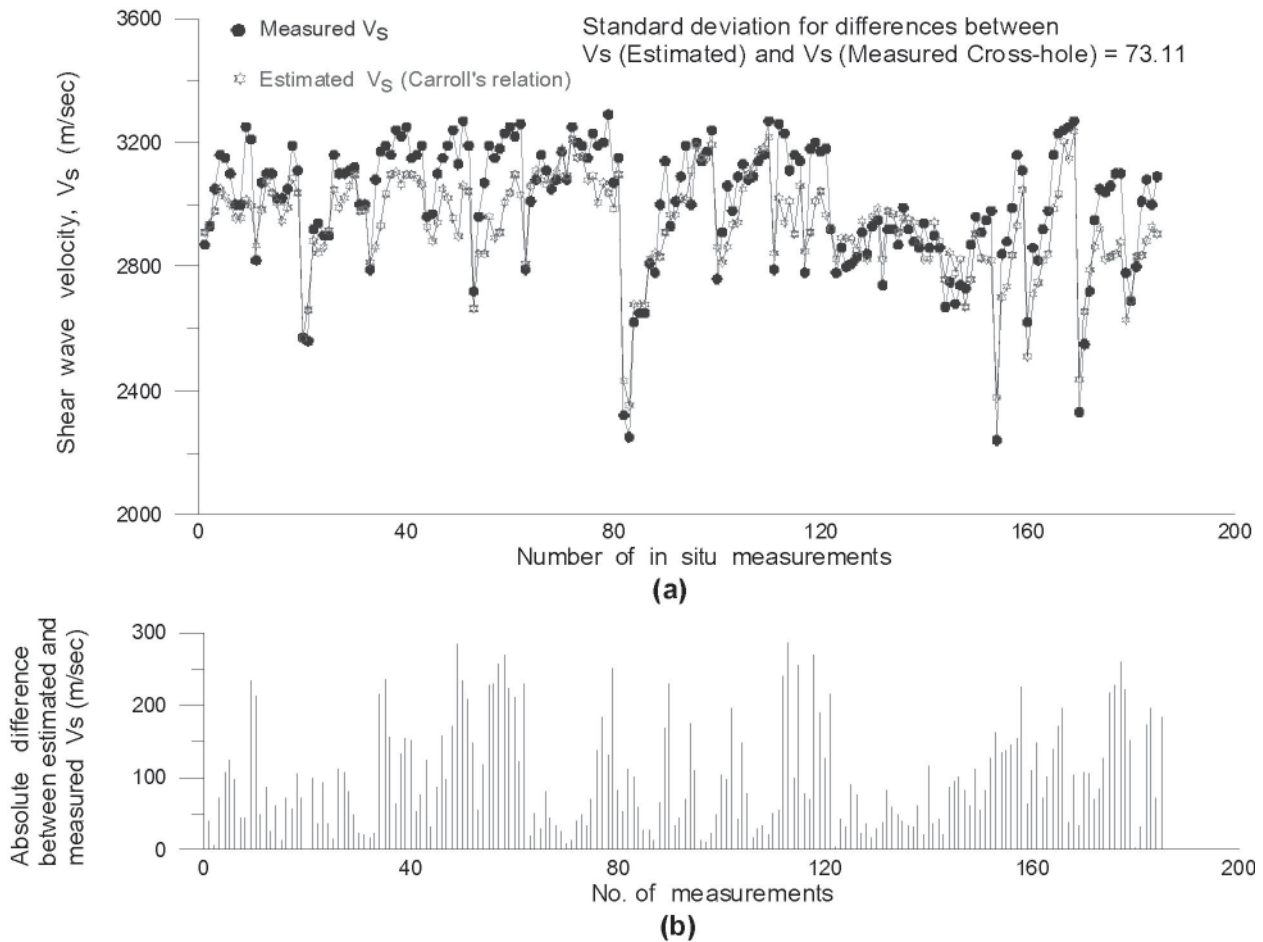
where velocities are in Kft/sec. Physical properties of rocks for which the relation was developed were; density 1.6 to 2.7 g/cm<sup>3</sup>, porosity 10 to 40 %; Compressional wave velocity 6000-20000 ft/sec. Standard error of estimate  $\log 1.055942 = 0.0236399$ . The equation (1) when velocities are expressed in km/sec can be written as

$$V_s = 0.756090 V_p^{0.81846} \quad \dots (2)$$

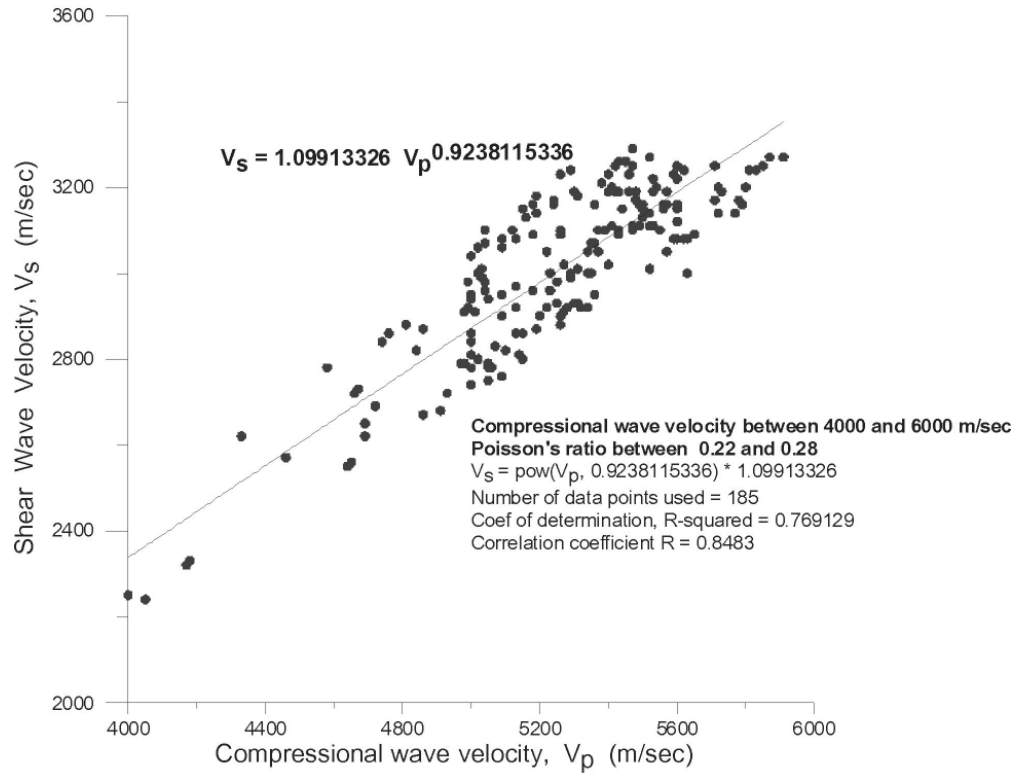
$$V_s = 1.09913326 V_p^{0.9238115336} \quad \dots (3)$$

The equation (2) was used for predicting shear wave velocities from compressional wave velocities using 185 data points of four nuclear power projects having host rock as basalt, granite-gneiss and quartzite. The in situ compressional wave velocity with depth at these projects varied between 4000 m/sec and 6000 m/sec. Fig. 6 shows the comparison between estimated and measured shear wave velocities. It is seen from this figure that estimated shear wave velocities are generally lower as compared to cross-hole measured values. The plot of measured in situ compressional and shear wave velocities up to 30 m depth for four nuclear power plant sites is shown in Fig.7. The 185 data pairs of  $V_p$  and  $V_s$  were used for regression analysis and a power fit model correlation was developed. The regression equation with a correlation coefficient of 0.8483 is :

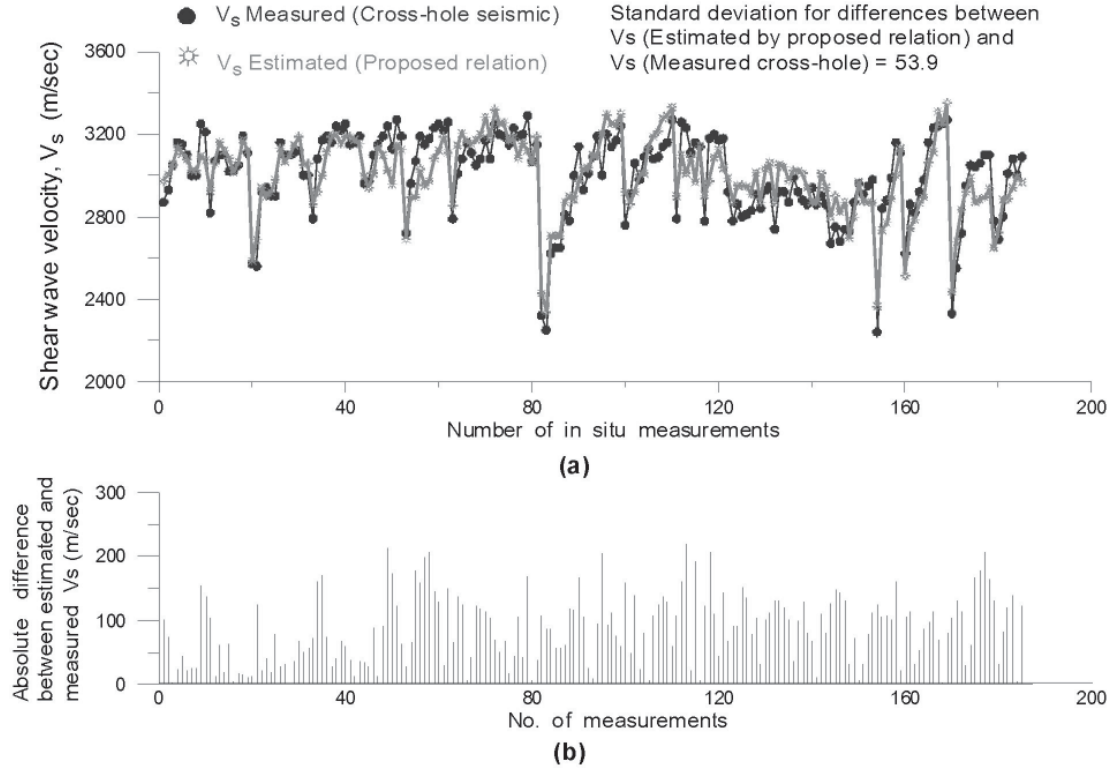
Fig.7 shows the plot of actual 185 data pairs of  $V_s$  and  $V_p$  along with the fitted equation. This proposed empirical relation is valid for any rock having compressional wave velocity between 4000 m/sec and 6000 m/sec. Poisson's ratio of rock should range from 0.22 to 0.28 which implies that  $V_p/V_s$  ratio should be between 1.61 and 1.85. The compressional wave velocity range 4000 – 6000 m/sec was selected because for basalt, granite gneiss and quartzitic sandstone, this range of velocities represents unweathered rock. Generally, velocities greater than 6000 m/sec are not encountered at shallow depths (< 100 m) which are of interest for foundation studies. Also, this was the range of velocities determined at four nuclear power projects where the cross-hole studies had been carried out. This proposed empirical relation was then used to estimate shear wave velocities from in situ



**Figure 6.** (a) Comparison between estimated (Carroll) and measured (crosshole) shear wave velocities (b) Error bars for Vs (est. by Carroll's and in situe measured)



**Figure 7.** Measured in situ compressional and shear wave velocities for Basalt, Granite gneiss and Quartzite rocks with the best fit line



**Figure 8.** (a) Comparison between estimated  $V_s$  (Proposed relation) and measured  $V_s$  (crosshole) (B) Error bars for estimated  $V_s$  (using proposed modified relation) and in situ measured  $V_s$



measured compressional wave velocities. The comparison between estimated shear wave velocity and measured shear wave velocities is shown in Fig. 8. A good match between the estimated and measured shear wave velocities is seen. A small difference noticed is acceptable and was attributed to measurement accuracies reported for shear wave velocities evaluation (Wadhwa, Subba Rao & Ghosh 2005). The proposed empirical relation is independent of rock type but is valid for any material having compressional wave velocity between 4000 m/sec and 6000 m/sec and Poisson's ratio between 0.22 and 0.28.

## CONCLUSIONS

Determination of in situ shear wave velocity with depth requires specialised generating and recording equipment as well as trained and experienced personnel to decide the shear phases and interpret them. An empirical relation proposed by Carroll (1969) on rock samples for estimating shear wave velocities from compressional wave velocities was tested. In situ compressional and shear wave velocities measured using Cross-hole technique at four nuclear power project sites having basalt, granite gneiss and quartzitic sandstone as host rock were used to generate a correlation between  $V_s$  &  $V_p$ . A power fit regression equation was developed for 185 pairs of  $V_s$  and  $V_p$  with a correlation coefficient of 0.84. The proposed empirical relation is valid for predicting shear wave velocity for rocks having compressional wave velocity ranging from 4000 m/sec to 6000 m/sec and Poisson's ratio between 0.22 and 0.28. The shear wave velocities estimated using proposed empirical relation matched well with those measured in situ using cross-hole seismic technique. The proposed relation can be effectively used to predict the shear wave velocity of rocks anywhere in India.

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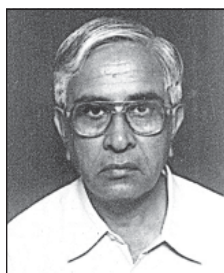
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