Predictability of solar activity using fractal analysis

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ABSTRACT

Various new techniques like neural networks, learning nonlinear dynamics and others are used by researchers to predict solar activity. But we are yet to obtain reasonably good results. This is mainly because the reason of the variation of solar activity is still unknown. Hence it is important to analyze the characteristics of the data. This paper considers sunspot numbers as an index of solar activity. The daily sun spot number data is analyzed using fractal technique and examined to determine the predictability of solar activity. For the period 1990 to 2004, the average fractal dimension for periods of 10 days or less was about 1.43. But during the same period, the average fractal dimension was 1.72 for periods longer than 10 days. Hence the result is encouraging for short-term prediction (i.e.) within about 10 days, but discouraging for medium-term prediction (longer than 10 days).

INTRODUCTION

The extent of solar activity has a good correlation with the sunspot numbers. Hence, sun spot numbers are widely used as an indicator of solar activity. The sun spot numbers finds its utility in selection of orbits for satellites, prediction of High Frequency propagation, prediction of weather and so on. Thus, the prediction of sun spot numbers gains importance. (Gorney 1990)

The recent techniques such as Neural networks (Higuchi 1988) and learning nonlinear dynamics (Koons & Gorney 1990) are being used to predict sunspot numbers. But, the mechanism of variation of solar activity is still unknown and hence the sun spot number is not clearly understood till date.

This paper uses the daily sun spot numbers as an index of solar activity and the data is subjected to fractal analysis. The fractal dimension values thus obtained is used as an indicator to examine the predictability of solar activity.

DATA

We have used the daily sun spot numbers for the a period of fifteen years from 1990 to 2004. The international sunspot number is produced by the Solar Influences Data Analysis Center (SIDC), World Data Center for the Sunspot Index, at the Royal Observatory of Belgium. The relative sunspot number is an index of the activity of the entire visible disk of the Sun. It is determined each day without reference to preceding days. Each isolated cluster of sunspots is termed a sunspot group, and it may consist of one or a large number of distinct spots whose size can range from 10 or more square degrees of the solar surface down to the limit of resolution (e.g., 1/25 square degree). The relative sunspot number is defined as R = K (10g + s), where g is the number of sunspot groups and s is the total number of distinct spots. The scale factor K (usually less than unity) depends on the observer and is intended to effect the conversion to the scale.

FRACTAL ANALYSIS

Method for calculating the Fractal Dimension

Various techniques are available to calculate the fractal dimension of the given time series data. They are

a. Power spectrum analysis method

b. Burlaga and Klein method [Burlaga and Klein, 1986]

c. Higuchi method [2]

The Higuchi method give relatively accurate results and hence we used the same to calculate the Fractal Dimension (D). The method is as described subsequently.

We now consider a finite set of time series

observations taken at a regular interval:

X(1), X(2), X(3),, X(N)

Where N is the total number of observations of the given time series.

From the given time series, we first construct a new time series, X_k^m , defined as follows:

$$X_{k}^{m}$$
; X(m), X(m+k), X(m+2k),....X

$$\left(m + \left[\frac{N-m}{k}\right] \cdot k\right) (m=1,2,\dots,k)$$

Where $[\]$ denotes the Gauss' notation and both k and m are integers.

m is the initial time and

k is the interval time.

For a time interval equal to k, we get k sets of new time series. In the case of k=3 and N=100, the three time series obtained by the above process are described as follows:

$$X_3^1$$
 X(1), X(4), X(7),.... X(97),X(100)
 X_3^2 ; X(2),X(5),X(8),.....X(98)

$$X_{3}^{3}$$
; X(3),X(6),X(9),.....X(99)

We define the length of the curve, Xkm as follows:

$$L_{m}(k) = \left\{ \begin{bmatrix} \left[\frac{N-M}{k}\right] \\ \sum_{i=1}^{N-M} X(m+ik) - X(m+(i-1)\cdot k) \end{bmatrix} \frac{N-1}{\left[\frac{N-m}{k}\right]\cdot k} \right\} / \binom{k}{k}$$

The term , N-1/[(N-m)/k].k represents the normalization factor for the curve length of the subset time series. We define the length of the curve for the time interval k, <L(k)>, as the average value over k sets of Lm(k). If <L(k)> α k^{-D}, then the curve is fractal with the dimension D.

Fractal dimension for daily sunspot numbers

First we make a new time series with daily sunspot numbers for a year and calculate <L(k)> as defined above in method for calculating the fractal dimension. Then we plot the logarithm of length, log <L(k)>, as a function of log k. The unit of k is a day in our case since we use the daily sun spot number data. If <L(k)> á k^{-D} , we judge that the curve is fractal. Then we deduce fractal dimension(D) from the slope of a plot.

Table 1. An example of calculated values of <L(k)> for the daily sun spot number, 1990

k	<l(k)></l(k)>	
1	5032	
2	2044.5	
3	1231.333	
4	844.375	
5	625.6	
6	484.1111	
7	389.7347	
8	321.6406	
9	271.3827	
10	231.54	
11	199.5207	
12	173.1945	
13	150.2308	
14	129.7551	
15	112.2445	
16	96.79297	
17	83.97578	
18	73.11112	
19	63.54847	
20	55.5725	
21	49.06575	
22	43.55785	
23	38.5482	
24	34.22396	
25	30.4784	
26	27.8432	
27	25.76818	
28	24.19133	
29	23.09275	
30	22.44778	
31	21.72112	
32	21.2041	
33	20.76401	
34	20.1436	
35	19.67429	

An example of the curve length $\langle L(k) \rangle$ for the time series of daily sunspot numbers in 1990, on a doubly logarithmic scale was plotted and is shown in fig.1. We can determine fractal dimension (D) from the slope of this plot.



Figure 1. An example of the $\langle L(k) \rangle$ for the time series of daily sunspot numbers in 1990 as a function of day on a doubly logarithmic scale.



Figure 2. Annual variation of Fractal Dimension for short term range and long term range.

In the case of 1990 data, the curve breaks at about 10 days. The fractal dimension is 1.33 within 10 days with a standard error [S.E.(D)] value of 0.0125 and is 1.70 for longer than 10 days with a standard error [S.E.(D)] value of 0.0425. A time series is originally one-dimensional data; hence fractal dimension is 1 for a regular time series, and it is 2 for a completely random time series. Fractal dimension for the shorter time scale expresses more regular variation than one for the longer time scale shown in Figure 1. Fractal dimension, 1.70, for the longer time scale is fairly close to 2. This implies randomness of the time series and difficulty of prediction for this time range.

Annual variation of fractal dimension

Yearly variation of fractal dimension, deduced by daily sunspot numbers is shown in fig. 2. Yearly sunspot numbers vary with the 11-year solar cycle. However, yearly values of fractal dimension do not change in correspondence to solar activity.

This result implies that solar activity for short and medium time scales is not affected by long-term solar variation such as the 11-year cycle.

CONCLUSIONS

The fractal analysis method was adopted to examine a time series of daily sunspot numbers. We can evaluate randomness of a time series by determining fractal dimension. The average fractal dimension between 1990 and 2004 was about 1.43 within 10 days and about 1.72 for periods longer than 10 days. It should be noted that the data exhibits Power Law (Fractal) behavior for shorter time scales and becoming completely random (or unpredictable) for longer time scale. This result reveals the possibility of short-term prediction and the difficulty of medium –term prediction (longer than 10 days).

Sunspot numbers vary according to the 11-year solar cycle. However, annual values of fractal dimension do not change in concert with this cycle. This may suggest that the physical mechanism producing short and medium-scale time variation does not change throughout the 11 - year cycle.

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