J. Ind. Geophys. Union (October 2010)
Vol.14, No.4, pp.259-264

GPS Position Error Analysis for Precise Surveying and GAGAN Applications over the Indian Subcontinent

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ABSTRACT

The multipurpose usage of NAVSTAR Global Positioning System (GPS) has developed enormously within the last three decades. With the elimination of Selective Availability (SA) on May 2nd, 2000, the usefulness of the system for civilian users was even more pronounced. The GPS accuracy relies in the precise knowledge of the satellite orbits and the time. In this paper a non recursive point solution approach algorithm is proposed for the receiver position estimation. The results are compared with a recursive least squares approximation and multipolynomial resultant approach methods. The position error analysis is made for all the three methods. The minimum, maximum, mean ($\bar{x}$), standard deviation (σ) and variance (σ^2) of the X position errors are 1.763m, 23.587m, 16.198m, 6.001m and 36.012m^2 respectively due to the proposed algorithm. The analysis clearly shows that the proposed algorithm gives a better user position than the other two methods and particularly helpful for surveyors and GPS Aided Geo Augmented Navigation (GAGAN) users over the Indian subcontinent.

INTRODUCTION

The GPS system consists of 24 satellites, evenly distributed in 6 orbital planes around the globe at an altitude of about 20,200 km. Each satellite transmits on two frequencies of signals (f1=1575.42 MHz and f2=1227.60 MHz). The GPS receiver calculates its position by precisely timing the signals sent by GPS satellites high above the Earth. Each satellite continually transmits messages which include the time the message was transmitted, precise orbital information (the ephemeris) and the general system health (Rao 2010, Strang & Borre 1997, Bradford 1996). The receiver utilizes the messages it receives to determine the transit time of each message and computes the distances to each satellite. These distances along with the satellite's locations are used with the possible aid of trilateration, depending on which algorithm is used, to compute the position of the receiver. Three satellites might seem enough to solve for position, since space has three dimensions and a position near the Earth's surface can be assumed. However, even a very small clock error multiplied by the very large speed of light—the speed at which satellite signals propagate—results in a large positional error. Therefore, receivers use four or more satellites to solve for the receiver's location and time (Liu, Tsai & Jung 1996, Leick 1995, Sardon, Rius & Zarraoa 1994).

The GPS pseudo-ranging four-point problem is concerned with the determination of the four unknowns comprising the three components of the receiver position and the stationary receiver range bias from four observed pseudo-ranges to four satellite transmitters of given geocentric position. Geometrically the four unknowns are obtained from the intersection of four spherical cones given by the pseudo-ranging equations (Pratap & Per 2001, Krause 1987). Several procedures have been put forward for obtaining closed form solution of the problem. In this paper, a recursive least squares approximation algorithm, multipolynomial resultant approach algorithm and a proposed non recursive point solution approach algorithms are used for receiver position estimation and comparison of their performance.

RECEIVER POSITION ESTIMATION ALGORITHMS

Recursive Least Squares Approximation Algorithm

The common way to solve the four nonlinear simultaneous equations is through linearization. But usually the number of satellites visible at particular instant will be more than four. When more than four satellites are available, a more popular approach to solve the user position is to use all the satellites. In this case, first the set of nonlinear equations are linearized. Now there are more number of equations than unknowns. The popularly known least squares approach algorithm can be used to find the solutions (Farrell & Barth 1999).

Multipolynomial Resultant Approach Algorithm

This method is an algebraic procedure. The advantage is that the procedure is straight forward and simple to apply. To solve the nonlinear GPS pseudo-ranging four-point equations, first they have to be converted into algebraic (polynomial) form and reduced to linear equations. The algebraic tools of Multipolynomial resultant provide symbolic solutions to the GPS four-point pseudo-ranging problem. The various forward and backward substitution steps inherent in the classical closed form solutions of the problem are avoided. Similar to the Gauss elimination technique in linear systems of equations, the Multipolynomial resultant approaches eliminate several variables in a multivariate system of nonlinear equations in such a manner that the end product normally consists of univariate polynomial equations (in this case quadratic equations for the range bias expressed algebraically using the given quantities) whose roots can be determined by existing functions in MATLAB (Joseph & Erik 2002).

Non-recursive Point Solution Approach Algorithm

The standard GPS positioning system of pseudorange nonlinear equations are

$$\hat{\rho}_i = \sqrt{(X_i - x)^2 + (Y_i - y)^2 + (Z_i - z)^2} + c\,dt \tag{1}$$

where, (X_i, Y_i, Z_i) is the satellite position of the i^{th} satellite, (x, y, z) is the receiver location and c dt is the range bias. C is speed of light and dt is receiver clock offset. To solve the system of nonlinear equations (1), different approaches exist (Seiji & Toshiyuki 2006). The proposed non-recursive closed form point solution method is described below. The advantage of this method is that it requires less computational time compared to the above two methods.

The pseudorange equation corresponding to the i^{th} satellite can be written as

$$\hat{\rho}_i - b = \sqrt{(X_i - x)^2 + (Y_i - y)^2 + (Z_i - z)^2} \tag{2}$$

where b = c dt. Squaring on both sides of Eq. (2) and regrouping terms, we get

$$(X_i^2 + Y_i^2 + Z_i^2 - \hat{\rho}_i^2) - 2(X_i x + Y_i y + Z_i z - \hat{\rho}_i b) = -(x^2 + y^2 + z^2 - b^2) \tag{3}$$

If the Lorentz inner product is defined as

$$\langle e, f \rangle = e^T K f \tag{4}$$

where, K=diagonal matrix [1,1,1,-1]

Equation (3) can be written as

$$\frac{1}{2}\left\langle \begin{bmatrix} L_i \\ \hat{\rho}_i \end{bmatrix}, \begin{bmatrix} L_i \\ \hat{\rho}_i \end{bmatrix} \right\rangle - \left\langle \begin{bmatrix} L_i \\ \hat{\rho}_i \end{bmatrix}, \begin{bmatrix} l \\ b \end{bmatrix} \right\rangle + \frac{1}{2}\left\langle \begin{bmatrix} l \\ b \end{bmatrix}, \begin{bmatrix} l \\ b \end{bmatrix} \right\rangle = 0 \tag{5}$$

where $L_i=[X_i, Y_i, Z_i]^T$ and $l = [x, y, z]^T$

Considering that each pseudorange generates an equation of the form Eq. (5), four equations are sufficient to solve for the unknowns. If we define matrix A with all known quantities as

$$A = \begin{bmatrix} X_1\ Y_1\ Z_1\ \hat{\rho}_1 \\ X_2\ Y_2\ Z_2\ \hat{\rho}_2 \\ X_3\ Y_3\ Z_3\ \hat{\rho}_3 \\ X_4\ Y_4\ Z_4\ \hat{\rho}_4 \end{bmatrix}$$

The four equations may be written in compact form as

$$\beta - AK\begin{bmatrix} l \\ b \end{bmatrix} + \delta e = 0 \tag{6}$$

where $e = [1\ 1\ 1\ 1]^T$

The j^{th} entry of β is given by

$$\beta_j = \frac{1}{2}\left\langle \begin{bmatrix} L_i \\ \hat{\rho}_i \end{bmatrix}, \begin{bmatrix} L_i \\ \hat{\rho}_i \end{bmatrix} \right\rangle \quad \text{and} \quad \delta = \frac{1}{2}\left\langle \begin{bmatrix} l \\ b \end{bmatrix}, \begin{bmatrix} l \\ b \end{bmatrix} \right\rangle$$

Then from Eq. (6), we get

$$\begin{bmatrix} l \\ b \end{bmatrix} = KA^{-1}(\delta e + \beta) \tag{7}$$

By substituting Eq. (7) into Eq. (6), we will get the quadratic equation in δ

$$\langle A^{-1}e, A^{-1}e \rangle \delta^2 + 2(\langle A^{-1}e, A^{-1}\beta \rangle - 1)\delta + \langle A^{-1}\beta, A^{-1}\beta \rangle = 0 \tag{8}$$

Equation (8) gives two possible values for δ. Now the two solutions are obtained by inserting the values of δ into Eq. (7) and one among them is correct one.

RESULTS AND DISCUSSION

The GPS data required for the paper was collected from a newly installed dual frequency GPS receiver (NovaTel make DLV3) at Andhra University College of Engineering, Visakhapatnam. The two data files (Navigation data and Observation data) corresponding to 15th February, 2010 were collected for implementing and comparing the performance of the three algorithms. The navigation data file consists of 38 parameters. But for the calculation of satellite position, elevation angle etc., only 23 parameters are used. The necessary data processing is done and the required parameters are kept in the files for the estimation of user position. The user position is estimated using all the three algorithms and the error analysis is carried out. The actual user position coordinates are X=706970.909 m, Y=6035941.022 m and Z=1930009.582 m. Fig. 1 shows local time vs. error in X, Y, Z positions estimated using recursive least squares algorithm over a day. The mean, standard deviation and variance values are also shown in Fig. 1. The position error data is smoothed by averaging over an hour. Fig. 2 shows the position

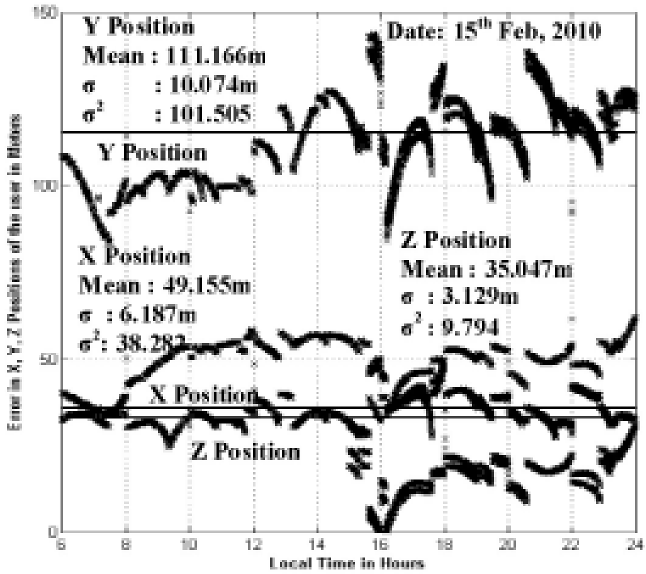

Figure 1. Local time vs. Error in user position (Recursive Least Squares method).

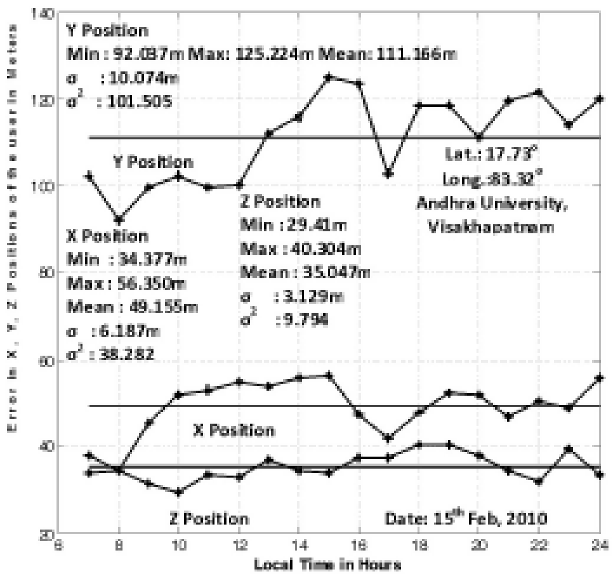

Figure 2. Local time vs. Error in user position smoothed over an hour (Recursive Least Squares method).

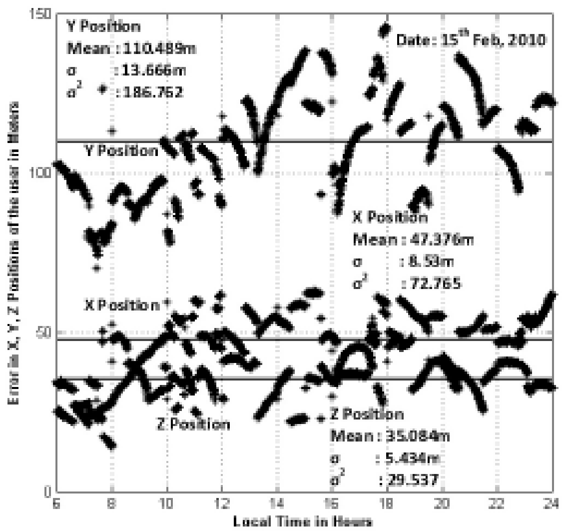

Figure 3. Local time vs. Error in user position (Multipolynomial Resultant method).

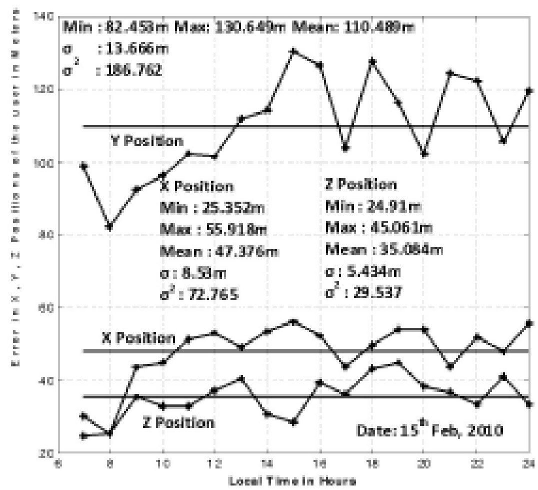

Figure 4. Local time vs. Error in user position smoothed over an hour (Multipolynomial Resultant method).

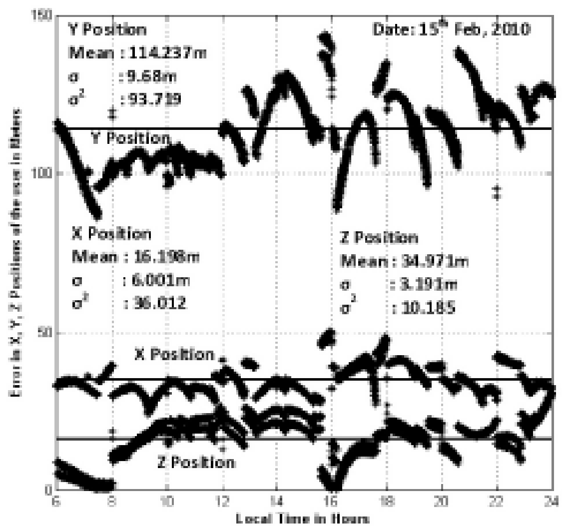

Figure 5. Local time vs. Error in user position (Proposed Point Solution method).

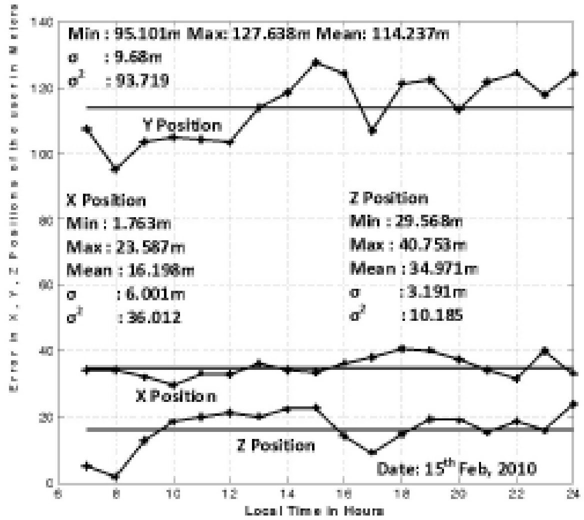

Figure 6. Local time vs. Error in user position smoothed over an hour (Proposed Point Solution method).

Table 1. Comparison of the position error (m) performance of all the three methods

S. No.	Parameter	Recursive least squares approximation method			Multipolynomial resultant approach method			Point solution approach method		
		X Position	Y Position	Z Position	X Position	Y Position	Z Position	X Position	Y Position	Z Position
1	Minimum	34.377	92.037	29.410	25.352	82.453	24.910	1.763	95.101	29.568
2	Maximum	56.350	125.224	40.304	55.918	130.649	45.061	23.587	127.638	40.753
3	Mean	49.155	111.166	35.047	47.376	110.489	35.084	16.198	114.237	34.971
4	Standard deviation (σ)	6.187	10.074	3.129	8.530	13.666	5.434	6.001	9.680	3.191
5	Variance (σ^2)	38.282	101.505	9.794	72.765	186.762	29.537	36.012	93.719	10.185

errors after smoothing over an hour.

Fig. 3 shows local time vs. error in X, Y, Z positions due to multipolynomial resultant approach algorithm over a day. The smoothed position errors are shown in Fig. 4. Fig. 5 shows local time vs. error in X, Y, Z positions due to non-recursive point solution approach algorithm over a day. The smoothed position errors are shown in Fig. 6.

From the results, it is observed that, the position estimation by recursive least squares algorithm gives better values than multi-polynomial resultant approach algorithm and the point solution approach algorithm gives better user position than the recursive least squares algorithm. The analysis is done for all the three methods and the results are presented in Table 1.

CONCLUSIONS

A new point solution algorithm based on non recursive approach is proposed for accurate GPS receiver position estimation. The results of the proposed method are compared with the recursive least squares method and multipolynomial resultant approach method. The pseudorange errors due to the atmosphere, multipath and satellite clock are not taken into consideration in the receiver position computation. The minimum, maximum, mean ($\bar{x}$), standard deviation (σ) and variance (σ^2) of the X position error are 1.763m, 23.587m, 16.198m, 6.001m and 36.012m^2 respectively due to the proposed algorithm and are less than the other two methods. Using the proposed algorithm, a better position accuracy in the order of ± 1meter can be obtained, if all the errors are taken into account. From the analysis it is also concluded that the proposed method gives best user position than the other two methods. Therefore, the proposed algorithm will be helpful to the precise accuracy requirement users over the Indian subcontinent like carrier phase based surveying and GAGAN users.

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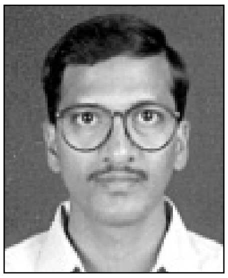
(Revised accepted 2010 October 9; Received 2010 August 20)



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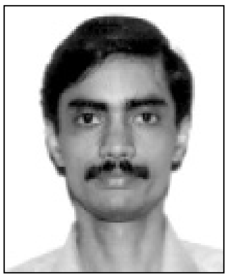

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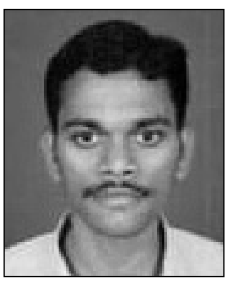

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