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Comparison of wave equation migration techniques over complex geological structures

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ABSTRACT
Wave-equation migration techniques play an important role in imaging complex geological structures and are becoming more acceptable as data processing tools. Usually, these subsurface geological structures provide complex seismic signature due to their geometrical set up. As a result, it is extremely difficult to interpret these seismic sections in terms of subsurface configuration. In order to image these complex geological features, three wave equation migration schemes such as Phase-Shift (PS), Phase-Shift Plus Interpolation (PSPI) and Stolt migrations schemes are applied to the synthetic time sections generated over the geological models. In addition, we have added Gaussian noise with signal to noise ratio (sn=30) on corresponding sections. On comparison, it is observed that the Stolt migration imaged the subsurface better than Phase-Shift (PS) and Phase-Shift Plus Interpolation (PSPI) migration schemes. Besides, Stolt migration generates minimum noise, in the form of numerical artifacts compared to other two techniques. Even after adding significant noise during migration process, it is observed that the reflectors are clearly delineated.

INTRODUCTION

Wave equation migration techniques for subsurface imaging are widely applied in hydrocarbon exploration and play an important role in imaging complex geological structures. In historical perspective, the seismic migration was implemented as early as the 1920's as a graphical method. Subsequently Hagedoorn (1954) introduced migration, using the concept of surfaces of maximum convexity. Later Mayne (1962) developed CMP stack and the application of digital signal processing techniques to seismic data. Schneider (1971) has proposed the first wave-equation-based digital migration method. This technique was the outcome of the work carried out by Claerbout in Stanford Exploration Project. He derived migration as a finite-difference solution of an approximate wave equation. Then Claerbout and Doherty (1972) gave the finite-difference solution of an approximate wave equation. This is followed by Kirchhoff wave–equation migration (Schneider, 1978). In the same year, frequency-wave number migration (Gazdag, 1978; Stolt, 1978) appeared in the oil industry. Since then migration techniques have been improved tremendously, depending upon the objectives and data quality based on wave equation. In our study, Phase-Shift (PS), Phase Shift Plus Interpolation (PSPI), and Stolt migration methods are considered for analysis on typical geological structures suitable for hydrocarbon accumulation. In addition, a comparison study has been undertaken to judge the imaging accuracy of these migration techniques with and without noise. The analysis has been carried out on synthetically generated seismic section, as a numerical study.

MATHEMATICAL BACKGROUND

We illustrate below, the basic mathematical formulation of the f-k migration techniques viz. PS, Stolt and PSPI algorithms. Details of notations used in the formulae are given in Table-1. We start with solution of the scalar wave equation for the zero-offset wave field as given by equation (1) and assume

$$P(k_x, z, \omega) = P(k_x, 0, \omega) \exp(-ik_z z) \qquad (1)$$

for a horizontally layered earth model with a varied velocity with depth v(z), where z is depth axis (positive downward). By inverse Fourier transforming equation (1), where k_x is replaced with k_y, where k_x is wave number in the lateral (i.e. X-) direction, where k_y is wave number in the horizontal (i.e. Y-) direction, we have:

$$P(y, z, t)=\iint P(k_y, 0, \omega) \exp(-ik_z z).\exp(-ik_y y+i\omega t)\, dk_y d\omega \quad (2)$$

$$\text{Where } k_z=2\frac{\omega}{v}\left[1-\left(\frac{vk_y}{2\omega}\right)^2\right]^{1/2} \quad (3)$$

where k_z is wave number in the vertical (i.e. Z-) direction, where v is velocity,

The imaging principle t=0 then is applied to get the migrated section p(y, z, t=0).

$$P(y, z, t=0)=\iint P(k_y, 0, \omega).\exp(-ik_y y - ik_z z)\, dk_y d\omega \quad (4)$$

This is the equation for Phase-Shift method (Gazdag, 1978). Equation (4) involves an integration over frequency and inverse Fourier transformation along mid point axis y.

We now consider the special case of v(z)=v=constant. Stolt (1978) devised a migration technique that involves an efficient mapping in the 2-D Fourier transform domain from temporal frequency ω to the vertical wave number k_z. We rewrite equation (3) to get:

$$\omega = \frac{v}{2}\frac{k_z}{(\sqrt{k_y^2+k_z^2})} \quad (5)$$

$$P(y, z, t=0)=\iint \left[\frac{v}{2}\frac{k_z}{(\sqrt{k_y^2+k_z^2})}\right].P\left[k_y, 0, \frac{v}{2}\sqrt{k_y^2+k_z^2}\right] .\exp(-ik_y y - ik_z z)\, dk_y dk_z \quad (6)$$

This is the equation for constant-velocity Stolt migration. It involves two operations in the f-k domain. First, the temporal frequency ω is mapped onto the vertical wave number k_x ia equation (5).

To maintain high accuracy for small dip, Gazdag and Sguazzero (1984), introduced a laterally varying time-shift in the (w, x) domain as a preprocessor for the input data. Specifically, they defined a modified field $P^*(z)$ in the space domain. Basic principle of PSPI migration consists of two steps:

The wave field is extrapolated by the phase-shift method (Gazdag, 1978) using n laterally uniform velocity fields. The intermediate result is n reference wave fields.

The actual wave field is computed by interpolation from the reference fields.

Solution to the wave equation:

$$P^*(z) = P(z)\, exp\left[\pm i \frac{\omega}{v(x)} dz\right] \quad (7)$$

$P^*(z)$ is transformed into the wave number domain with FFT. In the wave number domain, the influence of the previous time-shift is compensated by the ω/v_r dz term in the following formula.

$$P(z+dz) = P^{\wedge *}(z)\, exp\left[\left[\pm i \left(k_z \pm \frac{\omega}{v_r} dz\right]\right. \quad (8)$$

Here, $P^\wedge$ is the intermediate wave field, v_r is the reference velocity, k_z is the vertical wave number, defined as

$$\sqrt{\left(\frac{\omega}{v_r}\right) - k_x^2} \quad (9)$$

And the $P^{\wedge *}$ is the Fourier transform of P^* form (ω, x) to (ω, k_x). Such a time shift term is important in the implementation of the PSPI method. But it also computes the reference velocities according to the distribution of velocities. More reference velocities will be used when the lateral velocity variation is strong and fewer velocity values will be used when the velocity contrast is small.

METHODOLOGY

Conceptually, there are two distinct parts to migration—namely, extrapolation and imaging. By extrapolation, we mean reconstruction from surface data of the wave field at depth. By "imaging," we mean some formula or principle which allows us to obtain local reflection strength from extrapolated data. Since migration is an imaging procedure which takes seismic wave fields recorded at the surface of the earth as an input and then calculates the location and strength of reflectors, it can be based on the wave equation which governs the propagation of the recorded wave field. In our study, we have considered three migration wave equation techniques PS, PSPI and Stolt migrations respectively. The above migration methods are based on the downward continuation process propagating the wave field from one depth step to the next by a phase shift operation. To demonstrate the above migration techniques, we have constructed geological model that contains velocity-depth information. After constructing geological model, we have generated synthetic seismograms through forward modeling scheme. Subsequently, we have applied the above migration techniques in generating corresponding geological models. The entire analysis has been carried out using the resource of Seismic Unix (SU, 2012).

RESULTS AND DISCUSSIONS

In the present work, the phase-shift (PS), Phase Shift Plus Interpolation (PSPI) and Stolt migration techniques have been applied on the generated zero-offset sections in order to judge the imaging accuracy and background noise. In addition we have added Gaussian noise with signal to noise ratio (sn=30), on corresponding sections. The three models considered a salt dome, a pinch-out and a reef, which are suitable for hydrocarbon accumulation.

Model 1

Figure 1 (a) represents velocity-depth model for a salt-dome structure. The selected background velocity was 2.0 km/s with dv/dz = 0.005 and dv/dx = 0.005, indicating velocity variation in lateral as well as vertical direction. The top part of the model indicates one horizontal reflector followed by smoothly folded reflector. The salt- dome structure appears just after the folded bed with numerous faults. Figure 1 (b) represents zero offset section without noise. Fig 1(c) represents zero-offset section with addition of Gaussian noise. It is observed that significant diffractions are occurring from the edges of the reflectors corresponding to various faults. Moderate amplitude is noticed over the reflectors, along with noise. This noise might have generated during the numerical process. In general, the seismic signature has brought out the salt dome structure. In order to improve the resolution, three migration schemes have been applied to the zero-offset sections. Figures 1(d, e, f) represent the migrated sections. Subsequently, noise has been added and migrated sections are produced. Figures 1(g, h, i) represent the migrated sections with noise. On comparison, it is observed that Stolt migration provided the good image quality without artifacts. PSPI algorithm also yielded good sub salt structure with negligible amount of migration artifacts. Where as in the case of PS migration the reflectors are not properly imaged. In addition more diffraction events are observed. A comparison of generated models indicates that Stolt scheme is superior in comparison to other two migration schemes in terms of imaging capability, handling amplitude and phase-shift. Even after adding significant noise during migration process, it is observed that the reflectors are clearly delineated, by the Stolt migration technique.

Model 2

Figure 2 (a) represents velocity-depth model for a pinch-out structure. The selected background velocity was 2.0 km/s with dv/dz = 0.005 and dv/dx = 0.005, indicating velocity variation in lateral as well as vertical direction. The model shows one horizontal reflector followed by five inclined reflectors. Three pinch-out features have been incorporated in the velocity- depth model simulating actual subsurface

Table 1: Notations used in the text

Notations	Expanded form
PS	Phase-Shift migration
PSPI	Phase Shift Plus Interpolation migration
P	Actual wave field
P*	Modified wave field
P^	Intermediate wave field
k_x	Wave number in the lateral (i.e., x-)direction
k_y	Wave number in the horizontal (i.e., y-)direction
k_z	Wave number in the vertical (i.e., z-)direction
v	velocity
x	Horizontal spatial axis
z	Depth axis (positive downward)
t	Time

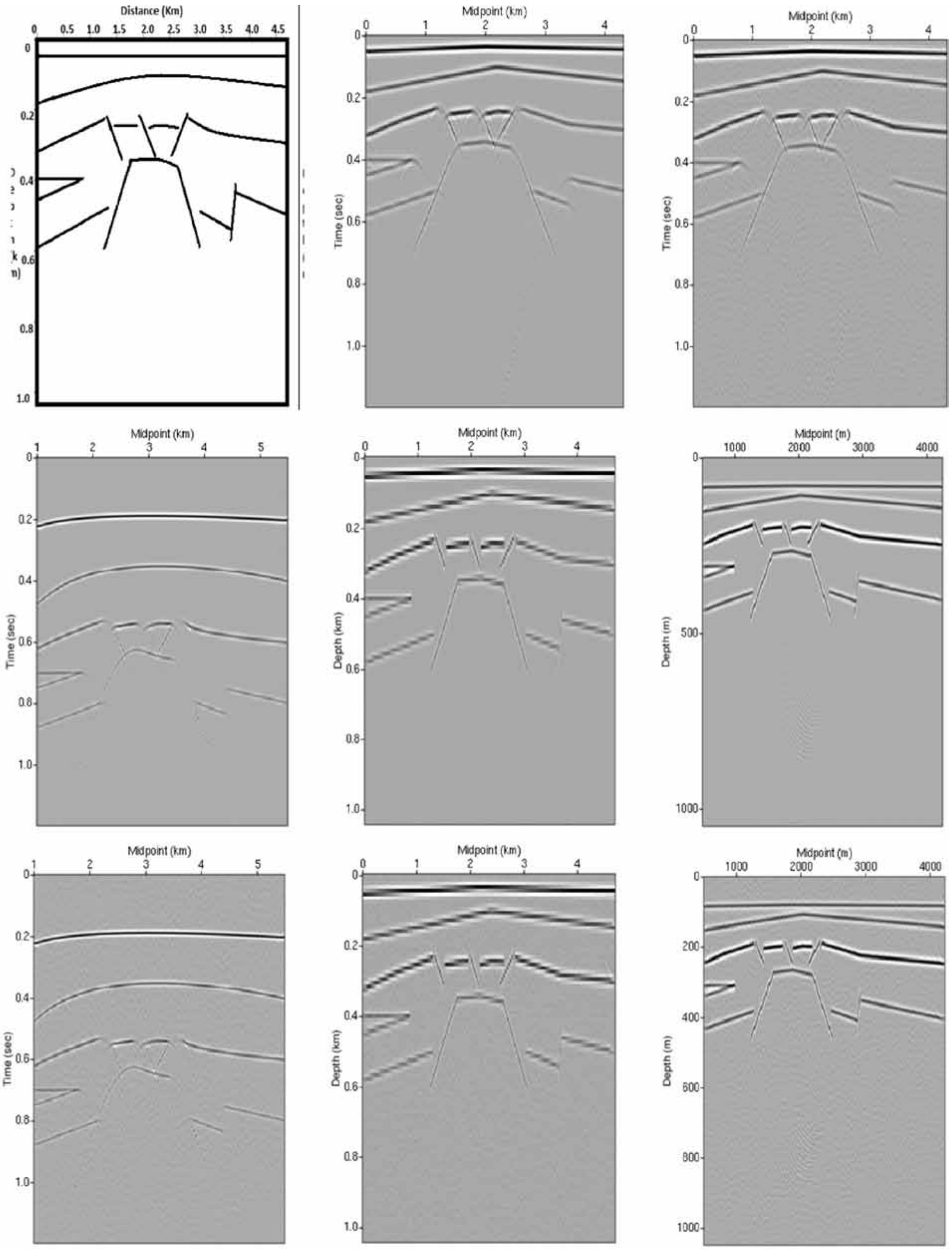

Figure 1. Comparison of wave equation migration over a salt-dome model (a) velocity-depth model (b) & (c) Zero-offset sections (without & with) noise, (d, e, f) are migrated sections (without noise) of PS, PSPI, Stolt methods, (g, h, i) are the migrated sections (with noise) of PS, PSPI, Stolt methods.

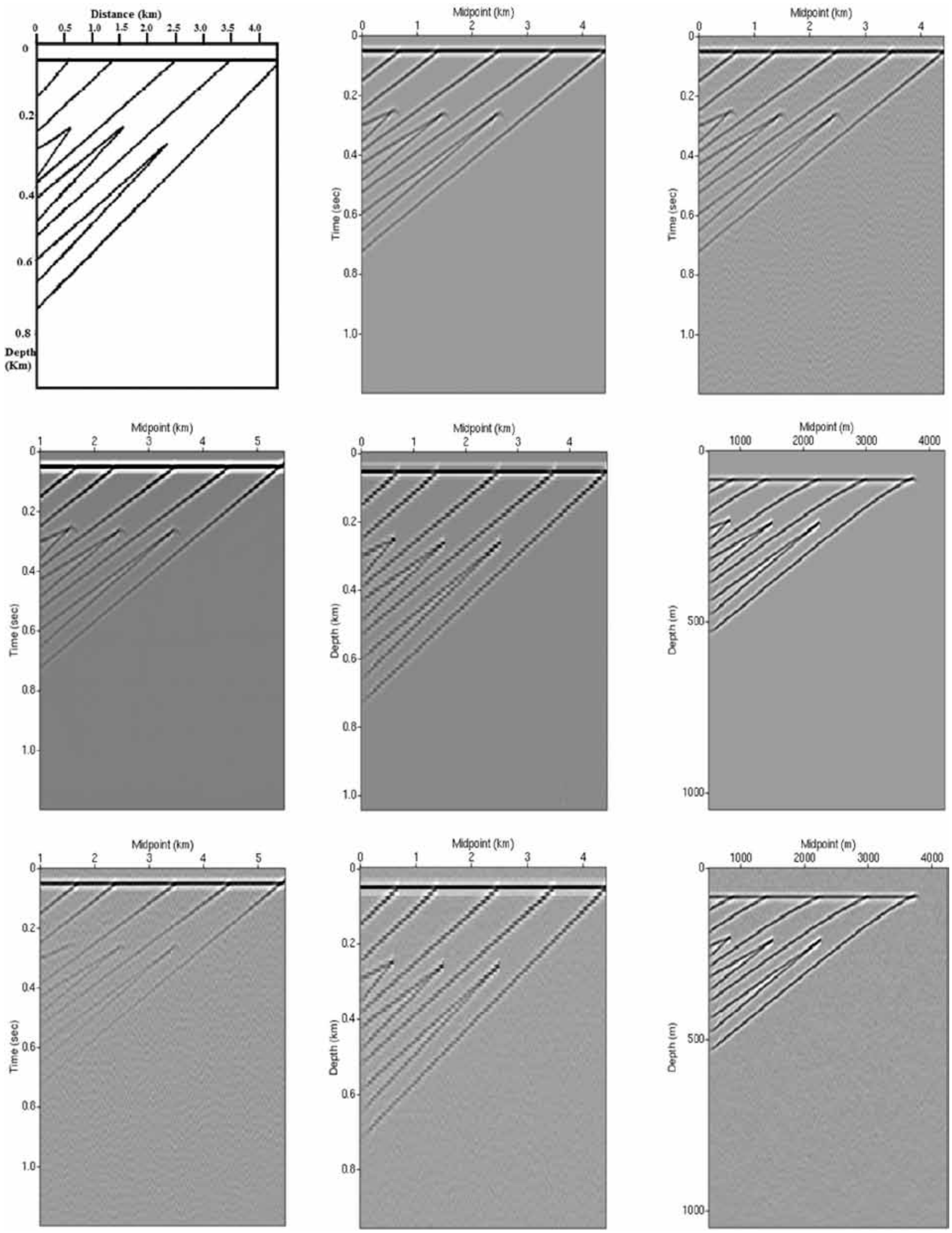

Figure 2. Comparison of wave equation migration over a pinch-out model (a) velocity-depth model (b) & (c) Zero-offset sections (without & with noise), (d, e, f) are migrated sections (without noise) of PS, PSPI, Stolt methods, (g, h, i) are the migrated sections (with noise) of PS, PSPI, Stolt methods.

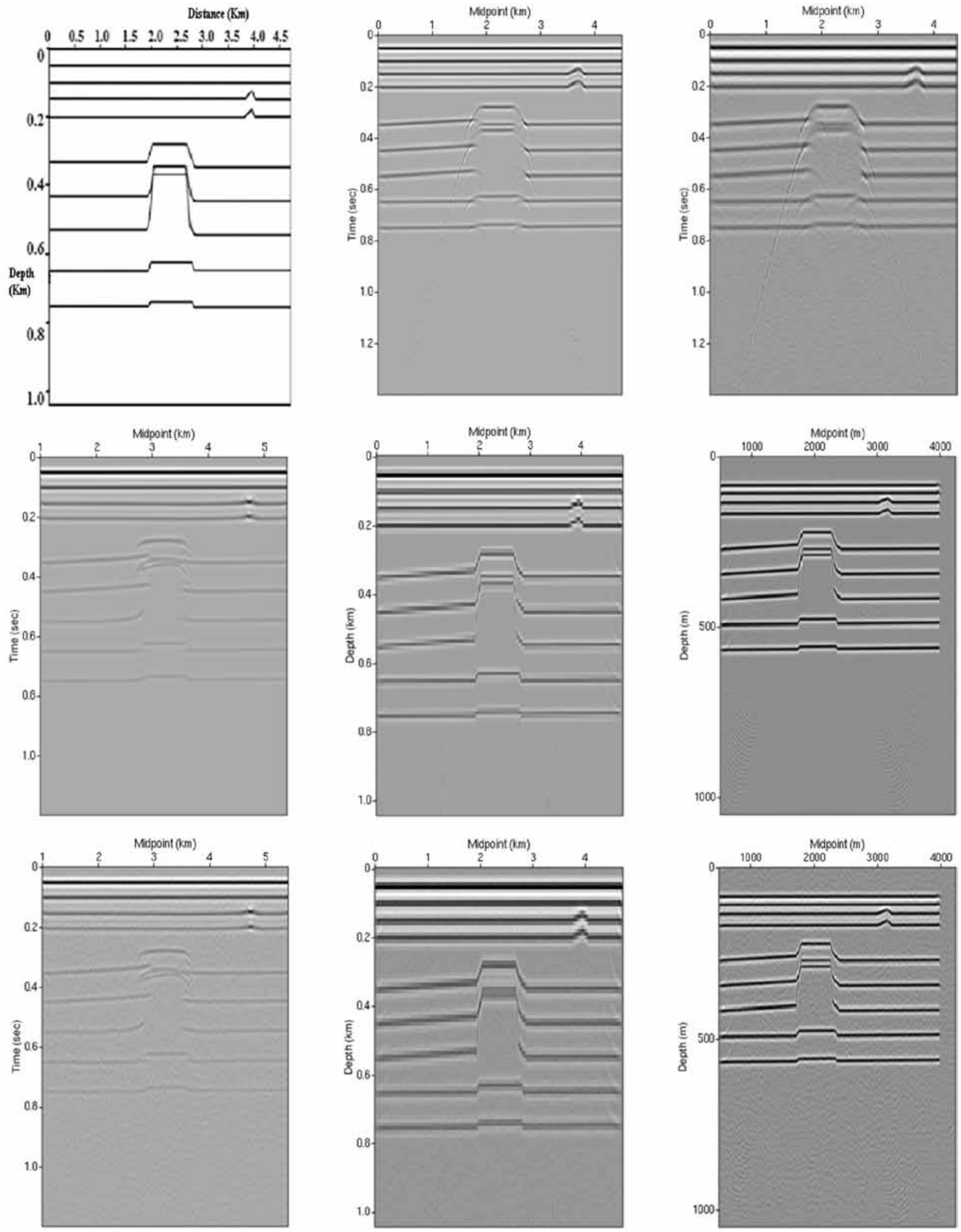

Figure 3. Comparison of wave equation migration over a reef model (a)velocity-depth model (b) & (c) Zero-offset sections (without & with) noise, (d, e, f) are migrated sections (without noise) of PS, PSPI, Stolt methods, (g, h, i) are the migrated sections (with noise) of PS, PSPI, Stolt methods.

configuration. Figure 2 (b) represents zero offset section without noise. Fig 2 (c) represents zero-offset section with addition of Gaussian noise. It is observed that significant diffractions are occurring from the edges of the reflectors corresponding to various faults. Moderate amplitude is noticed over the reflectors along with noise. This noise might have generated during the numerical process. In order to improve the resolution, three migration schemes have been applied to above zero-offset sections. Figures 2 (d, e, f) represent the migrated sections. Subsequently, noise has been added to the section and migrated sections are produced. Figures 2 (g, h, i) represent the migrated sections with noise. On comparison, it is observed that PS migration provided a negligible amount of diffraction, which has crept into the time section at the far offset. The PSPI migration provided good image quality with negligible amount of noise, with diminished amplitude. The Stolt migration provided superior image in comparison to other two migrations (in terms of imaging capability, handling amplitude, and phase-shift)

Model 3

Figure 3 (a) represents velocity-depth model for a reef structure. The selected background velocity was 2.0 km/s with dv/dz = 0.005 and dv/dx = 0.005, indicating velocity variation in lateral as well as vertical direction. The top part of the model indicates two horizontal reflectors followed by two smoothly folded reflectors associated with carbonate reef structure. Figure 3 (b) represents zero offset section without noise. Fig 3 (c) represents zero-offset section with addition of Gaussian noise. It is observed that significant diffractions are occurring from the edges of the reflectors corresponding to various faults. Moderate amplitude is noticed over the reflectors along with noise. This noise might have generated during the numerical process. In general, the seismic signature has brought out the reef structure. In order to improve the resolution, three migration schemes have been applied to the zero-offset sections. Figures 3 (d, e, f) represent the migrated sections. Subsequently, noise has been added to the section and migrated sections are produced. Figures 3 (g, h, i) represent the migrated sections with noise. On comparison, it is observed that Stolt migration provided the good image quality with insignificant artifacts. PSPI algorithm also delineated reef structure with diminished amplitude. Where as in the case of PS migration, the reflectors are not properly imaged. In addition, noise in form of diffraction is observed. A comparison of generated models indicates that Stolt scheme proved to be superior in comparison to other migrations in terms of imaging capability, handling amplitude and phase-shift. Even after adding significant noise during migration process, it is observed that the reflectors are clearly delineated, by the Stolt migration technique.

CONCLUSIONS

The following conclusions are arrived from the present study:

Three geological models, viz; a salt- dome, a pinch-out and a reef structure are considered for the present analysis.

i. On comparison, it is observed that the Phase-Shift migration could not bring out the reflectors clearly for salt-dome and reef structures where as the pinch-out structure is moderately delineated.
ii. The other two migration schemes corresponding to PSPI and Stolt have delineated the reflections. But there was a significant drop in amplitude for PSPI migrated section as compared to stolt migration.
iii. Computational noise in the form of numerical artifacts is more dominant in PS migrated section compared to PSPI and Stolt migrated sections.
iv. Overall, Stolt scheme provided the best migrated picture of the subsurface by properly delineating the reflections. In addition, the faults are clearly delineated with restoration of proper throw.
v. Out of the three migration techniques, Stolt migration scheme yielded the minimum noise in terms of numerical artifacts ,as compared to other two techniques.

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