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Fractal dimensional analysis of Cyclonic disturbances over the North Indian Ocean

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ABSTRACT
In this paper, we use fractal dimensional analysis to investigate the number of cyclonic disturbance that includes depressions, cyclonic storms and severe cyclonic storms over the North Indian Ocean (comprising Bay of Bengal and Arabian Sea) using the Hurst exponent. We use the rescaled range (R/S) analysis to estimate the Hurst exponent for a period of 104 years (1901-2004) of cyclone data. The value of the Hurst exponent is corroborated by computing the correlogram of the concerned time series. The results are validated by Detrended Fluctuation Analysis (DFA). The distinct value of the Hurst exponent shows the persistence nature of the cyclonic disturbances over the North Indian Ocean.

INTRODUCTION

The term "cyclone" is a generic term covering all the four atmospheric disturbances, namely, low pressure areas, depressions, deep depressions and cyclonic storms. There are some variations in the definition and names of their stages of storm's intensity in one region to other. In the North Indian Ocean (comprising Bay of Bengal and Arabian Sea), these stages are divided into six categories depending upon the maximum sustained surface winds associated with the system. The new nomenclature introduces by Indian Meteorological Department (IMD) in 1998 for description of cyclonic disturbances in the North Indian Ocean is given in table 1.

About 80 tropical cyclones (with wind speeds $\geq$ 34 knots) form in the world's waters every year (McBride, 1995) of these about 6.5% develop in the Bay of Bengal and Arabian Sea (Neumann, 1993). The frequency of occurrence of tropical cyclone in the Bay of Bengal is about 4 times the frequency of those in the Arabian Sea. When compared with the frequency of occurrence of the tropical cyclone in the world's water every year, the Bay of Bengal's share comes out to be about 5.5%. The tropical cyclones forming in the Bay of Bengal hit the coast of India every year, causing heavy loss of life and property. The globally-averaged annual variation of cyclone occurrence is only about 10%. For instance, in the Australian/Southwest Pacific region, the average number of tropical cyclones observed during 1950-1986 was 14.8, with an annual variation of 40% (Evans, **1990).** The quality of the tropical cyclone databases can be highly variable (Holland, 1981). Different definitions, techniques and observational approaches may produce errors and biases in these datasets which could have implications for the study of the natural variation of tropical cyclone activities and the detection of possible historical trends (Nicholls et al., 1998).

The very limited instrumental record makes extensive analyses of the natural variability of global tropical cyclone activities difficult in most of the tropical cyclone basins. Vulnerability to tropical cyclones is becoming more pronounced because the fastest population growth is in tropical coastal regions. Understanding tropical cyclone genesis, development and associated characteristic features has been a challenging subject in meteorology over the last several decades. In recent years, attempts to associate tropical cyclone trends with climate change resulting from greenhouse warming has led to additional attention being paid to tropical cyclone prediction (Emanuel, 1987, Evans, 1992, Lighthill et al., 1994).

Forecasting the tropical cyclonic disturbance is the most challenging goal in recent times with an era of global warming scenario. Exploring possible changes in tropical cyclone activity due to global warming is not only of theoretical but also of practical importance. Before actually attempting to forecast the tropical cyclonic behaviour using the models, it becomes necessary to subject the data into certain preliminary analysis such as persistence, spectral, fractal dimensional, correlation dimensional analysis, etc. A trend analysis of normalized insured damage from natural disasters can potentially be useful for attempts at detecting whether there has been an increase in the frequency and/or intensity of natural hazards, whether caused by natural climate variability or anthropogenic climate change.

Time series from many physical systems display some form of self similarity (Turcotte, 1997). In physics, as well as other scientific disciplines, the Hurst exponent is often considered as an indicator for correlations in time series analysis (Feder, 1998; Bansal and Dimri, 2005; Chamoli et al., 2007; Bansal et al., 2010). In this paper, the persistence analysis of the cyclonic behaviour over the North Indian Ocean is carried out by estimating the Hurst exponent and Detrended Fluctuation Analysis.

DATA

A time series is a collection of observations of well-defined data items obtained through repeated measurements over time. Data collected irregularly or only once are not time series. An observed time series can be decomposed into three components: the trend (long term direction), the seasonal (systematic, calendar related movements) and the irregular (unsystematic, short term fluctuations). In this paper, we use 104 year cyclonic disturbance data for the period from 1901-2004, which includes depressions, cyclonic storms and severe cyclonic storms over the North Indian Ocean. Persistence analysis has been carried out for annual frequency of cyclonic disturbances, frequency of occurrence during southwest and northeast monsoon seasons. The corresponding frequencies of the occurrences of cyclonic disturbances are illustrated in fig. 1 (a, b and c). The studies are carried out in three parts since they would be helpful for both short and long term planners towards disaster mitigation. The statistical information regarding the annual, monthly/seasonal frequencies of tracks of cyclones and depressions over North Indian Ocean from 1891-2011 are available in the official website of Regional Meteorological Centre, Chennai www.rmcchennaieatlas.tn.nic.in.

METHODOLOGY

Hurst Exponent

The Hurst exponent is a parameter that quantifies the persistent or antipersistent (past trends to reverse in future) behaviour of a time series. It determines whether the given time series is completely random or has some long term memory. Ruzmaikin et al. (1994) examined whether or not the nonperiodic variations in solar activity are caused by a white-noise, random process. They evaluated the Hurst exponent for a time series of ^{14}C data from 6000 BC to 1950 AD. They find a Hurst exponent of 0.8 indicating a high degree of persistence in the variations of solar activity. The reconstructed sunspot numbers for the past 11360 years (Solanki et al., 2004) are found to be correlated with a Hurst exponent of ≈ 0.8 (Xapsos et al., 2009). Xapsos et al. (2009) also showed the evidence of 6000-year periodicity in the reconstructed sunspot numbers. The oscillating characters of the intensity of ENSO event (ENSO-event-index) can be well studied with the trends of Hurst parameter anomaly. H can be an alternate ENSO index (Huang and Morimoto, 2006).

Rescaled Range Analysis (R/S):

The Hurst exponent is used as a measure of the long term memory of time series, i.e. the autocorrelation of the time series. To calculate the Hurst exponent, estimation of the dependence of the rescaled range on the time span n of observation is very important (Feder, 1998). A time series of full length n (104 years) is divided into a number of shorter time series of length $n = N$, $N/2$, $N/4$... The average rescaled range is then calculated for each value of n. For a (partial) time series of length n, X_1, X_2...Xn, the rescaled range is calculated as follows:

1. Calculate the mean: m.
2. Create a mean-adjusted series: $Y_t = X_t - m$ for t= 1, 2.., n
3. Calculate the cumulative deviate series Z_t

$$Z_t = \sum_{i=1}^{t} Y_i \quad \text{for} \quad t = 1, 2, ..., n.$$

4. Compute the Range.
5. Compute the standard deviation S.
6. Calculate the rescaled range R(n) / S(n) and average over all the partial time series of length w.

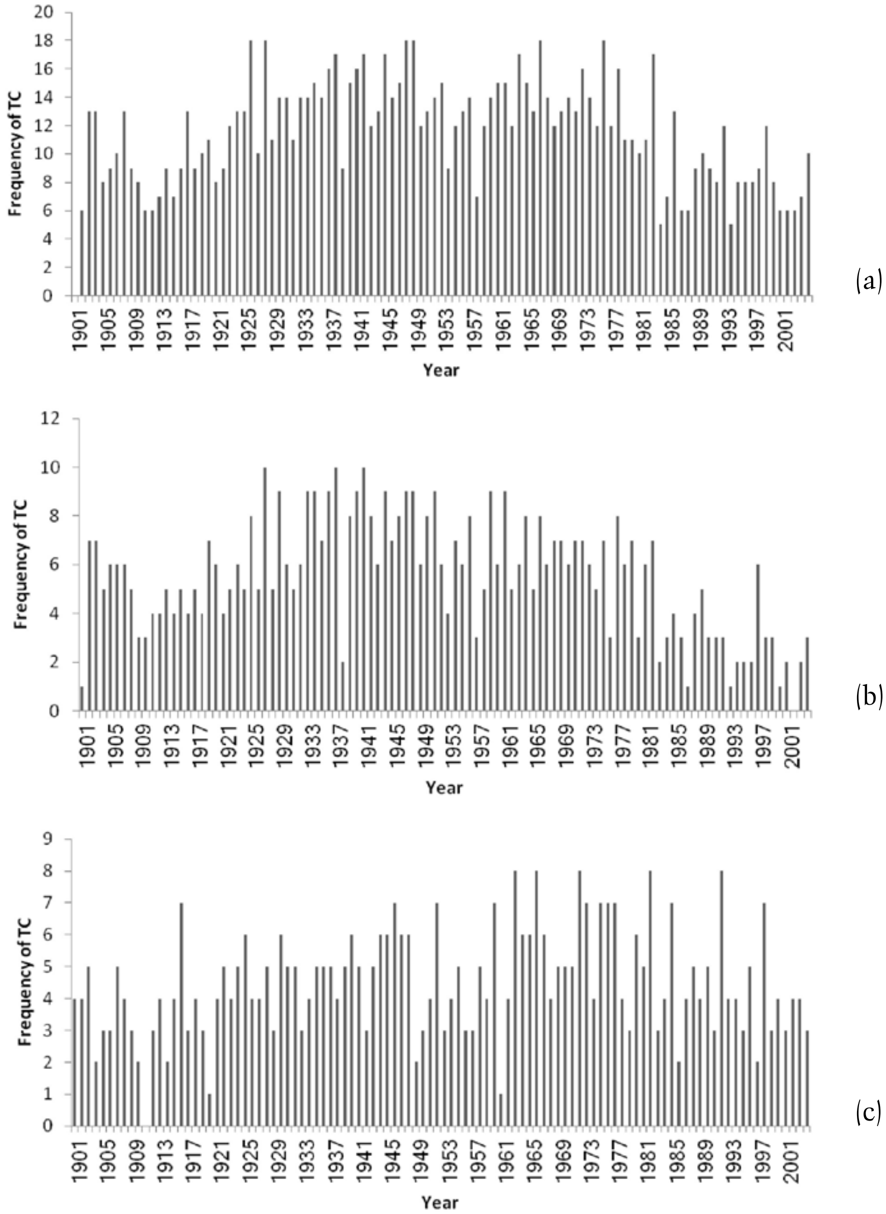

Figure 1. Frequency of occurrence of Cyclonic disturbances (including depressions, cyclonic storms and severe cyclonic storms) for (a) annual, (b) southwest and (c) northeast monsoon over the North Indian Ocean for the period 1901-2004

Hurst exponent is estimated by fitting the power law to the data.

In this present work, total number of data is 104. It divided into two sets of 52 each (N/2) then it is further divided into four sets of each 26 (N/4) and so on. The average rescaled range is then calculated for each value of n. Computing (R/S) (t_0, w) for time lag w the rescaled range for the time lag w is finally written as the average of those values (Here R and S is calculated for each time series of n). It has been observed that the rescaled range (R/S) over a time window of width w varies as a power law: $(R/S)_w = k\, w^H$, where k is a constant and H is the Hurst exponent. To estimate the value of the Hurst exponent, R/S is plotted against w on log-log axes. The slope of the linear regression gives the value of the Hurst exponent. A value of $0 < H < 0.5$ indicates a time series with negative autocorrelation (e.g. a

Table – 1. Terminology used by Indian Meteorological Department (IMD) to classify the intensity of Tropical Cyclones over the North Indian Ocean (Ramesh Kumar and Sankar, 2010)

Category	IMD classification	Wind speed	
		Knots	Kmph (approx.)
1	Low Pressure area	Less than 17	31
2	Depression	17 to 27	31 to 51
3	Deep depression	28 to 33	52 to 62
4	Cyclonic storm	34 to 47	63 to 87
5	Severe Cyclonic Storm	48 to 63	88 to 117
6	Very Severe Cyclonic storm	64 to 119	118 to 221
7	Super Cyclonic Storm	120 and above	222 and above

decrease between values will probably be followed by an increase), and a value of $0.5 < H < 1$ indicates a time series with positive autocorrelation (e.g. an increase between values will probably be followed by another increase). A value of $H=0.5$ indicates a true random walk, where it is equally likely that a decrease or an increase will follow from any particular value (e.g. the time series has no memory of previous values).

The Hurst exponent is related to the fractal dimension D of the time series curve by the formula

$$D=2-H$$

If the fractal dimension D for the time series is 1.5, there is no correlation between amplitude changes corresponding to two successive time intervals. Therefore, no trend in amplitude can be discerned from the time series and hence the process is unpredictable. However, as fractal dimension decreases to 1, the process becomes more and more predictable as it exhibits "persistence" - namely the process shows a clear trend. As the fractal dimension increases from 1.5 to 2, the process exhibits "anti-persistence".

DETRENDED FLUCTUATION ANALYSIS

DFA is a method for determining the statistical self-affinity of a signal or a time series. The DFA technique was introduced to investigate long-range power-law correlations. The obtained exponent is similar to the Hurst exponent, except that DFA may also be applied to signals whose underlying statistics (such as mean and variance) or dynamics are non-stationary (changing with time). Due to the simplicity in implementation, the DFA is now becoming a widely used method in physics and engineering. Sarkar and Barat (2005) investigated long time series of the rainfall records for all India and different regions of India and succeeded in finding evidence for power law distributions of the rainfall quantity. Peters et al. (2002) has presented power law behaviour in the distribution of rainfall over at least four decades. Orun and Koçak (2009) used DFA to calculate scaling exponent of daily temperature data for 52 stations in Turkey.

The DFA procedure is detailed below.

1. Calculate the cumulative sum Y(k) for the time series $x_1, X_2, \ldots X_n$ of length n

   $$Y(k) = \sum_{i=1}^{k} [x(i) - \langle x \rangle].$$

   here $< x >$ indicates the mean value of x (i)'s.

2. The profile Y (k) is divided into time windows of length w (n = N, N/2, N/4, ...)

3. The local trend for each segment is calculated by a least square fit of the data. The y coordinate of the fitted line is denoted by $Y_n(k)$. Then the Detrended time series for the segment duration 'n' as $Y_s(k) = Y(k) - Y_n(k)$.

4. The root-mean square fluctuation of the original time series and the Detrended time series is calculated by

$$F(n) = \{(\frac{1}{N}) \sum_{k=1}^{N} (Y(k) - Y_n(k))^2\}^{1/2}$$

Repeat this calculation to all segment sizes (n= 104, w= 104, 52, 21, 13) to obtain a relationship between *F* (n) and *w*. The double logarithmic plot of F (n) versus *w* is used to calculate the slope, which gives the scaling exponent β. The scaling exponent for the cyclonic disturbances over the North Indian Ocean has been calculated using DFA technique.

Correlogram

In time series analysis, a correlogram is a plot of the sample autocorrelations for data values at varying time lags which is also known as an autocorrelation plot. The correlogram is a commonly used tool for checking randomness in a data set. If random, such autocorrelations should be near zero for any and all time-lag separations. If non-random, then one or more of the autocorrelations will be significantly non-zero. Autocorrelation refers to the correlation of a time series with its own past and future values. Autocorrelation is also sometimes called "lagged correlation" or "serial correlation", which refers to the correlation between members of a series of numbers arranged in time. Positive autocorrelation might be considered a specific form of "persistence", a tendency for a system to remain in the same state from one observation to the next. Geophysical time series are frequently auto correlated because of inertia or carryover processes in the physical system. For example, the slowly evolving and moving low pressure

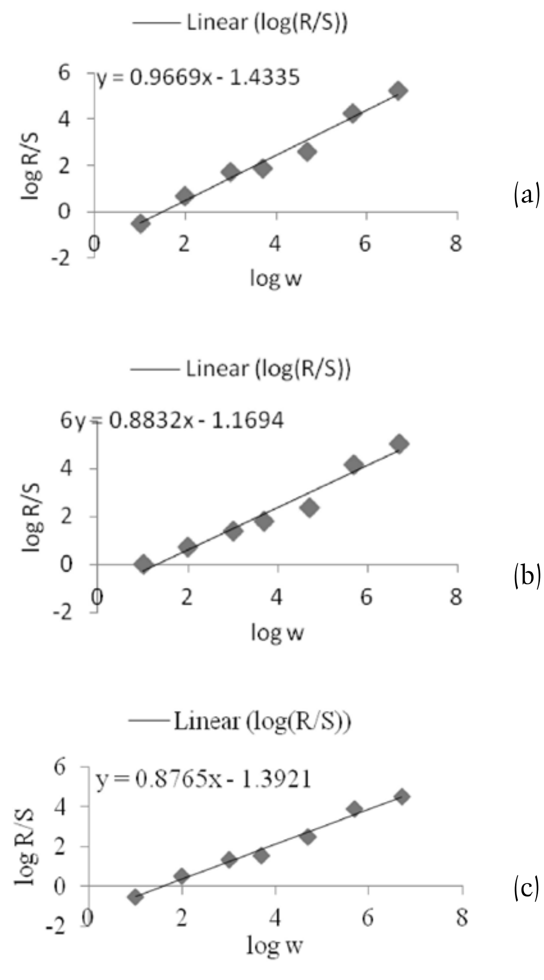

Figure 2. The slope of the linear regression that gives the value of the Hurst exponent for the (a) annual, (b) southwest and (c) northeast monsoon frequency of cyclonic disturbances over the North Indian Ocean.

systems in the atmosphere might impart persistence to daily rainfall. Autocorrelation complicates the application of statistical tests by reducing the number of independent observations. Autocorrelation can also complicate the identification of significant covariance or correlation between time series. Autocorrelation can be exploited for predictions: an auto correlated time series is predictable, probabilistically, because future values depend on current and past values.

RESULT AND DISCUSSION

The Hurst exponent analysis has been employed on 104 year data (from 1901-2004) of the frequency of cyclonic disturbances (comprising Bay of Bengal and Arabian Sea). R/S is plotted against w on log-log axes to determine the Hurst exponent. Fig. 2 (a, b and c) shows the slopes of the linear regression for the annual frequency, the number of southwest monsoon and northeast monsoon cyclonic disturbances over the North Indian Ocean that gives the value of the Hurst exponent. The values are found to be 0.9, 0.8 and 0.8 corresponding to the fractal dimension of 1.1, 1.2 and 1.2.

The validity of the above results has been examined by employing the DFA on the concerned time series data. Fig. 3 (a, b and c) gives the corresponding slopes of the linear regression that illustrates the DFA. The values are found to be 0.9, 0.8 and 0.8. It is vivid from the above observations that the results for the concerned time series data are highly corroborate among them. We further validated the results by computing the correlogram of the time series. Fig. 4 (a, b and c) illustrates the autocorrelation plots for the three categories as mentioned above. The autocorrelations were computed to a maximum of 26

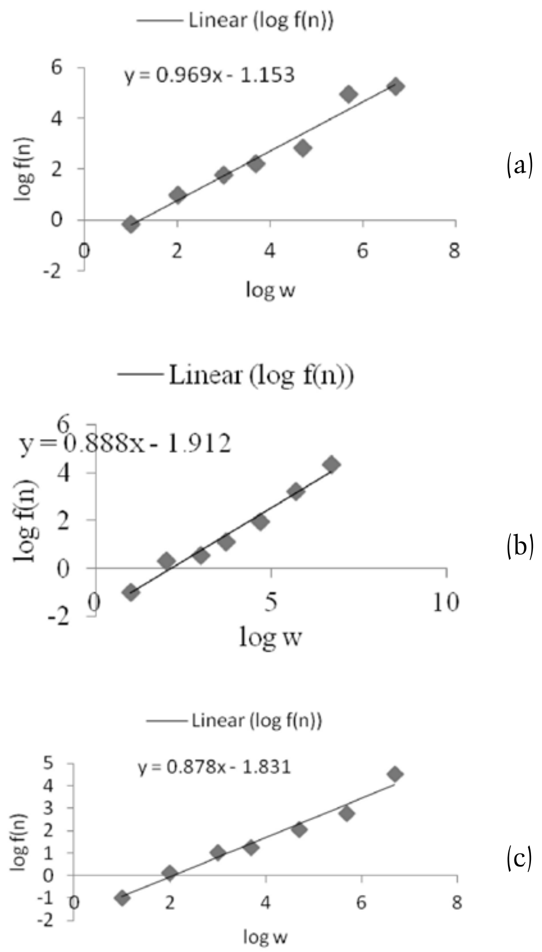

Figure 3. The slope of the linear regression that gives the Detrended fluctuation Analysis for the (a) annual, (b) southwest and (c) northeast monsoon frequency of cyclonic disturbances over the North Indian Ocean

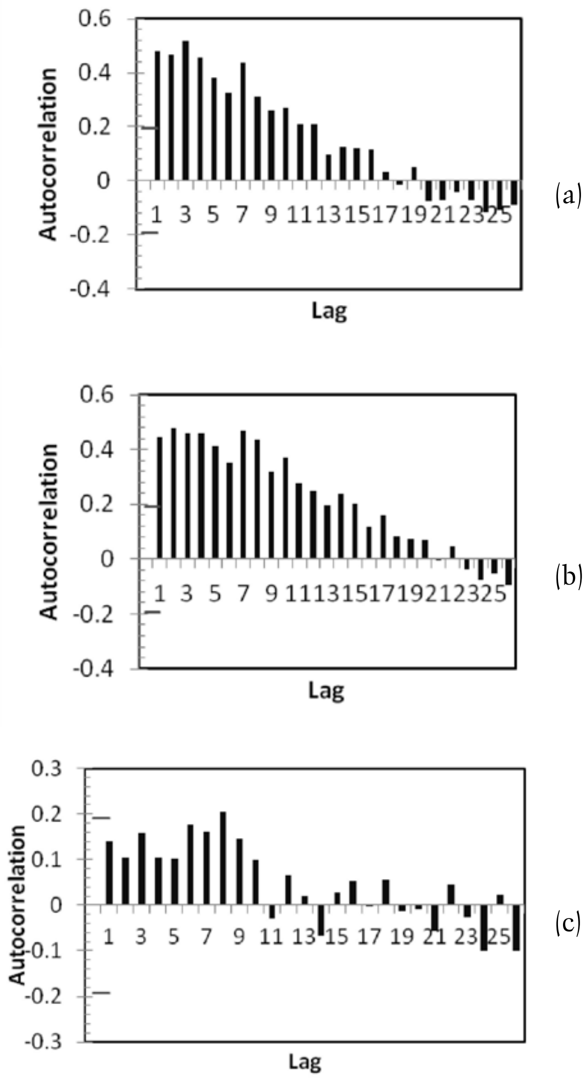

Figure 4. The autocorrelation plot for the (a) annual, (b) southwest and (c) northeast monsoon frequency of cyclonic disturbances over the North Indian Ocean

lags (n=104, n/4) (Chatfield, 2004). It can be readily seen that majority of the auto correlated values are positive in all the three cases.

CONCLUSION

The result from Hurst exponent analysis and Detrended Fluctuation Analysis shows that the scaling values are very much greater than 0.5. The plots of the autocorrelation factor are the evidence for the non-randomness of the time series. Hence it can be summarized that the cyclonic disturbances over the North Indian Ocean shows a very high degree of persistent behaviour for the period 1901-2004.

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