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Erlang Distribution Model for Ocean Wave Periods

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ABSTRACT

The Erlang distribution is derived as a model for ocean wave periods from the estimation of the functional form of the various mean wave periods. The relative and mean rms_{error} values of computed and theoretical mean wave periods are not more than 9% and 5% respectively. Further various characteristics of this distribution are derived to obtain some useful wave period estimations.

INTRODUCTION

While there is an abundance of literature on distributions of wave heights, there is a paucity on the distribution of wave periods. A few studies have been reported on the short-term distributions of wave periods (Longuet-Higgins,1975; Dattatri, Raman & Jothi Shankar 1979; Deo & Narasimhan 1979). Putz (1952) was the first to suggest that the distribution of wave periods, follow a Gamma-type distribution. Bretschneider (1959) suggested a model for simulating wave period distribution. Rayleigh model is also applied as an ocean wave period model (Baba & Harish 1985). By observing the distribution of visually estimated wave periods (NPOL 1978; NIO 1982) for Arabian Sea and Bay of Bengal for a period of 10 years and 5 years respectively, the exponential model along with the above given models were used to represent the wave periods and it was found that the Gamma distribution was superior among the other competing models (Muraleedharan, Unnikrishnan Nair & Kurup 1993). Hence the distributions of the recorded shallow water wave period data analysed by the zero up-crossing method (T_z) have been simulated by Gamma distribution model and the goodness of fit was tested using χ^2-test at 0.02 level of significance. (Unnikrishnan Nair, Muraleedharan & Kurup 2002). Recorded wave data obtained from wave records off Valiathura, Kerala coast using pneumatic wave recorder charts at 0900hrs, 1200hrs and 1500hrs for January 1981 (depth of recording,5.5m) carried out by Centre for Earth Science Studies, Thiruvananthapuram, Kerala are used. Wave recording was carried out for 10 days for about 15 minutes (CESS 1984). The model fits in 92.86% cases after appropriate grouping of the data. The wave data recorded at 0900hrs, 1200hrs and 1500hrs are treated as separate sets of data. As the distribution function of Gamma model derived by power series expansion is complicated, the predictive formulae for various wave period parameters are not analytically tractable to apply for practical purposes. Accordingly the shape parameter of the Gamma model is approximated to the nearest integer to arrive at Erlang distribution. This model is employed to derive various prediction formulae for estimating different wave period parameters theoretically. A modified Gamma and Erlang models are suggested for redefined significant wave periods by the method of characteristic functions and for predicting various statistical parameters of redefined significant wave periods. The various predicted parameters are comparable with the computed ones from the point of view of RMS_{error} values.

MATERIALS AND METHODS

One approach to determining the distribution of wave periods is to derive it as the marginal distribution of the joint distribution of wave heights and periods. By this method Longuet-Higgins (1975) obtained the model

$$f(t) = (1+c^2/4).\,(1/2ct^2)\{1+(1-1/t)^2.1/c^2\}^{-3/2},\ t > 0,$$

where 'c' is the spectral width and $t = T/\bar{T}$ with T representing the period and $\bar{T}$ its mean. An inherent weakness of the model is that the mean of the distribution is infinite and variance does not exist. Arhan, Cavanie & Ezraty (1976) and Kwon & Deguchi (1994) also provide a joint distribution of wave height and period. The period distributions are sensitive to the shape of the wave spectrum and further estimation of wave period parameters often turns out to be difficult for these models. A second and conventional approach is to fit known probability distributions based on the physical characteristics of the wave periods.

In the present paper we explore the possibility of finding the distributions of wave periods by modeling the function $m(t) = E(T/T > t)$ on the basis of the observations on T. Since m(t) determines the distribution of T uniquely, it is enough to find the appropriate functional form of m(t) consistent with the data. Based on such a model we examine some of the parameters such as mean, mean of maximum wave period, most frequent maximum wave period, extreme wave periods and analysis of return periods.

The concept of significant wave height as introduced by Sverdrup & Munk (1947) is the mean of the highest one-third of waves present in a sea. This definition is unrealistic since the number of waves 'n' is unspecified (Kinsman 1965). Further, it presents difficulties in finding the distribution of significant wave. Accordingly we modify the definition of significant wave as the average of one-third highest zero up-crossing waves of a constant number of consecutive waves(n) in a wave record. The distribution of redefined significant wave height in the new formulation is derived(Muraleedharan, Unnikrishnan Nair & Kurup 1999) and also the redefined significant wave period(n=6) (Unnikrishnan Nair, Muraleedharan & Kurup 2002) and it is shown that the modified definition compare favourably with the existing ones.

While designing floating marine systems it is of importance to know wave heights having periods that are close to the natural periods of the system in areas where it is operated. We study the mean wave period, mean maximum wave period, most frequent maximum wave period, significant wave period, extreme wave periods, return period of an extreme wave period and also the probability of a wave period of designated size is realized in a specified period of time for a given return period.

The probability that the period T exceeds a given value t is

$$\bar{F}(t) = P(T > t) = 1 - F(t).$$

Where F(t) is the distribution function of T. The average of such periods larger than t is $m(t) = E(T \mid T > t) =$

$$= (1/\bar{F}(t)).\int_t^\infty x.f(x).dx$$

$$= (-1/\bar{F}(t)).\int_t^\infty x.[dF(x)/dx].dx$$

$$= t + (1/\bar{F}(t)).\int_t^\infty \bar{F}(x)dx \qquad (1)$$

It is known that the knowledge of m(t) will enable one to determine the distribution of 'T' through the relationship

$$\bar{F}(t) = \exp[-\int_0^t [m'(x)/(m(x)-x).]dx \qquad (2)$$

where $m'(x)$ is the derivative of m(x)

Thus modeling the wave periods can be accomplished through the function m(t), provided sufficiently accurate information on the functional form of m(t) can be established from the data. This aspect is discussed in the next section.

Significant Wave Period

If the sample represents zero up-crossing consecutive waves in a record arranged in ascending order, the significant wave period is $T_s = E\,(T_{(2n/3)} + T_{((2n/3)+1)} + ... + T_{(n)}$ where (2n/3) is chosen as the largest integer value, if (2n/3) is not an integer. It is well known that the distribution of $T_{(r)}$ is specified by the density

$$h(t_r) = [n!/((r-1)!(n-r)!)].[F(t_r)]^{r-1}.[1-F(t)]^{n-r}.f(t),$$
$$r = 1, 2, 3, ..., n \qquad (3)$$

where F(.) and f(.) are the distribution and density function of T. From (3), theoretical expressions for T_s can be computed, once the distribution of T is identified.

Erlang distribution as a wave period model

The Gamma distribution function is given by

$$F(t) = [\lambda^\alpha/\Gamma(\alpha)]\,{}_0\!\int^t \exp\text{-}(\lambda t).t^{\alpha-1}.dt;\ t, \lambda, \alpha > 0$$

Power series expansion for this incomplete Gamma function is

$$F(t) = [1/(\alpha.\Gamma(\alpha))].(\lambda t)^\alpha.\exp\text{-}(\lambda t).[1 + (\lambda t/(\alpha+1)) + ((\lambda t)^2/(\alpha+1)(\alpha+2)) + ...]$$

Since the prediction formulae for the various wave period parameters derived from these are complicated, a simplifying assumption is made by treating α as a positive integer resulting in the Erlang distribution with density function

$$f(t) = \lambda^\alpha.\,t^{\alpha-1}.\,e^{-\lambda t}/(\alpha-1)!,\quad t > 0, \lambda > 0 \qquad (4)$$

as a model for wave periods. For this distribution, the expression for F(t) and

$\bar{F}(t)$ can be obtained in closed form. In fact

$$\bar{F}(t) = \sum_{i=0}^{\alpha-1} (\lambda t)^i.\,e^{-\lambda t}/i!,\ t \geq 0,$$

$$\text{and } \int_t^\infty \bar{F}(x).dx = \lambda^{-1}\sum_{i=0}^{\alpha-1}.\sum_{j=0}^{i} (\lambda t)^j.e^{-\lambda t}/j! \qquad (5)$$

Hence from equation (5)

$$m(t) = t + [\lambda^{-1}\sum_{i=0}^{\alpha-1}.\sum_{j=0}^{i} (\lambda t)^j./j!\,/\sum_{i=0}^{\alpha-1} (\lambda t)^i/i!\,] \qquad (6)$$

To be able to use (6) in a practical problem one needs estimates of λ and α

Initial values of λ and α can be obtained from the expressions

$$\hat{\lambda} = \bar{x}/s^2 \text{ and } \hat{\alpha} = \bar{x}^2/s^2$$

where $\bar{x}$ and s^2 are respectively the mean and variance of the sample values. The nearest integer value of $\bar{x}/s^2$ is chosen as the point estimates of α

The distributions of the various average wave periods derived from expression (6) are compared with the computed values for the recorded wave data. A few typical examples are given in fig.1(a-d). Two indicators of the overall accuracy of the expression (6) are computed,(Table.1)viz.the root-mean-square relative error and the relative bias (mean error). Since there is fairly good empirical support, from expression (6), it is inferred that the Erlang distribution (4) adequately represents the observations.

Thus the proposal of Erlang as an ocean wave period model from empirical and Putz's (1952) appears to have empirical and logical foundation.

Parametric relations could be derived from this expression for predicting various characteristics of wave period as follows.

Mean Wave Period

The mean wave period $\overline{T}$ is given by the expectation of t as $\overline{T} = E(T) = \int_0^\infty t.f(t).dt = \alpha/\lambda$ and variance as $\sigma^2 = \alpha/\lambda^2$

Mean Maximum Wave Period

Let T be the random variable representing a maximum wave period which is assumed to be continuous and non-negative with distribution function F(t) and probability density function f(t). Then T_{max} has distribution function

$$G(t) = [F(t)]^n \text{ , n-sample size} \quad (7)$$

and hence the maximum wave period has density

$$g(t) = G'(t) = n.F(t)^{n-1}.f(t) \quad (8)$$

The mean value of the maximum wave period is

$$E(T_{maxn}) = \int_0^\infty t.n[F(t)]^{n-1}.f(t)\,dt$$
$$= n.E[t.F(t)]^{n-1} \quad (9)$$

where the average is taken over the distribution of T. It is found to be

$$\overline{T}_{max} = [2\alpha/\lambda]-[1/(\lambda(\alpha-1)!)]\sum_{i=0}^{\alpha-1}(a+i)!/(2^{\alpha+i}.i!) \text{ , } n=2$$

Most Frequent Maximum Wave Period

On the other hand the most probable maximum period is the mode of (8) obtained as the solution of the equation g'(t) = 0 or

$$(n-1)[f(t)]^2 + F(t).f'(t) = 0 \quad (10)$$

It is the solution of the equation

$$\lambda t + \sum_{i=0}^{\alpha-1} \exp\text{-}(\lambda t)[(\alpha+i-1)(\lambda t)^i\text{-}2(\lambda t)^{i+1}/i!] = \alpha-1, n=2 \quad (11)$$

The Most Frequent Maximum Wave Period predicted using (11) are found to be closer to the observed values (Unnikrishnan Nair, Muraleedharan & Kurup 2002).

Extreme Wave Period

When the observed wave periods is a random variable following Erlang law, the distribution of the maximum wave periods is specified by G(t). Therefore the probability that we get a wave period exceeding T_{max} is $1\text{-}G(t) = P(T_L > t)$

In a series of observations on maximum periods, the probability that the r^{th} observation is the first that exceed T_{max} is $[1\text{-}G(t)][G(t)]^{r-1}$ and $E(r) = (1\text{-}G)^{-1}$. E(n) is infact the average number of periods between two exceedances and therefore represents the average time interval with which exceedances occur(Return period-R_p) ie.The extreme wave period is the solution of the expression

$$1\text{-}\sum_{i=0}^{\alpha-1}(\lambda t)^i.\exp\text{-}(\lambda t)/i! = (1\text{-}1/R_p)^{1/N} \text{ for } t \quad (12)$$

Analysis of Return Periods

The re-occurrence of an extreme wave period (t_{maxn}) is derived from (12) as

$$R_p = \{1\text{-}[1\text{-}\sum_{i=0}^{\alpha-1}(\lambda t)^i \exp\text{-}(\lambda t)/i!]^N\}^{-1} \quad (13)$$

where N is the time of observations.

Another question that is of interest at this juncture is given a period of time R_p(in days or years), what is the probability that level t_{maxn} is never realized in m period of time(in days or years, $m < R_p$). For this we assume that P is

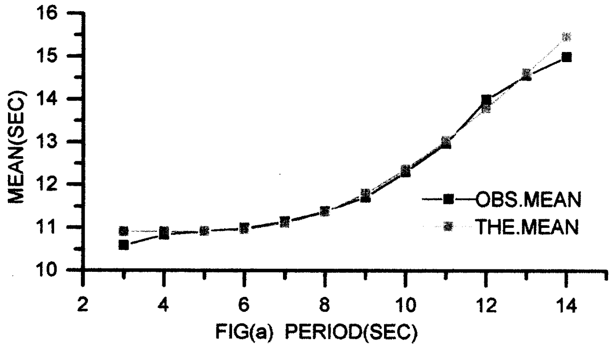
Table1. The relative and mean rms_{error} values for computed and theoretical distributions of mean values.26 tests

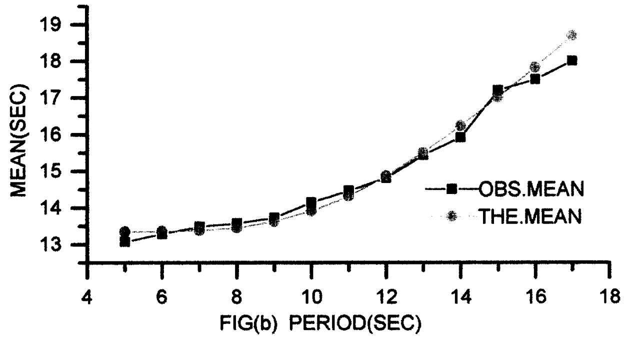
	relative rms_{error}	mean rms_{error}
1	0.064	-0.014
2	0.056	0.028
3	0.086	0.050
4	0.045	-0.036
5	0.036	0.011
6	0.035	-0.028
7	0.033	0.026
8	0.021	-0.005
9	0.046	-0.019
10	0.045	-0.020
11	0.048	0.024
12	0.046	0.002
13	0.018	0.005
14	0.034	0.005
15	0.085	0.006
16	0.030	0.008
17	0.025	0.008
18	0.043	-0.020
19	0.047	-0.006
20	0.015	0.006
21	0.054	-0.027
22	0.037	-0.022
23	0.034	0.027
24	0.036	0.030
25	0.043	-0.022
26	0.041	0.028

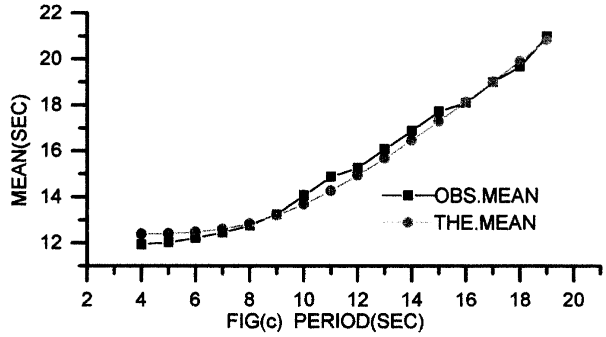
A few typical examples [Figs 1(a-d)] showing the goodness of fit between the computed and the theoretical values of mean wave periods are given in Table 2.

Table2. Typical values of computed and theoretical mean wave periods.

	t	E(T êT > t) Theoretical	Computed
Fig(a)	3	10.91	10.59
	4	10.91	10.84
	5	10.92	10.92
	6	10.96	10.99
	7	11.10	11.15
	8	11.37	11.39
	9	11.80	11.71
	10	12.36	12.30
	11	13.04	12.97
	12	13.79	14.00
	13	14.61	14.56
	14	15.47	15.00
Fig(b)	5	13.35	13.08
	6	13.36	13.29
	7	13.38	13.49
	8	13.45	13.58
	9	13.62	13.73
	10	13.91	14.15
	11	14.32	14.47
	12	14.87	14.81
	13	15.51	15.44
	14	16.23	15.92
	15	17.01	17.20
	16	17.83	17.50
	17	18.69	18.00
Fig(c)	4	12.40	11.94
	5	12.42	12.04
	6	12.47	12.22
	7	12.60	12.46
	8	12.84	12.75
	9	13.21	13.24
	10	13.69	14.07
	11	14.27	14.88
	12	14.94	15.27
	13	15.68	16.09
	14	16.46	16.88
	15	17.29	17.73
	16	18.14	18.11
	17	19.02	19.00
	18	19.92	19.67
	19	20.84	21.00
Fig(d)	3	11.67	11.15
	4	11.69	11.38
	5	11.76	12.05
	6	11.92	12.16
	7	12.19	12.56
	8	12.59	12.74
	9	13.09	12.90
	10	13.69	13.39
	11	14.37	13.72
	12	15.11	14.95
	13	15.89	15.93
	14	16.71	16.70
	15	17.57	17.13







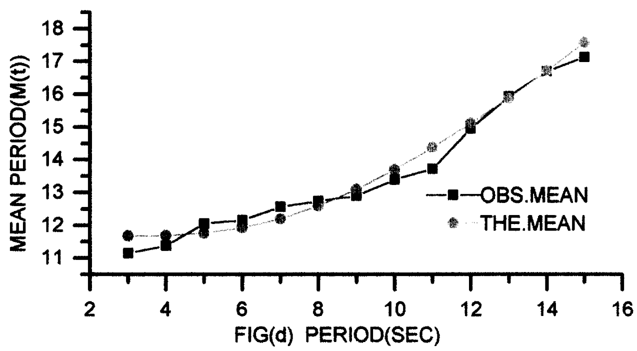


Table.3. Computed and predicted wave period statistics

Wave period Statistics(s)	0900hrs C	0900hrs P	1200hrs C	1200hrs P	1500hrs C	1500hrs P	Relative RMS_{error}	$Mean_{error}$
T	11.4	11.6	11.8	11.8	11.4	11.7	0.019	0.017
T_{max}	13.7	13.2	14.3	13.3	14.2	13.5	0.054	-0.051
T_s	14.0	14.9	14.3	14.7	14.4	15.2	0.051	0.048
T_{mfm}	13.2	12.6	15.0	12.7	13.0	12.7	0.101	0.078
$T_{extreme}$	17.5	19.3	19.7	18.6	19.0	20.1	0.076	0.033

the probability that a level larger than t_{maxn} will not be realized in consecutive time m is

$$P = (1-G)^{m/Rp} = \{1-[1-\sum_{i=0}^{\alpha-1} (\lambda t)^i \exp(-\lambda t)/i!]^N\}^{m/Rp} \quad (14)$$

The probability of realizing a wave period of 't' during any one of the m time is 1-p = q.

DISCUSSION AND RESULTS

The computer programs for these expressions are given as appendix with examples and the empirical validations of these for zero up-crossing wave period data have been made(Unnikrishnan Nair, Muraleedharan & Kurup 2002) which are reproduced in Table.3

Putz (1952) had obtained a gamma-type distribution function to represent wave periods. Baba & Harish (1985) suggested that the swell wave periods fit closer to the Bretschneider distribution given by

$$P(T) = 2.7 (T^3 / T^4).\exp[-0.675(T/\bar{T})^4]$$

T is the mean wave period. This distribution explains the wave periods satisfactorily when dominated by swells (Baba & Harish 1985). The strong base of this function is on the narrow band of the wave spectrum, which can be observed in the swell dominated sea state. They also suggest that the sea wave periods fit closer to the Rayleigh distribution of the form

$$P(T) = \exp(-\pi/4(\bar{T}/T)^2)$$

It is to be noted that of the various theoretical models available for modeling ocean wave periods are of empirical or semi-empirical origin. Hence the validity of these models are highly data based.

The computer programs for these expressions are given as appendix with examples.

It is to be noted that the mean maximum wave period(T_{max}) and most frequent maximum wave period(T_{mfm}) are less compared to the significant wave period(T_s).These are actually to be computed from the distribution of the maximum wave periods. Here it is computed from the individual zero up-crossing wave periods as a testing procedure for their parametric expressions and computer programs developed.

The conventional Significant wave is the one-third average of the highest waves in a wave record. Here various averages of the highest waves are computed from the data and simulated theoretically from the expression derived for 26 data sets. Since the maximum relative and mean rms error of the computed and theoretical values are of the order of less than 9% and 5%, it can be assumed that the functional form derived for various mean wave periods (6) is adequate. Thus from the distribution of the various mean wave periods we arrive at F(t) by (2), ie., the Erlang is the model for conventional significant wave period distribution.

It is to be noted that the visual observations on mean wave periods are the conventional significant wave ***periods*** (NPOL 1978) and they are not as accurate as the visually estimated conventional significant wave ***heights***. However, Muraleedharan, Unnikrishnan Nair & Kurup (1993) observed that the visually estimated mean wave period for sea and swell dominated state follow Gamma distribution. The data grids comprised of both southwest and northeast monsoon seasons off Valiathura and Mangalore, southwest coast of India. The present theoretical finding of the Erlang model as a wave period model reaffirms the earlier empirical support.

CONCLUSIONS

An expression has been derived for various mean zero up-crossing wave periods using Erlang distribution model. The comparability of the computed and the theoretically predicted mean wave period distributions has been tested using relative rms_{error} and mean rms_{error}. The errors are not more than 9% and 5% respectively. From the empirical support for the theoretically simulated mean wave periods, we arrive at the model for conventional significant wave period as an Erlang distribution. Parametric relations are derived from the model for predicting various wave period statistics.

Erlang model is a special case of Gamma when its shape parameter is an integer. By suggesting the Erlang distribution model and prediction formulae derived therefrom the complexity arising from considering Gamma model is eased.

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APPENDIX
1

```
C   PROGRAM FOR COMPUTING VARIOUS MEAN WAVE PERIODS
C   THEORETICALLY, m(t)=E(T\T>t)
C   INPUT PARAMETERS
C   ALPHA-LPHA, LAMDA-AMDA, TI-PERIOD.
C   OUTPUT PARAMETERS
C    m(t)
    READ(*,*)LPHA,AMDA
  2 READ(*,*)TI
    WRITE(*,5)TI
  5 FORMAT(///,4X,'t=',F5.2)
    SUM=0
    SUM1=0
    I=LPHA-1
    A=AMDA*TI
    IF(I.EQ.0) GO TO 20
    DO10II=0,I
    AJ=1
    AM=1
     KK=II
    DO50LL=1,KK
    AM=AM*A/LL
 50 CONTINUE
    SUM1=SUM1+AM
    DO15III=0,KK
     JJJ=III
    DO30IIII=1,JJJ
    AJ=AJ*A/IIII
 30 CONTINUE
    SUM=SUM+AJ
    AJ=1
 15 CONTINUE
 10 CONTINUE
    SUM1=SUM1
    SUM=SUM/SUM1/AMDA
    SUM2=TI+SUM
    WRITE(*,100)SUM2
100 FORMAT(///,4X,'m(t)=',F5.2)
    GO TO 2
 20 SUM2=TI+1/AMDA
    WRITE(*,110)SUM2
110 FORMAT(///,4X,'m(t)=',F5.2)
    GO TO 2
    STOP
     END
```

2

```
C  PROGRAM FOR COMPUTING MEAN MAXIMUM
C  WAVE PERIOD(FROM ERLANG DISTRIBUTION)
  2 READ(*,*)LPHA,AMDA
    WRITE(*,5)LPHA,AMDA
  5 FORMAT(///,4X,'ALPHA=',I3,3X,'LAMDA=',F6.4)
    E=0.0
    M=LPHA-1
    B=2.0*LPHA/AMDA
    AN=LPHA
    A=(AN)/2.0**AN
    DO10I=1,M
    AN=AN+1
    A=AN*A/2.0/I
    E=E+A
 10 CONTINUE
    E=E+(LPHA)/2.0**LPHA
    E=E/AMDA
    BB=B-E
    WRITE(*,20)BB
 20 FORMAT(///,4X,'MEAN MAXIMUM WAVE
    PERIOD=',F5.2)
    GO TO 2
    STOP
    END
```

3

```
C  PROGRAM FOR COMPUTING MOST FREQUENT MAXIMUM WAVE PERIOD C  USING THE EXPRESSION
C  EXP(-(LAMDA*T)).T**(ALPHA-2)=0
C  INPUT PARAMETERS
C  ALPHA-LPHA,LAMDA-AMDA,TMEAN-MEAN WAVE PERIOD(USED AS AN C  APPR).
C  VALUE TO RUN THE PROGRAM
C  OUTPUT PRAMETER
C  MOST FREQUENT MAXIMUM WAVE PERIOD
    H=0.1
    READ(*,*)LPHA,AMDA,TMEAN
    WRITE(*,5)LPHA,AMDA
  5 FORMAT(///,4X,'ALPHA=',I3,3X,'LAMDA=',F6.4)
 10 A=AMDA*TMEAN
    B=EXP(-(A))
    C=LPHA-2
    C=TMEAN**C
    D=B*C
    D=D+0.05
    L=10*D
    D=L/10.0
    WRITE(*,*)D
    IF(D.EQ.0.0)GO TO 30
    IF(D.GT.0.0)GO TO 20
 20 TMEAN=TMEAN+H
    GO TO 10
 30 WRITE(*,35)TMEAN
 35 FORMAT(///,4X,'MOST FREQUENT MAXIMUM WAVE PERIOD=',F11.8)
    STOP
    END
```

4

```
C    PROGRAM FOR PREDICTING MOST FREQUENT MAXIMUM
C    WAVE PERIOD USING THE EXPRESSION
C    ALPHA-1=LAMDA*T+SUM((ALPHA+i-1)*(LAMDA*T)**i-
C    2(LAMDA*T)**i)*EXP((-(LAMDA*T))/iFACTORIAL
C    INPUT PARAMETERS
C    ALPHA-LPHA,LAMDA-AMDA,TMEAN-MEAN WAVE
C    PERIOD(APPR. VALUE TO RUN THE PROGRAM)
        H=0.00001
   READ(*,*)LPHA,AMDA,TMEAN
   WRITE(*,5)LPHA,AMDA,TMEAN
  5 FORMAT(///,4X,'ALPHA=',I3,3X,'LAMDA=',F6.4,3X,'PERIOD=',F5.2)
   N=LPHA-1
 50 B=AMDA*TMEAN
   E=0.0
   C=EXP(-(B))
   BB=2*B
   DO10I=0,N
   DD=1.0
   J=I
   D=N+J-BB
   DO15II=1,J
   DD=DD*B/II
 15 CONTINUE
   EE=DD*D*C
   E=E+EE
 10 CONTINUE
   E=E+D*C
   E=E+B
   E=E-N
   E=E+0.000005
   L=100000*E
   E=L/100000.0
   WRITE(*,*)E
   IF(E.EQ.0.00000)GO TO 30
   IF(E.GT.0.00000)GO TO 20
   IF(E.LT.0.00000)GO TO 25
 20 TMEAN=TMEAN-H
   GO TO 50
 25 TMEAN=TMEAN+H
   GO TO 50
 30 WRITE(*,40)TMEAN
 40 FORMAT(///,4X,'MOST FREQUENT MAXIMUM WAVE
    PERIOD=',F5.2)
   STOP
   END
```

5

```
C   PROGRAM FOR EXTREME WAVE PERIOD FROM ERLANG MODEL
C   INPUT PARAMETERS
C   LPHA-ALPHA,LAMDA-ALAMDA,EXTREME WAVE
C   PERIOD(APPR.. THE HIGHEST
C   PERIOD IN THE DISTRIBUTION)-T,TIME 0F  OBSERVATION
C   (IN DAYS)-NN, RETURN PERIOD-NR(IN DAYS)
C   OUTPUT PARAMETERS
C   EXTREME WAVE PERIOD
      H=0.00001
  2 READ(*,*)LPHA,ALAMDA,T,NR,NN
    WRITE(*,5)LPHA,ALAMDA,T,NR,NN
  5 FORMAT(///,4X,'ALPHA=',I3,3X,'LAMDA=',F6.4,3X,'EXTREME WAVE
   1PERIOD(APPRO)=',F5.2,1X,'SECS',3X,'RETURN
    PERIOD=',I2,1X,'DAYS',13X,'TIME OF
    OBSERVATION=',I6,1X,'DAYS')
 10 E=0.0
    B=ALAMDA*T
    N=LPHA-1
    A=EXP(-(B))
    IF(N.EQ.0)GO TO 15
    DO20J=1,N
    D=A
    JJ=J
    DO40I=1,JJ
    D=D*B/I
 40 CONTINUE
    E=E+D
 20 CONTINUE
    E=E+A
    E=1.0-E
    EE=1.0/NN
    EEE=(1.0-1.0/NR)**EE
    Z=E-EEE
    Z=Z+0.000005
    L=100000*Z
    Z=L/100000.0
    WRITE(*,*)Z
    IF(Z.EQ.0.00000)GO TO 30
    IF(Z.GT.0.00000)GO TO 15
    IF(Z.LT.0.00000)GO TO 25
 15 T=T-H
    GO TO 10
 25 T=T+H
    GO TO 10
 30 WRITE(*,50)T
 50 FORMAT(///,4X,'EXTREME WAVE PERIOD=',F11.8,1X,'SECS')
    GO TO 2
    STOP
    END
```

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```
C   PROGRAM FOR RETURN PERIOD OF AN EXTREME WAVE
C    PERIOD FROM ERLANG DISTRIBUTION FUNCTION
C   INPUT PARAMETERS
C    LPHA-ALPHA,ALAMDA-LAMDA, EXTREME WAVE PERIOD- T
C    TIME OF OBSERVATION(IN MINUTES OR DAYS)-N
  2 READ(*,*)LPHA,ALAMDA,T,NN
   WRITE(*,5)LPHA,ALAMDA,T,NN
  5 FORMAT(///,4X,'ALPHA=',I3,3X,'LAMDA=',F6.4,3X,'EXTREME WAVE
   1PERIOD=',F5.2,'SECS',3X,'TIME OF OBSERVATION=',I6,1X,'DAYS')
    E=0.0
   B=ALAMDA*T
   N=LPHA-1
    A=EXP(-(B))
   IF(N.EQ.0)GO TO 15
   DO20J=1,N
   D=A
   JJ=J
   DO40I=1,JJ
   D=D*B/I
 40 CONTINUE
    E=E+D
 20 CONTINUE
 15 E=E+A
    E=1.0-E
    E=E**NN
    E=1.0-E
    E=1.0/E
    WRITE(*,10)E
 10 FORMAT(///,4X,'RETURN PERIOD=',F5.1,1X,'DAYS')
    GO TO 2
    STOP
    END
```

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```
C   PROGRAM FOR REALISING AN EXTREME WAVE PERIOD IN A
C   TIME LESS THAN THE DESIGNATED RETURN PERIOD
C   INPUT PARAMETERS
C   LPHA-ALPHA,ALAMDA-LAMDA, EXTREME WAVE PERIOD-
C   T,TIME OF OBSERVATION(DAYS)-NN,RP-RETURN
C   PERIOD(DAYS),TM-TIME LESS THAN THE DESIGNATED RETURN
C   PERIOD(DAYS)
C  OUTPUT PARAMETERS
C  PROBABILITY PERCENTAGE OF REALISING AN EXTREME WAVE
C  PERIOD IN A TIME LESS THAN THE DESIGNATED RETURN PERIOD
   2 READ(*,*)LPHA,ALAMDA,T,NN,RP,TM
   WRITE(*,5)LPHA,ALAMDA,T,NN,RP,TM
  5 FORMAT(///,4X,'ALPHA=',I3,3X,'LAMDA=',F6.4,3X,'EXTREME WAVE
  1PERIOD=',F5.2,'SECS',3X,'TIME OF OBSERVATION=',I6,1X,'DAYS',

                 13X,'RETURN PERIOD=',F5.2,1X,'DAYS',3X,'TIME LESS THAN THE
  1DESIGNATED RETURN PERIOD=',F5.2)
   E=0.0
   B=ALAMDA*T
   N=LPHA-1
   A=EXP(-(B))
   IF(N.EQ.0)GO TO 15
   DO20J=1,N
   D=A
   JJ=J
   DO40I=1,JJ
   BB=1.0/I
   D=D*BB*B
 40 CONTINUE
   E=E+D
 20 CONTINUE
 15 E=E+A
   E=1.0-E
   E=E**NN
   WRITE(*,*)E
   P=TM/RP
   E=(1.0-E)**P
   E=1.0-E
   E=100.0*E
   WRITE(*,10)E
 10 FORMAT(///,4X,'PROBABILITY OF REALISING A WAVE
  1PERIOD IN A TIME  LESS THAN THE DESIGNATED
  1RETURN PERIOD=',F6.2,1X,'%')
   GO TO 2
   STOP
   END
```