J. Ind. Geophys. Union (2003)
Vol.7, No.4, pp.247-248

Research Note

A Model for Long Term Climate Change

B.G.Sidharth and B.S.Lakshmi*
Centre For Applicable Mathematics and Computer Sciences,
B M Birla Science Center, Adarsh Nagar, Hyderabad
*Department of Mathematics, JNTU College of Engineering, Kukatpally, Hyderabad.

ABSTRACT

In this paper we consider the well-known cyclical model for climate change for example from glaciation to the present day and back, and compare this with a two-state model in Quantum Mechanics. We establish the full correspondence.

INTRODUCTION

In the Northern Hemisphere, the last glaciation took place 18,000 years ago. [1] The ice layer was several kilometers thick. It covered up to the middle of US and Europe and right up to Paris. Infact the current situation is one in which the continental ice extends only up to Greenland and this was established 10,000 years ago. That is, the earth has in a few thousand years undergone a tremendous transition between two completely different states in a very short span of time - geologically speaking. The earth can be seen as a point object in space receiving solar radiation and emitting infra-red radiation back into space. We consider the only important state variable, the mean temperature T, with respect to time given by the heat balance equation

$$\frac{dT}{dt} = \frac{1}{C}\left(Q(1-a(T)) - \varepsilon_B \sigma T^4\right) \qquad (1)$$

σ, the Stefan constant ε_B, emissivity factor, C, the heat capacity of the system Q the Solar Constant and a, the albedo, which expresses the part of solar radiation emitted back into space. The equation (1) admits two steady states T_a, T_b, T_a, being the present day climate and T_b, the glaciation time. A third state T_0 is unstable and separates the above two stable states. As is well known, in systems involving only one variable, U the kinetic potential is given by [2]

$$U = -\int dx F(x)$$

In the present case, the kinetic potential U(T) is given by

$$U(T) = \frac{1}{C}\int dT\left(Q(1-a(T)) - \varepsilon_B \sigma T^4\right) \qquad (2)$$

Now climatic systems like any other physical system are continuously subject to statistical fluctuations, the random deviation from deterministic behavior. We include the effect of the fluctuations in a random force F(t). The energy balance equation, (1), and (2) now become a stochastic differential equation of the form

$$\frac{dT}{dt} = -\frac{\partial U(T)}{\partial T} + F(T) \qquad (3)$$

The important new element introduced by this enlarged description is that different states become connected through the fluctuation. That is, starting from some initial state the system will reach any other state, sooner or later. This is true for the stable states T_a and T_b, taken as initial states, which become some sort of meta-stable states.

The time scale of this phenomenon is determined by two factors: the potential barrier

$$\Delta U_{a,b} = U(T_0) - U(T_{a,b})$$

and the strength of the fluctuations as measured by the variance, q^2 of F(t) in (3).

The mean transition time from state T_a or T_b via the unstable state T_0 is given by [1]

$$T_{a,b} \sim e^{\frac{\Delta U_{a,b}}{q^2}} \qquad (4)$$

The Model

We will now model the above situation in terms of a Quantum Mechanical two state system. In the Quantum Mechanical world a two state system [3] could be represented by the equations.

$$ih\frac{dC_1}{dt} = H_{11}C_1 + H_{12}C_2$$

$$ih\frac{dC_2}{dt} = H_{21}C_1 + H_{22}C_2 \qquad (5)$$

where the coefficients H_{ij} are the Hamiltonian matrix and C is given by the vector,

$C \equiv (C_1, C_2)$

We now identify C with t and its two states T_a and T_b and write

$C \equiv (C_1, C_2) \equiv (T_a, T_b) \equiv T$

This reduces (5) to

$$ih \frac{dT}{dt} = HT \qquad (6)$$

where H is the 2x2 matrix. Taking $t = \frac{t}{ih}$ reduces (6) to

$$\frac{dT}{dt} = HT \qquad (7)$$

Reverting back to equation (3) and replacing t with T we have

$$\frac{dT}{dt} = -\frac{\partial U\ (T)}{\partial t} + F(t)$$

Taking

$-\frac{\partial U\ (T)}{dt} + F(T) = H(T)$, we get

$$\frac{dT}{dt} = HT \qquad (8)$$

(8) is identical to (7). We note that from Quantum Mechanical Theory,

$$\Delta t \ \alpha \ \frac{h}{H} \qquad (9)$$

To proceed further and establish the correspondence fully, we now observe $H_{12}=H_{21}$ is proportional to the transition probability of C_1 to C_2 i.e., T_a to T_b In fact [4]

$H_{12} \ \alpha \ e^{-U/kT}$

So the transition time in Quantum Theory is given by

$$\Delta t \ \alpha \ \frac{1}{H} \ \alpha \ e^{-U/kT} \qquad (9)$$

(10) can be identified with equation (4). This establishes the required result. The reduction of the Climate problem to the analogous Quantum Mechanical problem has interesting consequences which need to be studied further. This is all the more so because in recent years "Scaled" Quantum effects have been studied in macroscopic systems and even a Bode-Titius type relation for planetary distances has been deduced on the basis of "quantized" energy levels. [5,6,7,8]

REFERENCES

[1] I. Prigogine, G.Nicolis, "Exploring Complexity", R.Piper, Gmbh, Munich, 1989, pp-226 ff.

[2] A.Berger.,(ed) "Climate Variations and Variability: facts and theories", Reidel, Dordrecth.1981.

[3] R.P.Feynman,R.B.Leighton, and M.Sands, "The Feynman Lectures on Physics", Vol.III, Addison-Wesley Publishing Co., Inc., Massachussets, 1965, Chapter 8 ff.

[4] F.Reif, "Statistical and Thermal Physics", McGraw-Hill, Singapore, 1965.

[5] B.G.Sidharth, Chaos, Solitons and Fractals, 12, 2001, 613-616.

[6] B.G.Sidharth, Chaos, Solitons and Fractals, 12, 2001, 1371-1373.

[7] L.Nottale, "Fractal Space-Time and Microphysics: Towards a Theory of Scale Relativity", World Scientific, Singapore, 1993, p.312.

[8] A.G. Agnese and R.Festa, Phys. Lett.A., 227, 1997, p.165-171.