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Determining sharp layer boundaries from straightforward inversion of resistivity sounding data

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ABSTRACT
The main purpose of this paper is to develop methodology to determine geologically inspired layer boundaries from the inverse model obtained using inversion techniques like straightforward Inversion Scheme (SIS) and Occum's wherein sharp boundaries are missing. These techniques essentially solve for an over parameterized model and through regularized minimum norm solution a smooth model achieved. We present two different methodologies to identify sharp boundaries of the model inherent in the smooth model. In first, the solution of linear inverse problem is improved iteratively through weighted minimum norm inverse, the weight being taken from the current solution. The technique is referred as Iterative Straightforward scheme (ISIS). The second method is an analytical, based on the application of smoothing filter and referred as Edge-Preserving smoothing (EPS). The implementation of these methodologies are demonstrated by choosing different test models from the published literature and over a field data set. These methodologies also reduce the conspicuous oscillations in the smooth solutions caused due to conversion of sharp boundaries to the smooth one.

INTRODUCTION

The quantitative interpretation of Vertical Electrical Sounding (VES) data has a long cherished history. The separate grouping of indirect (Flathe 1955; Onodera 1960; Roman 1963; Van Dam 1964; Mooney et at. 1966; Ghosh 1971b) and direct (Langer 1933; Pekeris 1940; Vozoff 1958; Ghosh 1971a) methods are no longer required. Due to improved methods of computations based on linear filter theory (Kunetz 1966; Ghosh 1971b; O' Neill 1975; Anderson 1975; Guptasarma 1982) and exponential approximation of kernel function (Santini & Zambrano 1981; Sri Niwas & Israil 1986) coupled with fast computers and inversion techniques (Inman, Ryu & Ward 1973; Inman 1975; Jupp & Vozoff 1975; Johensen, 1977; Constable, :arker & Constable 1987; Gupta, Sri Niwas &Gaur 1997; Porsani. Sri Niwas & Niraldo Ferriera 2001) the quantitative interpretation is completely revolutionized. In this endeavor, it is important to know how accurately the layer parameters are determined from VES data.

It is well known that the non-uniqueness and instability of inverse solution are major concern as yet. The problem of instability of least square inverse solution in case of erroneous' data is chronic and requires fresh approach other than the classical ones. Depending on the ratio of layer thickness and layer resistivity/conductivity in a vertical profile, layer parameters are non-unique to a degree due to the principle of equivalence. There are broadly two classes of inversion techniques being employed to solve the nonlinear resistivity inverse problem. The more frequently used is the iterative one that requires quasi-linearization of non-linear problem and adjusts the model parameters iteratively to bring its response into some degree of coincidence with the data. Obviously it requires an educated guess of the initial model to start the iterative process and can he defined using the principle of minimum numbers of layers (Muiuane & Pederson 1999). The problem of instability is contained through regularization process. However, in these techniques containment of the instability does not necessarily mean containing non-uniqueness also. The non-uniqueness due to equivalence can only be contained if one of the layer parameters are a priori known or assumed to be known. Thus in the alternative approach like Occam's inversion (Constable, Parker & Constable 1987) and Straightforward Inversion Scheme (Gupta, Sri Niwas & Gaur 1997), in which the thicknesses are assumed to be known, making over-parameterization of the model and some kind of smooth solutions are obtained. However, in these solutions identifying the sharp boundaries is a problem. In case sharp boundaries are needed for delineating the real geological structures, some methodology for delineating the layer boundaries are required for meaningful implementation of smooth inversion techniques.

The focus of this paper is to delineate the layer boundaries from the smooth solutions using either of the two procedures discussed herein. In first procedure, the smooth solution is improved iteratively using weighted minimum norm universe wherein the weights are obtained from the current solution. The second procedure is analytical that has been adopted from the edge-preserving technique used by Yi Luo et al. (2002). For the sake of convenience, we would be demonstrating the implementation of these procedures using SIS only by choosing several synthetic data set over known typical test models.

The SIS is based on the model of uniform layer thickness and is the natural extension of the concept of exponential approximation of the resistivity transform function corresponding to the air-earth interface (Sri Niwas & Israil 1986), there by transforming the non-linear inverse problem to linear one. Regularized minimum norm inversion provides nearly continuous smooth variation of resistivity-depth model that adequately fits the apparent resistivity data. The idea of determining sharp boundaries is based on the compact inversion (Last & Kubik 1983) which not only reduces the number of layers used in SIS but also focus the inversion to delineate the boundaries corresponding to the real geological structure. The following section discusses the formulation of proposed iterative straightforward inversion (ISIS) technique.

ITERATIVE STRAIGHTFORWARD INVERSION SCHEME

The expression for the potential an arbitrary point placed at a distance r on the surface of the N- layered earth model, due to a point electrode of current strength I is given by (Stefanescu 1930)

$$u(s) = \frac{I}{2\pi}\int_0^\infty T_1(\lambda)\, J_0(\lambda s) d\lambda \tag{1}$$

where $J_0(\lambda s)$ is the zeroth order Bessel function of first kind $T_1(\lambda)$ is the electrical impedance at the surface of layered earth.

By writing $T_1(\lambda)$ in the form of series expansion as

$$T_1(\lambda) = \sum_{j=0}^{\infty} T_{1j} e^{-\varepsilon_j^{(\lambda)}}, \varepsilon_0 = 0.0 \tag{2}$$

Sri Niwas & Israil (1986) obtained the following expression for the apparent resistivity (ρ_a) for symmetrical electrode configuration as

$$\rho_a(s) = \sum_{j=0}^{\infty} T_{1j} G_j^{(2)}(s), \tag{3}$$

where

$$G_j^{(2)}(s) = \frac{m}{m+1} G_j^{(1)}(s) - \frac{1}{m-1} G_j^{(1)}(ms) \tag{4}$$

with,

$$G_j^{(1)}(s) = \frac{s}{(\varepsilon_j^2 + s^2)^{1/2}}, \tag{5}$$

Which is a green's function of a point source, ε_j being a real constant. The parameter m in equation (4) defines specific electrode configuration, e.g., Wenner(m=2), Schlumberger (1<m<1.1), pole-pole (m= ∞). In straightforward inversion the concept of uniform layer thickness (d) has been introduced so as the choice of ε_j = 2jd would make T_{1j} a function of layer resistivity alone. It has been shown that the coefficients T_{1j} are the function of reflection function (R_{ij}), which is obtained from reflection coefficients define at the each interface. Thus concept of uniform layer thickness allows us to rewrite the equation (2) as

$$T_1(\lambda) = \sum_{j=0}^{\infty} T_{1j} u^j, \tag{6}$$

where $u = e^{(-2\lambda d)}; T_{10} = \rho_1$ and for j>0 , the coefficient are related to reflection coefficients (Gupta, Sri Niwas & Gaur 1997).

Thus by introducing the concept of uniform layer thickness (d), the coefficients T_{1j} becomes a function of layer resistivity alone. In equation (3) the apparent resistivity values can be interpreted as superposed contributions from a number of sources (images), each of strength T_{1j} with the corresponding Green function as weights. Thus resistivity inverse problem is required to estimate T_{1j}, which can be used to compute the reflection coefficients and subsequently resistivity of the pre-assumed uniform layers. The regularized minimum norm inversion of equation (3) has been used in SIS to obtain the set of coefficients $T_{1j,}$ which in turns resulted the smoothed model corresponding to the apparent resistivity data set. It has also been noticed that the smoothed, set of coefficients generate unwanted reflection function and oscillation at the sharp boundaries. In some practical application where the main objective of inversion is to delineate a model with a few geological boundaries and thus requiring the few non zero coefficients (T_{1j}) corresponding to the real electrical boundaries, the smooth inversion techniques such as SIS and Occum's may not be helpful. With the objective to obtain sharp boundary model the SIS algorithm is modified so as the undesired coefficients (T_{1j}) reduces to zero iteratively and enhance the values of the coefficients corresponding to the actual boundaries. Such approach has been used earlier by Last & Kubik

(1983) for the compact gravity inversion and Portniaguine & Zhdanov (1999) in focusing geophysical inversion. These approaches disperse the smoothed distribution of parameters with the well-focused distribution. In the present case, the coefficients (T_{1j}) are the set of parameters and the objective is to obtained true set of coefficients corresponding to the actual boundaries. So the concept of compact inversion is applied here accordingly equations (3) is solved using weighted minimum norm method, the weights being defined as

$$W_{tj} = \left(T_{1j}^{2} + \varepsilon^2\right)^{-1} \quad (7)$$

where e is chosen sufficiently small value

This is equivalent to maximizing its compactness. By using the stabilizer (equation 7), as weighting function the solution of equation (3) can be written as

$$T = W_t^{-1} A^t \left(A W_t^{-1} A^t + W_e^{-1}\right)^{-1} R \quad (8)$$

where the column vector R consists of apparent resistivity values and coefficient matrix A can be written as

$$A = \begin{bmatrix} 1 & u_1 & u_1^2 & u_1^3 \cdots\cdots & u_1^j \cdots\cdots \\ 1 & u_2 & u_2^2 & u_2^3 \cdots\cdots & u_2^j \cdots\cdots \\ . & & & & \\ . & & & & \\ 1 & u_i & u_i^2 & u_i^3 \cdots\cdots & u_i^j \cdots\cdots \\ . & & & & \\ . & & & & \\ 1 & u_m & u_m^2 & u_m^3 \cdots\cdots & u_m^j \cdots\cdots \end{bmatrix}$$

and W_e is the weight function defined for priori estimated noise level.

Solution can be obtained iteratively with the application of equation (7) at each step. The solution in the first iteration is not known. Therefore for the first iteration weighting matrix is set to unit matrix and the solution give direct SIS solution.

Edge-preserving smoothing

The method is based on the recent technique introduced for the noise reduction applied to the seismic data (Yi Luo et al 2002. The technique looks for the most homogenous fragment around each point in the model parameters set and assigns the average value of the selected fragment to that parameter. This may be explained by considering the following five-point Edge-preserving smoothing (EPS) operator. For any depth location at index i, one first calculate standard deviations for five shifted windows given by

Window 1: (R_{i-4}, R_{i-3}, R_{i-2}, R_{i-1}, R_{i+0}),
Window 2: (R_{i-3}, R_{i-3}, R_{i-1}, R_{i+0}, R_{i+1}),
Window 3: (R_{i-2}, R_{i-1}, R_{i+0}, R_{i+1}, R_{i+2}),
Window 4: (R_{i-1}, R_{i+0}, R_{i+1}, R_{i+2}, R_{i+3}),
Window 5: (R_{i-0}, R_{i+1}, R_{i+2}, R_{i+3}, R_{i+4}),

Here, R_i represent the resistivity of i^{th} layer of the model obtained using SIS inversion. Next, we select the window that has the minimum standard deviation, calculate the average over the selected window, and assign the average as output at the i^{th} layer resistivity. Repeating this process for all layer resistivity will yield the output model. The procedure will not be applicable to the first few layers and last few layers. The number of first and last layers depends upon the window width. Thus the resistivity of the first and the last layers are not modified. The technique has been used successfully to the smoothed model obtained from SIS solution. The output model presents a fewer boundaries with sharp discontinuity.

NUMERICAL RESULTS

Following examples demonstrate the applications of the techniques discussed in Iterative Straightforward Inversion Scheme to determine the layer boundaries from smooth model solution. Typical synthetic models have been used to demonstrate the capabilities of resistivity inversion techniques. The model parameters are:

Model I

ρ_1 = 20 Ohm – m, d_1 = 20 m;
ρ_2 = 10 Ohm – m, d_2 = 20 m;
ρ_3 = 1 Ohm – m,

Model II

ρ_1 = 10 Ohm – m, d_1 = 20 m;
ρ_2 = 50 Ohm – m, d_2 = 40 m;
ρ_3 = 150 Ohm – m,

Model III

ρ_1 = 1 Ohm – m, d_1 = 1 m;
ρ_2 = 0.2 Ohm – m, d_2 = 5 m;
ρ_3 = 1 Ohm – m,

Model IV

ρ_1 = 10 Ohm – m, d_1 = 10 m;
ρ_2 = 2 Ohm – m, d_2 = 10 m;
ρ_3 = 5 Ohm – m d_3 = 20 m;
ρ_4 = 2 Ohm – m, d_4 = 10 m;
ρ_5 = 100 Ohm – m

Model V

ρ_1 = 10 Ohm – m, d_1 = 20 m;
ρ_2 = 50 Ohm – m, d_2 = 100 m;
ρ_3 = 150 Ohm – m,

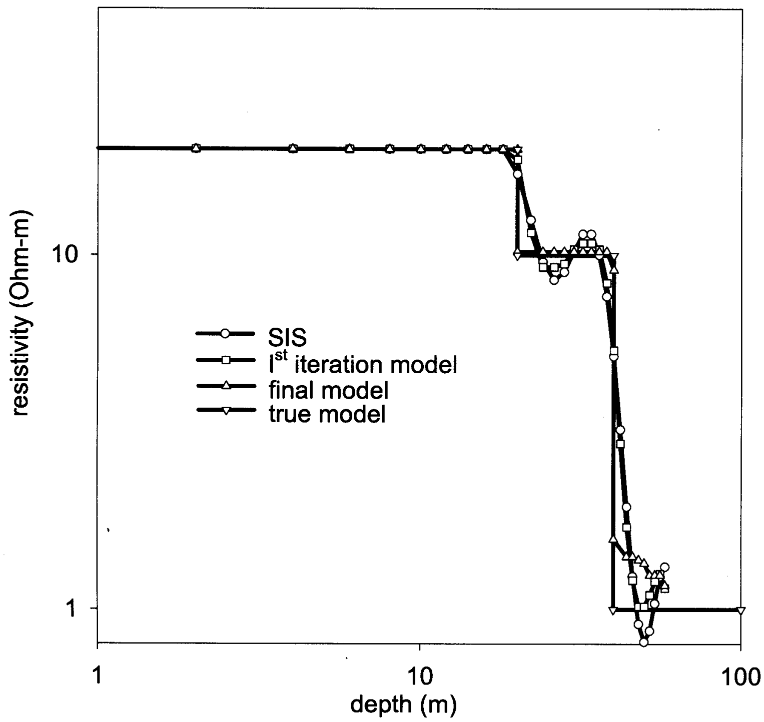

Figure 1. Comparison of inverted models obtained using straightforward inversion and proposed iterative straightforward inversion along with true model (Q-type model).

Model I, a three layer model taken from Gupta et al. (1997), which is also studied by Gai-Shan (1985) who has interpreted this in terms of composite parameters: total longitudinal conductance, total transverse resistance, and the substratum resistivity. This is due to the difficulty in obtaining solution using quasi-linearized technique. Fig.1 shows the comparison of models obtained using direct SIS technique (zeroth iteration), 1st iteration, the final model obtained at 8th iteration along with the true model. It may be seen that the oscillation presents in SIS solution (zeroth iteration) is largely reduced in final model. First iteration results show the speed of convergence.

Model II is a three layer model with resistivity increasing with depth. The increasing resistivity variation constitute an ideal model for testing resistivity inversion technique (Muiuane et al. 1999). The inverted solution using SIS (zeroth iteration), 1st iteration and at 7th iteration (final model) along with the true model are shown in Fig.2. In final solution boundaries are very close to the true boundaries and the oscillation are reduced to a minimum level.

Model III is again a three layer model studied by Simms & Morgan (1992) to study the phenomenon of the equivalence in resistivity data inversion. It may be mentioned here that our objective in this paper is not to resolve the equivalence problem in resistivity data inversion neither the technique discussed here addresses the equivalence problem. This example only demonstrate the performance of the technique over the equivalence model. Fig.3 shows the SIS solution (zeroth iteration), 1st iteration, final model (8th iteration) along with true model. This show that the technique is capable of handling of equivalent problem up to a limited extent.

Model IV is a five layer model in which apparent resistivity curves yields no indication about the correct number of layer in the true model. For such model quasi-linear method would not succeed as it needed an initial guess model (Gai-Shan 1985). However the SIS technique (Fig.4) successfully indicated the presence of five layer in the apparent resistivity data. The improvement in model using present' technique in Ist iteration, 15th iteration (final model) is shown in Fig.4, which also demonstrate SIS solution along with the true model. The final solution delineate all five layers and is very close to the true model.

Edge preserving smoothing method discussed in section (Edge-preserving smoothing) can be applied for all smooth model solution, the performance has been demonstrated over a three layer Model V. SIS

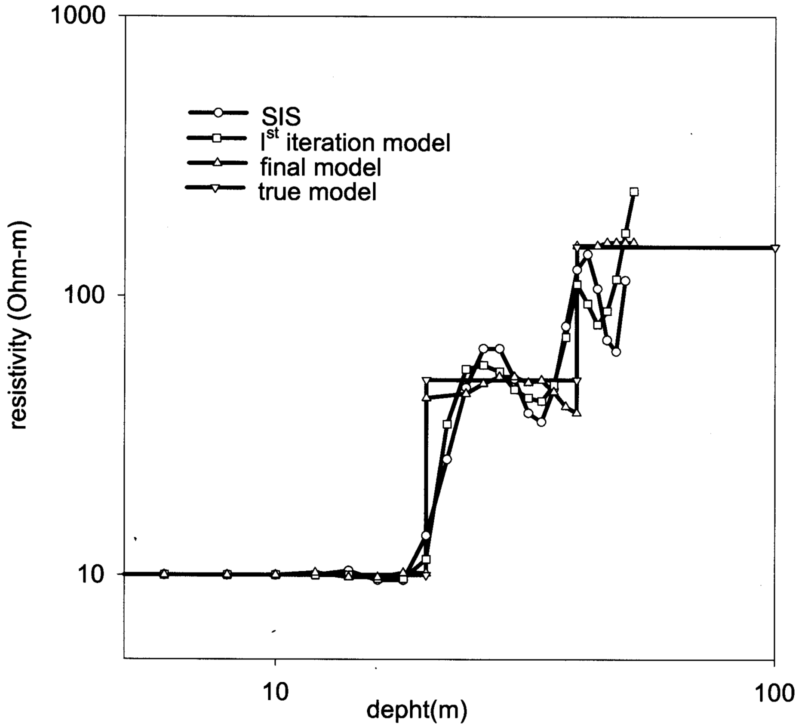

Figure 2. Comparison of inverted models obtained using straight forward inversion and proposed iterative straightforward inversion along with true model (A-type model).

generated smooth model is highly oscillatory with 2m unit layer thickness. Edge preserving smoothing technique with window width 5 has reduced these oscillations to a minimum value and also retains the boundary of the model. It may be mentioned here that first and last three layer resistivity of 2m thickness remain unchanged for 5 points window width. Therefore last layer resistivity is same as obtained by direct SIS method. Random Gaussian noise has been added to the apparent resistivity data and output model is compared with the true model. Fig.5 demonstrate output model with 5%, 10% and 20% Gaussian noise added in the data along with the true model. It has been observed that up to 10% error level output model closely follows the true model.

Field Example:

The techniques discussed in the present paper can be applied to the field apparent resistivity data. To demonstrate this aspect a vertical electrical sounding data set recorded using Schlumberger configuration near a borehole site, Roorkee (Uttaranchal) area has been used. The lithological boundaries from borehole have been correlated with the boundaries delineated using present method. Fig.6 shows the measured apparent resistivity curve, smooth inverted model, layered model obtained using present method and the borehole data available for the site. It may be seen that no boundary is visible clearly in the smooth inverted model, whereas boundaries delineated using present method are very close to the real boundaries present in the borehole data. In layered model, the first boundary from the surface appears at shallower depth (1.5m) whereas in borehole data this boundary is at 3m depth. The increase in resistivity at shallower depth is due to the increase in compactness of the top surface layer below the depth of 1.5m and thus forming resistivity boundary. Other lower boundaries delineated by the present method from the resistivity data are closely matching with the boundaries present in the borehole data.

CONCLUSIONS

Two techniques have been presented here for deriving layer boundaries from the smooth model obtained using SIS technique. The technique can also be used for the smooth models obtained from other smooth inversion techniques such as Occum's inversion. The

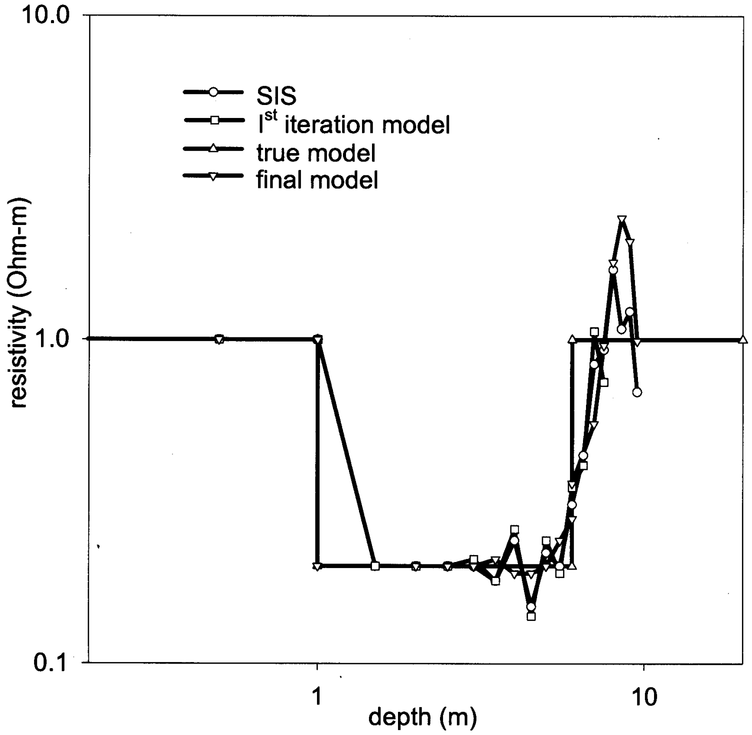

Figure 3. Comparison of inverted models obtained using straight forward inversion and proposed iterative straightforward inversion along with true model. The technique has been used successfully to the smoothed model obtained from SIS solution. The output model presents a fewer boundaries with sharp discontinuity.

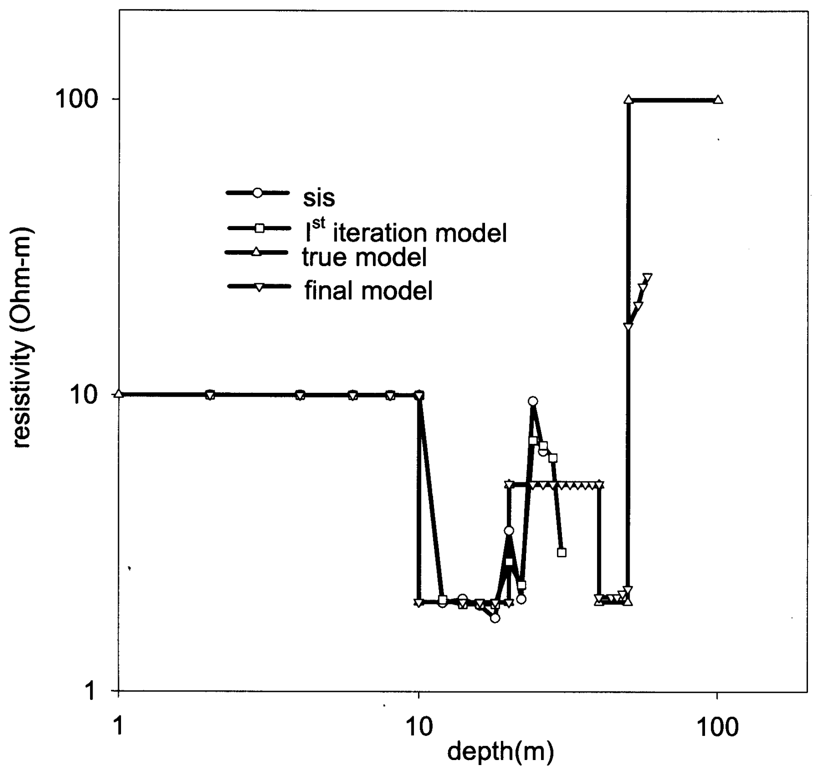

Figure 4. Demonstration of the performance of the proposed iterative straightforward inversion technique over a typical five layer model along with the results of straightforward inversion and true model.

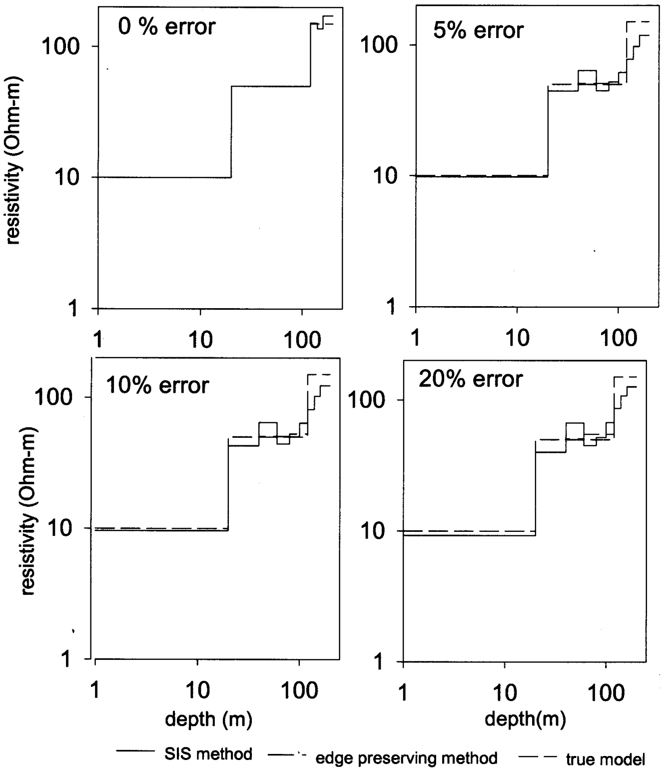

Figure 5. Performance of proposed edge-preserving smoothing technique over a three layer model with noise free and noise are added to the synthetic data.

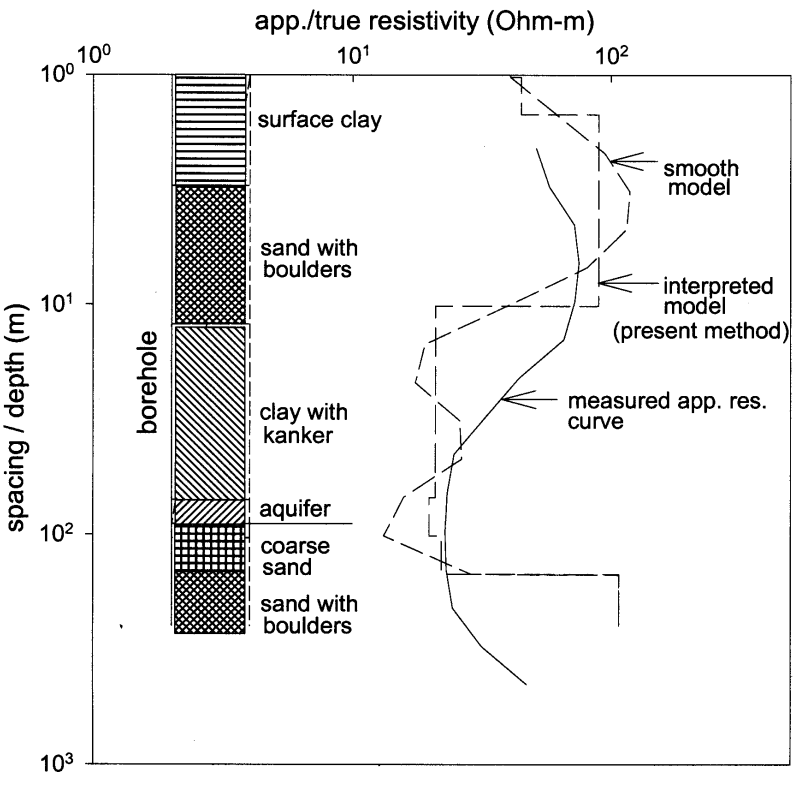

Figure 6. Performance of the sharp boundary method over field Schlumberger apparent resistivity data collected from the Roorkee (Uttaranchal) area and its comparison to the borehole data.

performance of these techniques have been demonstrated over test synthetic models showing the resistivity variation typically observed in variety of real geological situations and over a field data set. These test models include common three layer case (Model-I, 2 and 5), equivalent case (model-3) and a five layer model. Results show that the proposed techniques are capable of delineating layer boundaries very close to the boundaries present in the true model. The first technique is a extension of the regularised inversion algorithm and is referred as Iterative Straightforward Inversion (ISIS) whereas the second method is an analytical one and based on the application of smoothing filter referred as Edge Preserving and Smoothing (EPS). In ISIS the convergence rate is fast and final model is obtained in less than 20 iteration in all the cases studied.

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