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Modified Werner deconvolution technique for inversion of magnetic anomalies of horizontal circular cylinders

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ABSTRACT
The classical Werner deconvolution technique tries to solve the body parameters of geophysical models by framing linear equations with the help of magnetic anomalies and their locations on a profile and solving them. In the present paper, a method is designed which uses positions of point pairs with equal anomaly values, against the individual points in the Werner deconvolution technique. This method is presented with reference to the magnetic anomalies of a horizontal circular cylinder. On the field profile, two positions X_1 and X_2 for a chosen anomaly value ΔF are identified, and ΔF, X_1 and X_2 are used to frame linear equations and to calculate the depth to the center of the cylinder, its position and the dip of its effective magnetization.

INTRODUCTION

Werner (1953) has introduced a method of interpreting magnetic anomalies of two-dimensional sheet like bodies, wherein the anomalies and distances of observation points along a profile are arranged to form a linear equation, with its coefficients related to the parameters of the sheet. This method is popularly called as the Werner deconvolution technique. When compared to conventional methods, such as the method of characteristic curves, the Werner deconvolution method is expected to provide more reliable results, since all the available data on the profile is used in interpretation. When compared to normal inversion schemes, which assume initial values of the parameters and improve them iteratively, the Werner's method does not require any initial values of the parameters to be fed to the computer. There has been, however, an increasing tendency to apply the Werner deconvolution method to trace basement structures (for example see Hartman, Teskey & Preidberg 1971; Ku & Sharp 1983; Malleswara Rao et al. 1993; Sarma et al. 1994 and Thakur et al. 2000), based on clusters of fictitious positions of sheets. But Radhakrishna Murthy, Swamy & Rama Rao (2000) pointed that such clusters are confined to contacts and faults in the basement only when they are wide apart and very steep. For moderately dipping, close, not very shallow and smooth edged structures, the Werner's method does not provide any reliable interpretation. In essence, the method is simple, fast and reliable, when it is confined to interpret the anomalies of isolated simple geometric bodies.

Although introduced with reference to magnetic anomalies of sheets, the method of Werner deconvolution can generally be applied to a variety of geophysical models. According to Rao, Radhakrishna Murthy & Visweswara Rao. (1973), the observed anomaly, or its horizontal derivative, of a large number of geophysical models can be arranged to form a linear equation of the type

$$f_0(X_k,V_k) = C_1 f_1(X_k,V_k) + C_2 f_2(X_k,V_k) + ... + C_n f_n(X_k,V_k)$$

where V_k is the anomaly or its horizontal derivative, X_k is the distance of the anomaly or the derivative measured from a fixed reference, $f_0(X_k,V_k)$, $f_1(X_k,V_k)$, ... $f_n(X_k,V_k)$ are the functions that can be easily calculated from X_k and V_k, and C_1, C_2, ..., C_n are the unknown coefficients that are related to the parameters of the model. Linear equations of the above type can be framed one for each observation point and n normal equations are derived and solved for the unknown coefficients. The coefficients C_1, C_2, ..., C_n can be finally used to deduce the parameters of the model under question.

The serious drawback with this method is that the linear equation has generally more coefficients than the parameters, their number increasing with the presence of regional components of the anomaly in the profile. In the present paper, the Werner deconvolution method is modified, using the positions of point pairs with equal anomaly values, instead of individual points in the Werner deconvolution technique. Such a modification reduces the number of linear equations to a large extent. The modified method is applied to synthetic and actual field data of a horizontal circular cylinder.

FORMULATION OF THE METHOD

The generalized equation for the magnetic anomaly ΔF in any component (vertical, total or horizontal field) along a profile of a horizontal circular cylinder (Fig.1A) is given by Radhakrishna Murthy (1998),

$$\Delta F = C \frac{\left[Z^2 - (X-d)^2\right]A - 2B\left[(X-d)\right]Z}{\left[(X-d)^2 + Z^2\right]^2} \qquad (1)$$

where Z is the depth to the centre of the cylinder and X is the distance of the anomaly point P scaled from an arbitrary reference R in the profile, d is the horizontal distance from the reference to the position of the cylinder and ϕ' is the dip of effective magnetization. $C = 2\pi R^2 J'$, $A = \sin\phi'$ and $B = \cos\phi'$, where J' and ϕ' are generalized parameters connecting the magnitude of effective magnetization J, its dip ϕ and direction D_m along which the anomalies are measured by the equations

$$J' = J(1 - \cos^2\alpha \cos^2 D_m)^{1/2}, \text{ and}$$

$$\phi' = \phi - \tan^{-1}(\sin\alpha / \tan D_m).$$

D_m takes the values of i, $\pi/2$ and 0 for anomalies in the total field and its vertical and horizontal components respectively. α is the strike of the cylinder measured positive towards east from the magnetic north.

An inversion scheme tries to determine the values of d, Z and ϕ', besides the size parameter C. Cross multiplying and arranging the terms of Eq. (1), one gets a linear equation of the form

$$X^4 \Delta F = C_1 X^3 \Delta F + C_2 X^2 \Delta F + C_3 X \Delta F + C_4 X^2 + C_5 X + C_6 \Delta F + C_7, \qquad (2)$$

where

$$C_1 = 4d,$$
$$C_2 = -(6d^2 + 2Z^2),$$
$$C_3 = 4d(d^2 + Z^2),$$
$$C_4 = -A,$$
$$C_5 = 2(Ad - BZ),$$
$$C_6 = -(d^2 + Z^2)^2,$$

and $C_7 = (AZ^2 - Ad^2 + 2BdZ)$.

This is a seventh order equation against four unknown model parameters. However, on an anomaly profile, atleast two points can be located at which the anomaly values are equal. If X_1 and X_2 are the distances to these points of equal anomaly $\Delta F = \Delta F(X_1) = \Delta F(X_2)$ (Fig.1B), then X_1 and X_2 should satisfy Eq. (2) independently. Writing such equations and by subtracting one from the other, one obtains

$$(X_2^2 + X_1^2)(X_2 + X_1)\Delta F = C_1(X_2^2 + X_1^2 + X_1X_2)\Delta F + C_2(X_2 + X_1)\Delta F + C_3\Delta F + C_4(X_2 + X_1) + C_5, \qquad (3)$$

which is a fifth order equation for the four model parameters. This linear equation connects the

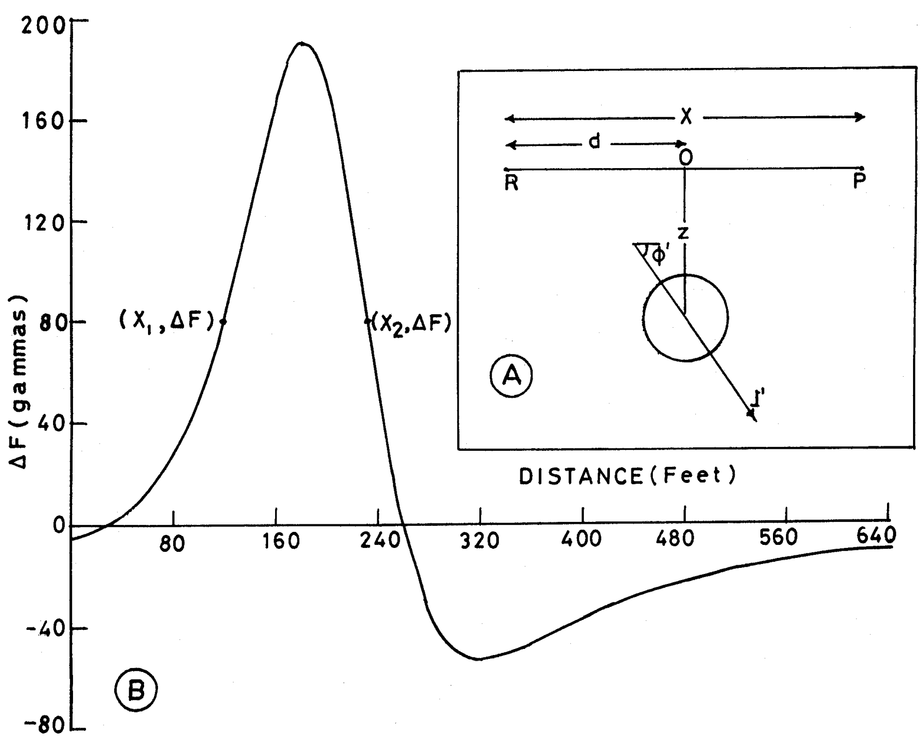

Figure 1 (A) Parameters of the vertical horizontal cylinder, (B) A synthetic vertical magnetic anomaly profile over a horizontal circular cylinder magnetized at 60°.

calculable function $(X_2^2 + X_1^2)(X_2 + X_1)\Delta F$ to other calculable functions $(X_2^2 + X_1^2 + X_1 X_2)\Delta F$, $(X_2 + X_1)\Delta F$, ΔF and $(X_2 + X_1)$. Several sets of values of ΔF, X_1, X_2 can be scaled from the field profile (Fig.1B), as many linear equations of the type in Eq. (3) framed as the number of these sets, five normal equations constructed and the coefficients viz., C_1 to C_5 solved for. These coefficients are then used to calculate sequentially the values of d, Z and ϕ'.

The values of d and Z are obtained from C_1 and C_2, while A and B are solved from C_4 and C_5 in succession as detailed below:

$$d = 0.25C_1,$$
$$Z = \sqrt{-0.5\,(C_2+6d^2)},$$
$$A = -C_4,$$
$$B = -0.5(C_5 + 0.5C_1C_4)/Z,$$
$$\phi' = \tan^{-1}(A/B),$$

and the size parameter $C = \sqrt{A^2 + B^2}$.

The method can be easily programmed to determine the model parameters of the horizontal cylinder.

A FEW EXAMPLES OF INTERPRETATION

The validity of the modified method was examined by trying it on synthetic profiles and two examples are shown in Figs 1 and 2, and the results compared in Table 1. Evidently, the performance of the method is highly satisfactory with the interpreted values closely matching with the synthetic data. Even the minor errors are due to digitization and measuring the distances correct to a couple of meters.

Fig.3 shows the example of interpretation of the vertical magnetic anomaly over a narrow band of quartz-magnetite in Manjampalli near Karimnagar town (Radhakrishna Murthy. Visweswara Rao & Gopala Krishna 1980) in India. Thirty one equispaced anomalies are sampled along this profile at an interval of eighteen feet. Only seven pairs of anomaly points with equal anomaly values were sampled and used in the interpretation. The interpreted parameters (viz., d, Z and ϕ' are compared in Table 2 with those reported by Radhakrishna Murthy, Visweswara Rao & Gopala Krishna (1980) by the conventional iterative inversion scheme. The interpretations generally agree, with the discrepancy between them being around 10%. This discrepancy between both the interpretations

Table 1. Interpretation of synthetic magnetic anomalies by the modified method

Parameter	Example in Figure 1		Example in Figure 2	
	Assumed values	Calculated values	Assumed values	Calculated values
d	200 feet	200 feet	200 feet	202 feet
Z	100 feet	103 feet	100 feet	100 feet
ϕ'	60 °	61 °	0 °	-1 °

Table 2. Interpretation of magnetic anomalies of a quartz-magnetite band

	Example in Figure 3	
Parameter	Present method	Inversion
d	335 feet	298 feet
Z	87 feet	78 feet
ϕ'	37 °	49 °

arose because of the actual tabular shape of the anomaly body, which in the present case is approximated to a cylinder.

CONCLUSIONS

The proposed method utilizes positions of point pairs of anomalies with equal anomaly values as against the positions of isolated points in the Werner deconvolution technique. The method, after scaling the distances X_1 and X_2 to the points of equal anomaly of value of ΔF, constructs linear equations of the type shown in equation (3), frames the necessary normal equations, solves them for the coefficients and finally calculates the parameters of the horizontal cylinder from the coefficients. All the steps are easily computerized. The modified Werner method was successfully applied to synthetic and field data. It can

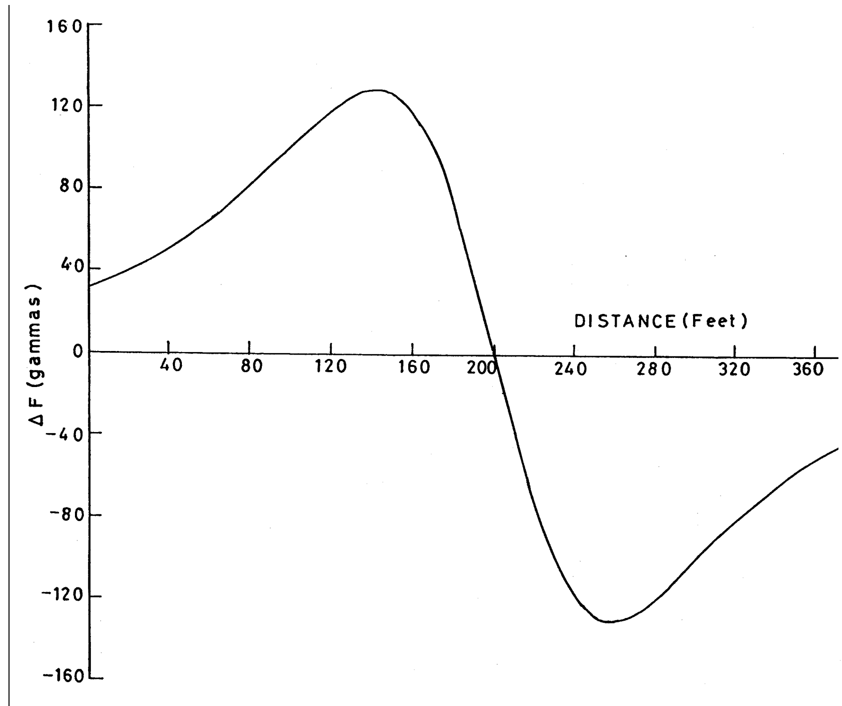

Figure 2. A synthetic vertical magnetic anomaly profile over a horizontally magnetized horizontal circular cylinder.

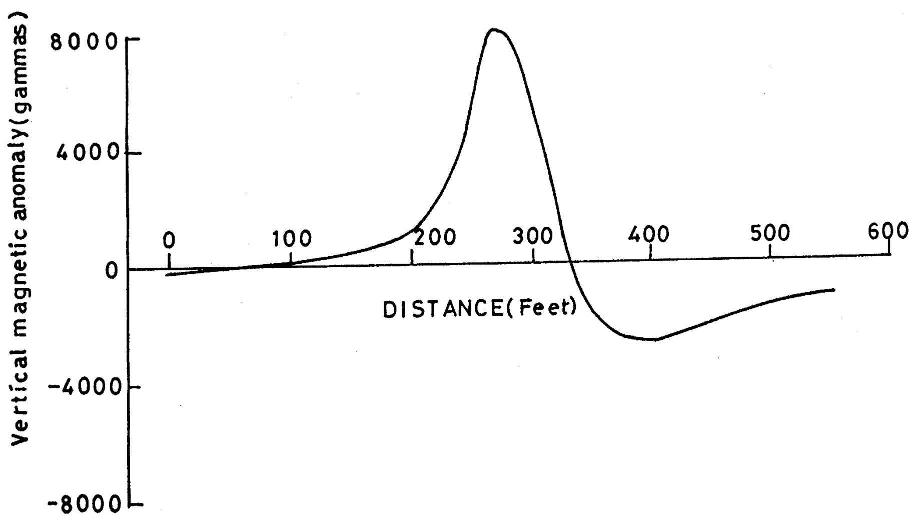

Figure 3. Vertical magnetic anomaly over a narrow band of quartz-magnetite in Manjampalli near Karimnagar town (Radhakrishna Murthy et.al., 1980).

be extended to gravity and magnetic anomalies of several geophysical models, with reference to the anomalies or their derivatives, and will be of academic interest.

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