J. Ind. Geophys. Union (October 2004)
Vol.8, No.4, pp.243-252

Analytical computation of Aquifer Potentials in a three-layered Porous Medium

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ABSTRACT
Layered aquifer systems are common. Seepage from surface water bodies can recharge a layered aquifer system depending upon the hydrogeological set up. In artificial recharge practices, which are on the rise all over the world, it is useful to have a prior understanding of the aquifer system. It may be helpful in planning sustainable watershed development projects in complex terrains. In this context, analytical solutions are presented for steady state hydraulic heads at any point within a three layered aquifer system. The analytical solutions are obtained with a point source of known strength, on the earth's surface. The recharging due to the point source would result in the build-up of hydraulic potentials in the layers beneath. By considering the analogy between electrical flow and groundwater flow, the geoelectric principles have been suitably invoked in formulating the presented methodology. An algorithm has been designed to compute the hydraulic potentials. The mode requires only recharge rate of the source, (aquifer) grid dimensions, hydraulic parameters for layers and depth to the various layers. Several examples are provided to demonstrate the effectiveness of the analytical solutions. Performance of the analytical model compares well with that of a numerical model, MODFLOW. The algorithm presented is much faster and easy to execute.

INTRODUCTION

Recharging is predominantly governed by the hydrogeological set-up, and the hydraulic and flow parameters of the porous medium. Quantitative assessment of replenishment of an aquifer system, subjected to artificial recharging, is essential to ascertain the efficiency of recharging schemes. The recharging process is much complex in a hard-rock terrain, where the aquifers are characterised by the presence of discrete faults and joints, compared to that in an alluvial set-up. Such investigations are uncommon, particularly, in the case of multi-aquifer systems. Nevertheless, it is desirable to have a proper understanding of the flow regime and distribution of potentials in an aquifer system, where recharging being undertaken, in order to perform quantitative assessment of a recharging scheme.

Groundwater flow models are powerful tools for such investigations. However, most numerical flow models demand elaborate data and computing facilities, which might not be available always. On the other hand, when simple solutions are sought as part of initial investigations, such elaborate modelling results are not necessary. Analytical models, if exists, can provide easy-to-use solution in similar field situations. Analytical models are handy compared to numerical models when a steady state solution of hydraulic potentials is sought in an aquifer having simple boundaries.

The analogy between electrical current flow (Ohm's law) and groundwater flow (Darcy's law) is well established (Hubbert 1940; Freeze & Cherry 1979). Groundwater flow models were used in simulating electrical current flow through porous media (Osiensky & Williams 1996). Also, discussions on the use of direct current electrical resistivity methods and electrical analog models in Groundwater applications are available in the literature (Walton 1970; Zohdy et al. 1974; Prickett 1975).

Objective of this study is to develop a set of analytical solutions for hydraulic potentials in a stratified aquifer system. Analytical expressions for steady state hydraulic potential, due to a point-source, would be derived specifically for a three-layered homogeneous isotropic aquifer system with infinite depth extent. The hydraulic conductivity values can be different for the different layers. The flow in the aquifer system is three-dimensional around a point-recharge source. An algorithm for solution procedure, using the derived analytical expressions, would be coded to enable computation of hydraulic potentials. Finally, the results obtained through the analytical solution would be compared with that obtained from MODFLOW (Mc Donald & Harbaugh, 1984).

THEORETICAL CONSIDERATIONS

Analytical expressions for steady state hydraulic potential would be derived, by considering advancements made in the theory of vertical electrical sounding (Bhattacharya & Patra 1968; Koefoed 1979), specifically for a three-layered aquifer with infinite extent. The methodology available in the theory of vertical electrical sounding for computing electric potential on the surface of the earth has been suitably modified and extended to the case of layered groundwater aquifers by re-defining relevant parameters and boundary conditions.

Development of Basic Equation

Let us consider a horizontally stratified aquifer system with *n* layers in a cylindrical coordinate framework with its origin at the point recharge source, A (Fig. 1). Let K_1, K_2,....., K_n be the hydraulic conductivities, and h_1, h_2,....., h_n be the depths to the bottom of respective layers from the surface. It is assumed that the last layer extends to infinity. Then, steady state hydraulic potential, ϕ_i (r,z) at any point in Layer i (for i=1,2,. . . ,n) is described by the following Laplace equation:

$$\frac{\partial^2\phi_i}{\partial r^2} + \frac{1}{r}\frac{\partial\phi_i}{\partial r} + \frac{\partial^2\phi_i}{\partial z^2} = 0 \qquad (1)$$

Following Bhattacharya and Patra (*1968*), the general solution of eqn. (1) is given by:

$$\phi_i(r,z) = \frac{q}{2\pi K_1}\left\{\frac{1}{(r^2+z^2)^{1/2}} + \int_0^\infty [A_i(\lambda)e^{-\lambda z} + B_i(\lambda)e^{\lambda z}]J_0(\lambda r)d\lambda\right\} \qquad (2)$$

where q [m³/s] is the recharge rate from the point source, $A_i(\lambda)$ and $B_i(\lambda)$ are unknown functions, and $J_0(\lambda r)$ is the Bessel function of zeroth order.

Using the Lipschitz Integral (*Watson, 1944*), viz.,

$$\int_0^\infty e^{-\lambda z} J_0(\lambda r)d\lambda = \frac{1}{(r^2+z^2)^{1/2}} \qquad (3)$$

eqn. (2) can be rewritten as:

$$\phi_i(r,z) = \frac{q}{2\pi K_1}\left\{\int_0^\infty [\, e^{-\lambda z} + A_i(\lambda)e^{-\lambda z} + B_i(\lambda)e^{\lambda z}\,] J_0(\lambda r)d\lambda\right\} \qquad (4)$$

Now, the unknown functions $A_i(\lambda)$ and $B_i(\lambda)$ in eqn. (4) may be evaluated subjected to the following boundary conditions (*Koefoed, 1979*):

(i) At the air-earth interface the vertical component of flow must be zero; i.e.,

$$K_1 \frac{\partial\phi_i}{\partial z}\Big|_{z=0} = 0 \qquad (5a)$$

This implies that, in eqn. (4),

$$A_1(\lambda) = B_1(\lambda) \qquad (5b)$$

(ii) At each of the boundary planes in the subsurface, the hydraulic potential as well as flow must be continuous. So, at any i^{th} interface within the medium,

$$\phi_i = \phi_{i+1} \text{ and } K_1\frac{\partial\phi_i}{\partial z} = K_{1+1}\frac{\partial\phi_{i+1}}{\partial z} \qquad (6a)$$

Applying these conditions to eqn. (4), we obtain the following:

$$A_i(\lambda)e^{-\lambda h_i} + B_i(\lambda)e^{-\lambda h_i} = A_{i+1}(\lambda)e^{-\lambda h_i} + B_{i+1}(\lambda)e^{-\lambda h_i} \qquad (6b)$$

$$K_i[1+A_i(\lambda)e^{-\lambda h_i} - B_i(\lambda)e^{-\lambda h_i}] = K_{i+1}[\{A_{i+1}(\lambda)e^{-\lambda h_i} + B_{i+1}(\lambda)e^{-\lambda h_i}] \qquad (6c)$$

(iii) At infinite depth/ distance, the hydraulic potential must approximate to zero, i.e.,

$$\lim_{z\to\infty} \phi(r,z) = 0 \text{ and } \lim_{r\to\infty} \phi(r,z) = 0 \qquad (7a)$$

It necessitates $B_n(\lambda)$ $e^{\lambda z}$ to vanish in eqn. (4). Therefore,

$$B_n(\lambda) = 0 \qquad (7b)$$

By using the eqns. 5(b), 6(b), 6(c) & 7(b), a system of *2n* equations in *2n* unknown functions $A_i(\lambda)$ and $B_i(\lambda)$ can be obtained. By introducing the following notations,

$$\begin{aligned} u_i &= e^{-\lambda h_i} \\ v_i &= e^{+\lambda h_i} \\ P_i &= K_i+1/K_i \\ A_i(\lambda) &= A_i \\ B_i(\lambda) &= B_i \end{aligned} \qquad (8)$$

the corresponding system of equations in $A_i(\lambda)$ and $B_i(\lambda)$ can be written as:

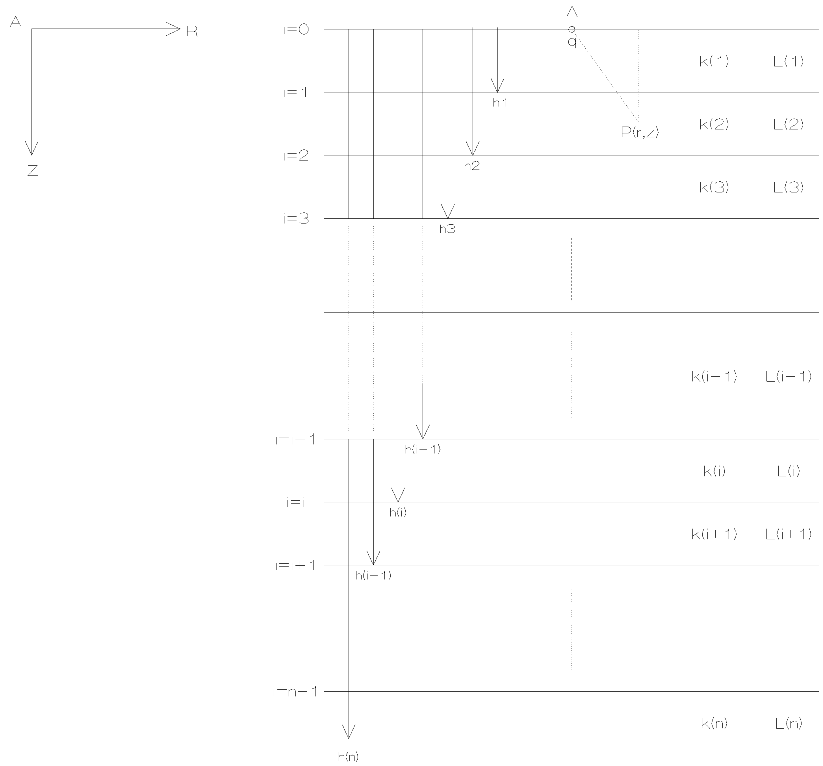

Figure 1. Schematic diagram of a layered aquifer system with reference to cylindrical coordinate system. A point-source is located at A; P(r,z) is any arbitrary point in the medium.

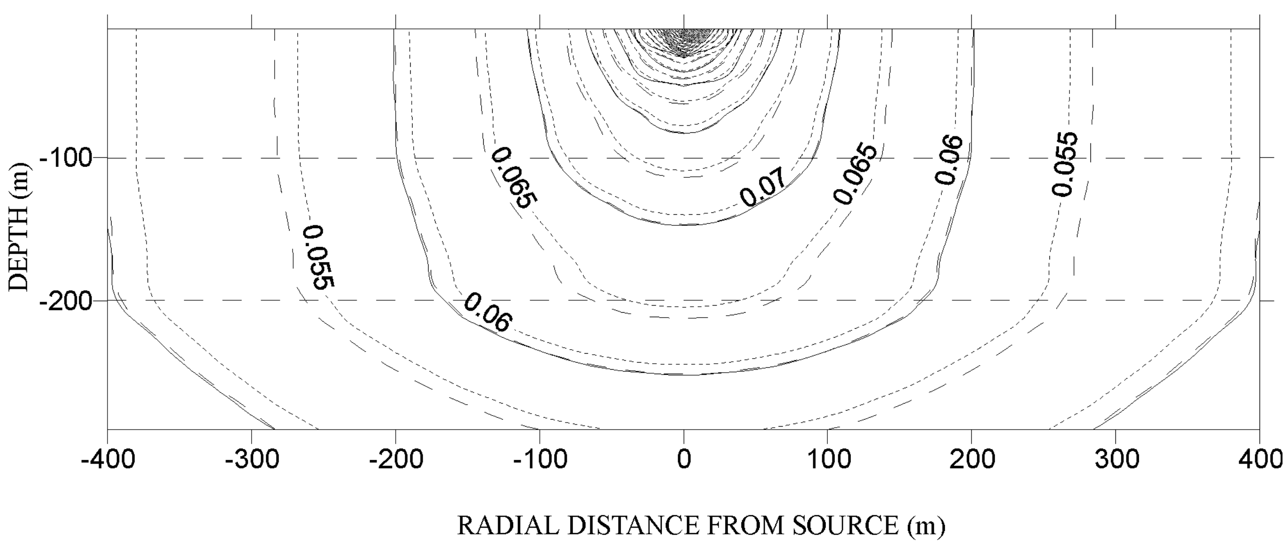

Figure 2. Convergence of the series expression for hydraulic potentials computed by analytical method with 100 terms (dotted lines), 200 terms (dashed lines) and 300 terms (solid lines) for a case where the layer set-up with respect to hydraulic conductivities is of the type: $K_1 > K_2 > K_3$.

$$
\begin{aligned}
&(u_1 + v_1)A_1(\lambda) - u_1 A_2(\lambda) - v_1 B_2(\lambda) = 0 \\
&(v_1 - u_1)A_1(\lambda) + p_1 u_1 A_2(\lambda) - p_1 v_1 B_2(\lambda) = (1-p_1)u_1 \\
&u_2 A_2(\lambda) + v_2 B_2(\lambda) - u_2 A_3(\lambda) - v_2 B_3(\lambda) = 0 \\
&-u_2 A_2(\lambda) + v_2 B_2(\lambda) - p_2 u_2 A_3(\lambda) - p_2 v_2 B_3(\lambda) = (1-p_2)u_2 \\
&\qquad \ldots\ldots\ldots\ldots\ldots\ldots \\
&\qquad \ldots\ldots\ldots\ldots\ldots\ldots \\
&u_{n-1}A_{n-1}(\lambda) + v_{n-1}B_{n-1}(\lambda) - u_{n-1}A_n(\lambda) = 0 \\
&-u_{n-1}A_{n-1}(\lambda) + v_{n-1}B_{n-1}(\lambda) + p_{n-1}u_{n-1}A_n(\lambda) = (1-p_{n-1})u_{n-1}
\end{aligned}
\tag{9}
$$

ANALYTICAL RESULTS

In geoelectrical prospecting, the depths and electrical resistivities of horizontal layers of earth are determined by using the electrical potential and electrical field at any point on the surface of the earth, and the analytical solution for electrical potentials on the surface is available (*Bhattacharya and Patra, 1968*). However, for geohydrological applications, the hydraulic potential needs to be computed at various points within the aquifer system and not on the surface. Therefore we have to derive a set of expressions for that purpose. The following analysis is an attempt towards achieving this objective, in the case of a three layered aquifer system.

Solution Procedure

The unknown functions $A_i(\lambda)$ and $B_i(\lambda)$ for a three layered aquifer system ($n=3$) were obtained, by solving the system of eqns. (9) using the method of determinants, as:

$$A_1(\lambda) = B_1(\lambda) = \frac{c_1 e^{-2\lambda h_1} + c_2 e^{-2\lambda h_2}}{1 - c_1 e^{-2\lambda h_1} - c_2 e^{-2\lambda h_2} + c_1 c_2 e^{-2\lambda(h_2-h_1)}} \tag{10}$$

$$A_2(\lambda) = \frac{c_1 + c_1 e^{-2\lambda h_1} + c_2 e^{-2\lambda h_2} - c_1 c_2 e^{-2\lambda(h_2-h_1)}}{1 - c_1 e^{-2\lambda h_1} - c_2 e^{-2\lambda h_2} + c_1 c_2 e^{-2\lambda(h_1-h_2)}} \tag{11}$$

$$B_2(\lambda) = \frac{c_2 e^{-2\lambda h_2} + c_1 c_2 e^{-2\lambda h_2}}{1 - c_1 e^{-2\lambda h_1} - c_2 e^{-2\lambda h_2} + c_1 c_2 e^{-2\lambda(h_2-h_1)}} \tag{12}$$

$$A_3(\lambda) = \frac{c_2 + c_2 e^{-2\lambda h_2} - c_1 c_2 e^{-2\lambda h_1} - c_1 c_2 e^{-2\lambda(h_2-h_1)} + \frac{2c_1 K_2}{K_2 + K_3}\left(1 + e^{-2\lambda h_1}\right)}{1 - c_1 e^{-2\lambda h_1} - c_2 e^{-2\lambda h_2} + c_1 c_2 e^{-2\lambda(h_2-h_1)}} \tag{13}$$

and

$$B_3(\lambda) = 0 \tag{14}$$

where $c_1 = (1 - p_1)/(1 + p_1)$ and $c_2 = (1 - p_2)/(1 + p_2)$.

It may be noted that as $A_i(\lambda)$ and $B_i(\lambda)$ are functions of λ, it is necessary to express them in suitable mathematical forms to enable evaluation of integral in eqn. (4). Therefore, the solution for hydraulic potential given by eqn. (4) is to be obtained using the following procedure (Bhattacharya & Patra 1968), which has been adopted to express the hydraulic potential, ϕ_i (r,z) (given by eqn. 4) in a convenient analytical form for computation:

Let $h_1 = s_1 h_0$ and $h_2 = s_2 h_0$ where s_1 and s_2 are integer multipliers, and h_0 has some fixed value. Also, let $\mathrm{Exp}(-2\lambda h_0) = g$ for the sake of brevity. Then, $A_i(\lambda)$ and $B_i(\lambda)$ for i=1, 2, and 3 can be rewritten in terms of s_1, s_2 and g. Also, since s_1 and s_2 are integers, $A_i(\lambda)$ and $B_i(\lambda)$ are rational functions of g. Thererfore, $A_i(\lambda)$ and $B_i(\lambda)$ can be expressed as polynomial series expansions in g with appropriate coefficients. Thus, eqns. (10-14) can be rewritten and equated to the respective polynomial series as:

$$A_1(\lambda) = \frac{c_1 g^{s_1} + c_2 g^{s_2}}{1 - c_1 g^{s_1} - c_2 g^{s_2} + c_1 c_2 g^{(s_2-s_1)}} = \sum_{m=0}^{\infty} a_m g^m \tag{15}$$

$$A_2(\lambda) = \frac{c_1 + c_1 g^{s_1} + c_2 g^{s_2} + c_1 c_2 g^{(s_2-s_1)}}{1 - c_1 g^{s_1} - c_2 g^{s_2} + c_1 c_2 g^{(s_2-s_1)}} = \sum_{m=0}^{\infty} b_m g^m \tag{16}$$

$$B_2(\lambda) = \frac{c_2 g^{s_2} + c_1 c_2 g^{s_2}}{1 - c_1 g^{s_1} - c_2 g^{s_2} + c_1 c_2 g^{(s_2-s_1)}} = \sum_{m=0}^{\infty} d_m g^m \tag{17}$$

$$A_3(\lambda) = \frac{\left(c_2 + \frac{2c_1 K_2}{K_2 + K_3}\right) + \left(\frac{2c_1 K_2}{K_2 + K_3} - c_1 c_2\right) g^{s_1} + c_2 g^{s_2} + c_1 c_2 g^{(s_2-s_1)}}{1 - c_1 g^{s_1} - c_2 g^{s_2} + c_1 c_2 g^{(s_2-s_1)}} = \sum_{m=0}^{\infty} f_m g^m \tag{18}$$

where a_m, b_m, d_m, and f_m are the coefficients of the respective polynomial expansions.

Equating coefficients of various orders of g on both sides of the above equations will yield a_m, b_m, d_m, and f_m when $m \le s_2$ as the highest order of g on the left hand side is s_2. The higher order ($m > s_2$) coefficients of g on the right hand side must be zero, which leads to the following recurrence formula for the rest of the coefficients:

$$R_{s_2+j} = c_1 R_{s_2-s_1+j} - c_2 R_j - c_1 c_2 R_{s_1+j}\,; \; for\ j = 1,2,3,\ldots \tag{19}$$

where R_m is the notation for any of the unknown coefficients a_m, b_m, d_m, or f_m.

Thus, the unknown functions $A_i(\lambda)$ and $B_i(\lambda)$ in the equation for hydraulic potential were reduced to a set of rational functions in terms of $\mathrm{Exp}(-2\lambda h_0)$ as:

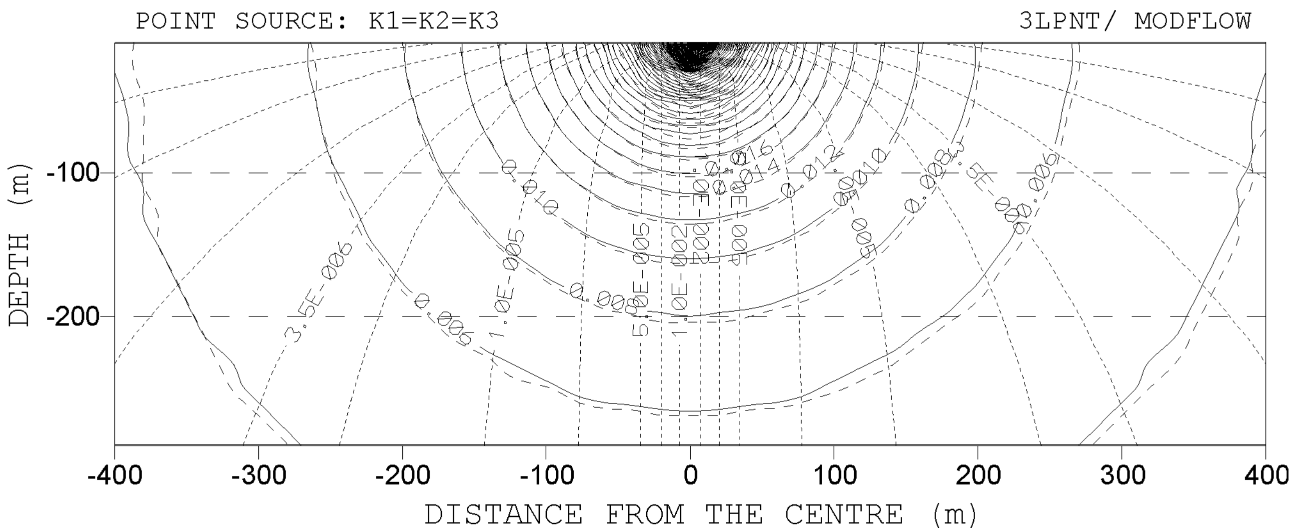

Figure 3a. Equipotentials in the vertical section of a homogeneous aquifer system ($K_1=K_2=K_3$); computed by 3LPNT (solid contours) and MODFLOW (dashed contours). Dotted contours are the stream lines computed by 3LPNT.

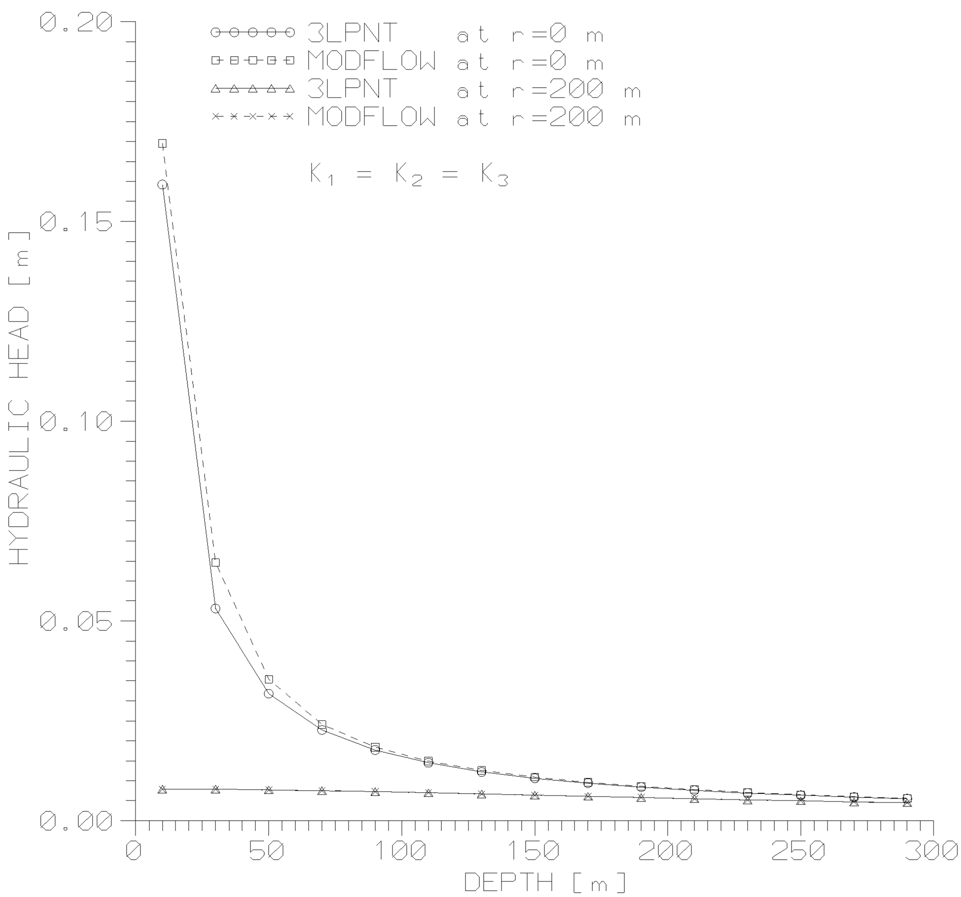

Figure 3b. Vertical distribution of hydraulic potential obtained from 3LPNT (solid line) and MODFLOW (dashed line) at the source and at a distance 200m from the source for $K_1=K_2=K_3$

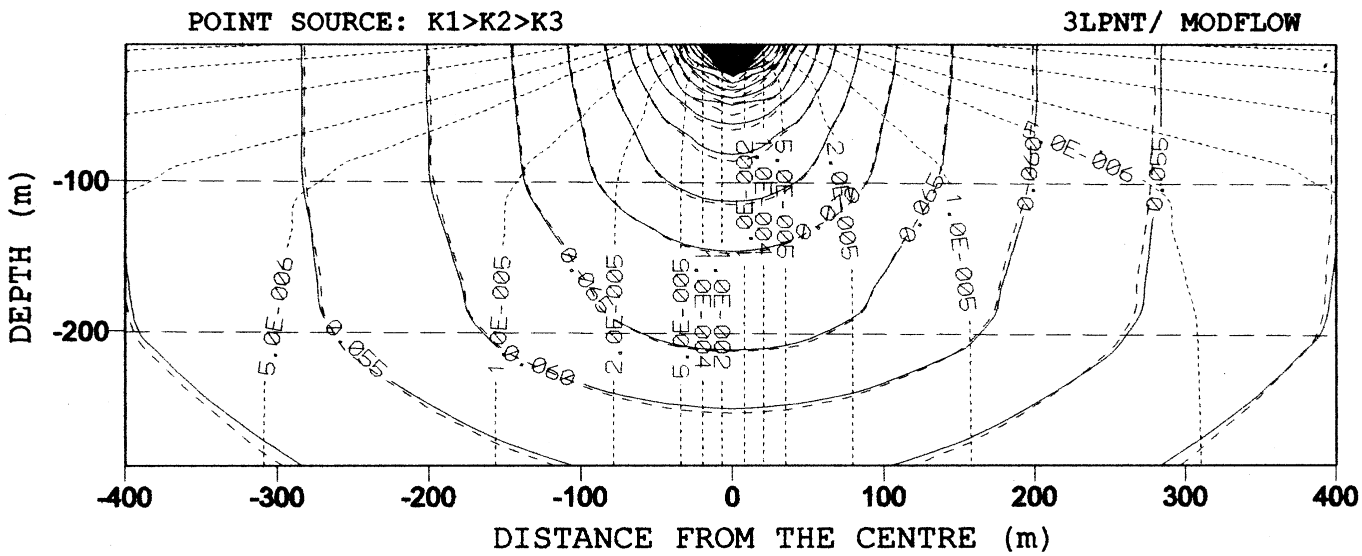

Figure 4a. Equipotentials in the vertical section of a homogeneous aquifer system ($K_1>K_2>K_3$); computed by 3LPNT (solid contours) and MODFLOW (dashed contours). Dotted contours are the stream lines computed by 3LPNT.

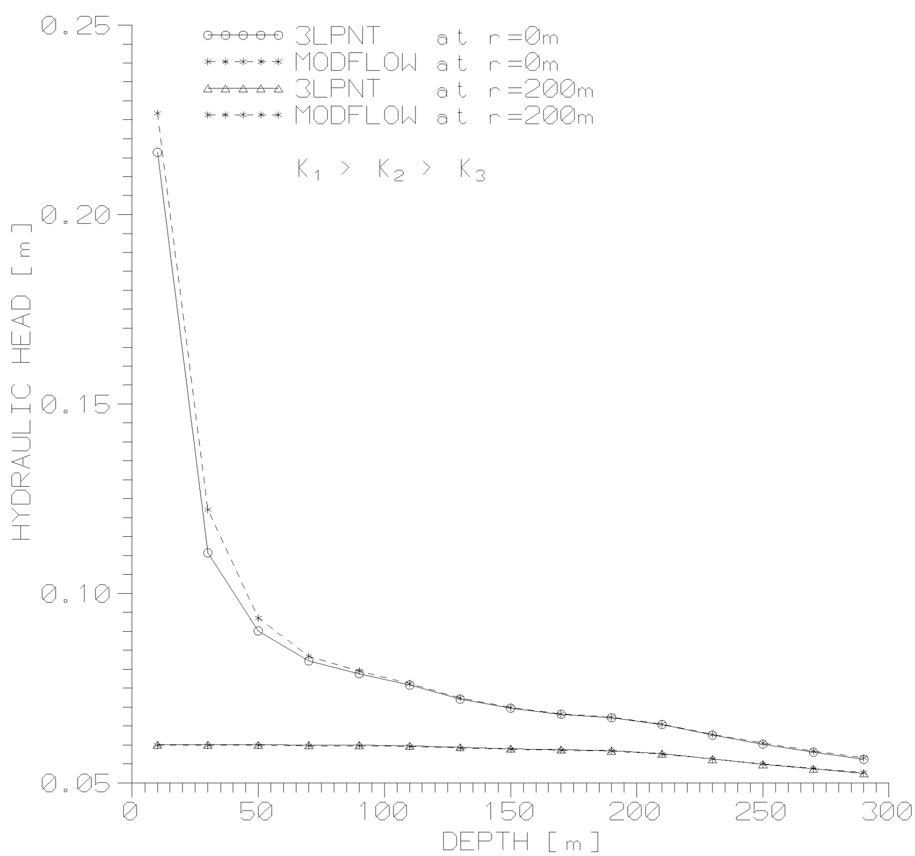

Figure 4b. Vertical distribution of hydraulic potential obtained from 3LPNT (solid line) and MODFLOW (dashed line) at the source and at a distance 200m from the source for $K_1>K_2>K_3$

$$A_1(\lambda) = B_1(\lambda) = \sum_{m=0}^{\infty} a_m e^{-2\lambda m h_0} \tag{20}$$

$$A_2(\lambda) = \sum_{m=0}^{\infty} b_m e^{-2\lambda m h_0} \tag{21}$$

$$B_2(\lambda) = \sum_{m=0}^{\infty} d_m e^{-2\lambda m h_0} \tag{22}$$

$$A_3(\lambda) = \sum_{m=0}^{\infty} f_m e^{-2\lambda m h_0} \tag{23}$$

$$B_3(\lambda) = 0 \tag{24}$$

Now, substituting the expressions given by eqns. (20) to (24) for $A_i(\lambda)$ and $B_i(\lambda)$ for i=1, 2, and 3, respectively in eqn. (4) followed by application of the Integral formula of Lipschitz will provide the hydraulic potential function, ϕ_i (r,z) for the computation of hydraulic potential at any point in the i^{th} layer. Thus, the hydraulic potential in Layer 1, Layer 2, and Layer 3, respectively are given by:

$$\phi_1(r,z) = \frac{q}{2\pi K_1}\left[\frac{1}{\sqrt{r^2+z^2}} + \sum_{m=0}^{\infty}\frac{a_m}{\sqrt{r^2+(2mh_0-z)^2}} + \sum_{m=0}^{\infty}\frac{a_m}{\sqrt{r^2+(2mh_0+z)^2}}\right] \tag{25}$$

$$\phi_2(r,z) = \frac{q}{2\pi K_1}\left[\frac{1}{\sqrt{r^2+z^2}} + \sum_{m=0}^{\infty}\frac{d_m}{\sqrt{r^2+(2mh_0-z)^2}} + \sum_{m=0}^{\infty}\frac{b_m}{\sqrt{r^2+(2mh_0+z)^2}}\right] \tag{26}$$

$$\phi_3(r,z) = \frac{q}{2\pi K_1}\left[\frac{1}{\sqrt{r^2+z^2}} + \sum_{m=0}^{\infty}\frac{f_m}{\sqrt{r^2+(2mh_0-z)^2}}\right] \tag{27}$$

where h_0 is a chosen fixed value, and a_m, b_m, d_m, and f_m are coefficients in polynomial expansions which can be determined as explained earlier. Thus, the coefficients in the above expressions are:

$a_0 = 0,$ $\quad b_0 = c_1,$

$a_1 = c_1,$ $\quad b_1 = c_1(1-c_2)(1+c_1),$

$a_2 = c_2-c_1^2(c_2-1)$ $\quad b_2 = (1+c_1)[c_2+c_1^2(1-c_2)^2]$

$d_0 = 0,$ $\quad f_0 = c_2 + 2c_1/(1+p_2),$

$d_1 = 0,$ $\quad f_1 = 2c_1/(1+p_2)[1-c_1(c_2-1)]- c_1c_2(1+c_2),$

$d_2 = c_2(1+c_1)$ $\quad f_2 = c_2 - f_1c_1(c_2-1) + f_0c_2$

The coefficients of higher order terms, such as a_i, b_i, d_i and f_i for i=3,4,5,... are obtained using the recurrence relation (eqn. 19). The computation of coefficients as well as potentials at any point in the medium is accomplished through a computer algorithm.

IMPLEMENTATION OF ANALYTICAL MODEL

Using the derived analytical solutions [eqns. (25), (26), (27)] an algorithm has been devised for computing the steady state hydraulic potential in a hypothetical three layered aquifer system.

Description of the Model

The hydraulic potential has been computed using the devised algorithm, at the nodes of a rectangular grid of dimensions 1180 m by 580 m in the RZ-plane (of Fig. 1). The vertical plane has been discretised into 59 columns and 29 layers (vertical discretistion) of dimension 20m each. The model parameters of the system are: h_1=100m, h_2=200m and $h_3=\infty$ by letting n=3, s_1=1, s_2=2, and h_0=100 m. A point source of strength, q=0.01 m^3/s has been used to recharge the aquifer. Hydraulic conductivities for the layers (K_1, K_2 and K_3) have been so chosen as to form four different types of layered aquifer systems viz., Type-I: $K_1=K_2=K_3$, Type-II: $K_1>K_2>K_3$, Type-III: $K_1<K_2>K_3$, and Type-IV: $K_1>K_2<K_3$.

It may be observed that the analytical solutions consist of Cauchy convergent sequences. Therefore, the number of terms required for convergence may be chosen based on some closure criterion. The convergence pattern is demonstrated by comparing equipotentials computed by the algorithm with different number of terms in the sequences (see Fig. 2). The difference in computed values with 200 and 300 terms of the sequences was only of the order of 10^{-4} m, for all the four types aquifer system. Hence, for the present computations only 200 terms of the sequences have been used.

Comparison of the Model

For validation purpose, steady state simulation of hydraulic potential has been performed by using a three dimensional finite difference groundwater flow model, MODFLOW (Mc Donald & Harbaugh 1984) with identical grid set-up and boundary conditions as that of the analytical solution procedure. Thus, the three dimensional model grid has been discretised into 59 rows, 59 columns and 29 layers (vertical discretisation) of dimension 20m each. A recharge well (with q=0.01m^3/s) has been introduced at the central node (L1, R30, C30) for source. Constant head boundary condition has been assigned to the boundaries of the model grid with near-zero head values. Model parameters have been assigned cell-wise through the data files of basic package (BAS), block-centred package

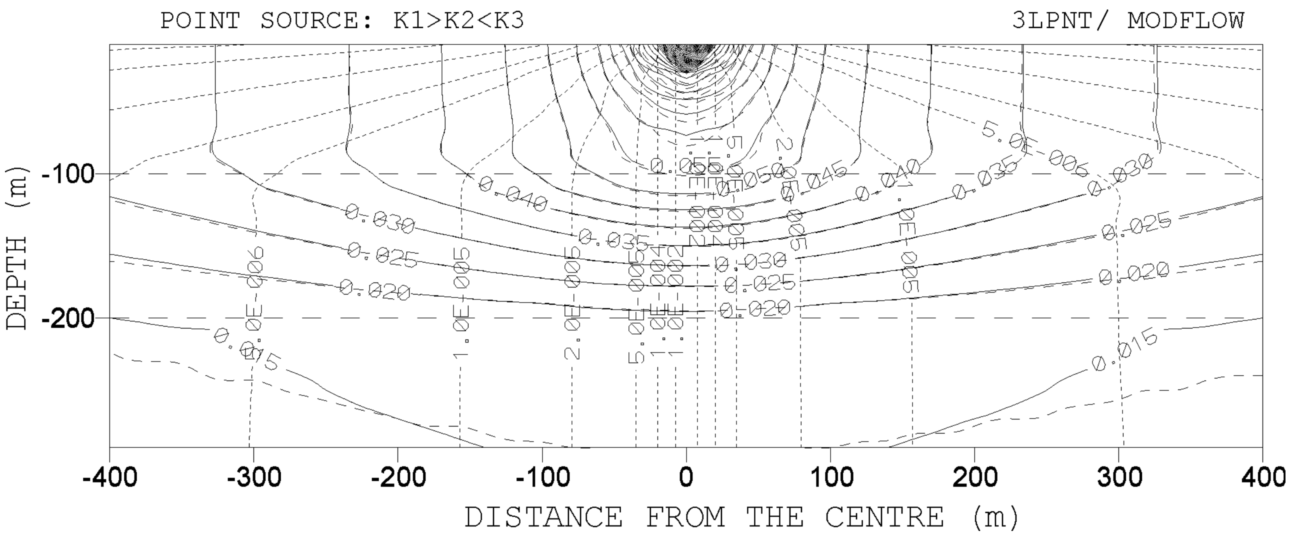

Figure 5a. Equipotentials in the vertical section of a homogeneous aquifer system ($K_1<K_2>K_3$); computed by 3LPNT (solid contours) and MODFLOW (dashed contours). Dotted contours are the stream lines computed by 3LPNT.

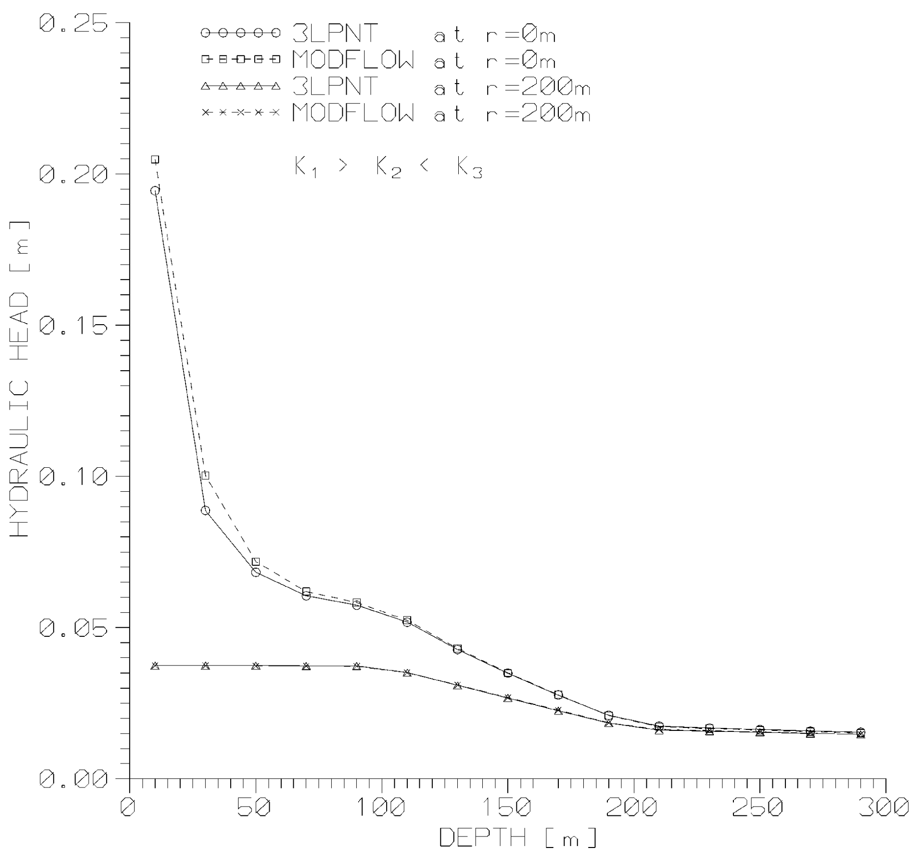

Figure 5b. Vertical distribution of hydraulic potential obtained from 3LPNT (solid line) and MODFLOW (dashed line) at the source and at a distance 200m from the source for $K_1<K_2>K_3$

(BCF), iteration package (SIP) and well package (WEL) of the model. While MODFLOW uses an iterative procedure to compute hydraulic head, computation by the presented algorithm is straight forward.

Simulations

The hydraulic potential has been computed in four different types of aquifer systems (see Table-1) using the presented algorithm and MODFLOW. The contour plots of hydraulic potential has been presented for the various cases (see Figs 3a, 4a, 5a, and 6a). For the sake of clarity, a region bound by -400 m to 400 m in the R- direction and 0 to 300m in the Z- direction has been used for the plots. The dashed horizontal lines in the plots demarcate the interfaces between layers. A uniform contour interval of 0.005 has been chosen in the case of the homogeneous aquifer (Type I). For all other cases, the equipotentials are spaced with 0.01. The distribution of hydraulic potential in the vertical obtained through the given algorithm and MODFLOW, respectively have also been provided for comparison (Figs 3b, 4b, 5b, and 6b).

Table 1: Hydraulic conductivity values for different types of aquifer systems

Aquifer Type		K_1(m/s)	K_2(m/s)	K_3(m/s)
Type I:	$K_1 = K_2 = K_3$	0.001	0.001	0.001
Type II:	$K_1 > K_2 > K_3$	0.001	0.0001	0.00001
Type III:	$K_1 < K_2 > K_3$	0.0001	0.001	0.00001
Type IV:	$K_1 > K_2 < K_3$	0.001	0.00001	0.0001

DISCUSSION OF RESULTS

Type-I is a homogeneous aquifer. When $K_1=K_2=K_3$, the coefficients a_m, b_m, d_m, and f_m reduced to zeroes as evident from their respective expressions, and the summation terms of eqns. (25), (26), and (27) vanish. Then, the solution for hydraulic potential reduces to: $\phi = q/[2\pi K_1(r^2 + z^2)^{1/2}]$, which is nothing but the exact solution for steady state potential in a homogeneous aquifer (Hubbert 1940). Contour plots of hydraulic potentials computed by analytical method and MODFLOW have been merged in single plot to facilitate visual comparison (see Fig. 3a). The depth-wise distribution of hydraulic potential at the centre (at the point source) and at a distance of 200 m away from the source is given in Fig. (3b). Similar plots are presented for the other three types of layered aquifer systems also. A performance comparison of computed hydraulic potentials (analytical and numerical) can be made by inspection of Fig. (4a), for equipotentials and (4b), for vertical distribution in the case of Type-II aquifer. Similar comparison can be made with Fig. (5a) and (5b) for Type-III aquifer, and Fig. (6a) and (6b) for Type-IV aquifer. It is seen that there is fairly good agreement between the computed hydraulic potentials of the two methods. The percentage difference of MODFLOW results with respect to analytical results is about 2% in most of the region. However, it is about 5% around the source. The deviations are attributed to inaccuracies in assigning near-zero constant head boundary values for the MODFLOW simulations.

CONCLUSIONS

Based on the geoelectrical sounding theory, an analytical solution procedure has been developed and demonstrated with numerical examples for the computation of steady-state hydraulic potentials due to a point source, in a three layered aquifer system. The major assumptions involved in the derivation of the analytical solutions are: (i) the stratified aquifer is of infinite extent, and (ii) there is cylindrical symmetry in the aquifer with respect to the Z-axis. A comparison was made between the analytical solution and MODFLOW. The equipotentials obtained through analytical technique and MODFLOW matched well, there by establishing the computational validity and effectiveness of the presented analytical solution procedure.

It can be seen that analytical technique requires only a few input parameters such as source strength, layer conductivities, layer thicknesses and grid information for computation. Therefore, when steady state hydraulic potential is to be determined in a layered aquifer like the one described, these analytical solutions provide a means for easier computation. It is possible to derive the corresponding stream lines from the analytical expressions for hydraulic potential, for flow computations, unlike many numerical flow models. Further, the solution procedure could be extended to the case of line-sources and an area-sources. Also, the derived set of analytical solutions may be modified by effecting suitable coordinate transformation to obtain solutions in an aquifer system with inclined strata.

ACKNOWLEDGEMENT

The authors acknowledge Prof. P.V.Seethapathi, Director, NERIWALM, Tezpur for helpful discussions and constant support.

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(Accepted 2004 April 30. Received 2004 April 4; in original form 2004 January 28)