J. Ind. Geophys. Union (April 2005)
Vol.9, No.2, pp.121-126

Relative Assessment of 3D – Analytic Signal Definitions – A Numerical Study

Rambhatla G.Sastry and Paras R.Pujari[1]
Department of Earth Sciences, IIT Roorkee, Roorkee – 247 667
[1]GEM Division, National Environmental Engineering Research Institute, Nagpur-440 020
E-Mail: rgss1fes@iitr.ernet.in / rgssastry@yahoo.com

ABSTRACT

Two alternate definitions for 3D- analytic signal exist in geophysical literature and their relative performance needs an in-depth study for a proper option. So, a numerical study involving a conductive two-prism model is undertaken in this study. FDM generated secondary pole-pole potential data due to this model served as input to our stabilized analytic signal algorithm, RES3AS, which outputs several analytic signal parameters conforming to both the definitions of 3D-analytic signal. Analysis of analytic signal parameters has provided means of assessing position location, lateral widths and depths to top surfaces of both the prisms. Thus assessed model parameters for both options of 3D-analytic signal have suggested that the Nabighian's (1984) definition for 3D-analytic signal is more effective than that due to Roest, Verhoef & Pilkington (1994) in the interpretation of secondary pole-pole data.

INTRODUCTION

The 3D analytic signal (AS) concept (Nabighian 1984., Roest, Verhoef & Pilkington 1992) is increasingly being used in the interpretation of gravity and magnetic anomalies (Blakely & Simpson 1996., Debeglia & Corpel 1997., Hsu, Sibuet & Shyu 1996, Agarwal & Shaw 1996., Shaw & Agarwal 1997). The concept of AS has been successfully transferred to DC resistivity for 2D targets (Sastry & Pujari 1997., Pujari 1998., Pujari & Sastry 2003) and 3D targets (Sastry & Pujari 1997., Pujari 1998).

The 2D – analytic signal's (Nabighian 1972, 1974) definition is unique while that for 3D- case two alternate definitions (Nabighian 1984., Roest, Verhoef & Pilkington. 1992) exist. Sastry & Pujari (1997) and Pujari (1998) have established the usefulness of the Nabhighian's 3D analytic signal definition in deciphering the position location and lateral extent estimation for 3D targets. However, no in-depth analysis is made by them to establish the relative effectiveness of available definitions for 3D analytic signal in the interpretation of secondary pole-pole data. So, the present effort is devoted to this aspect. Here, the numerical study model involves two conductive 3D prisms and pole-pole array. The relevant details of our stabilized 3D analyitic signal algorithm, RES3AS based on Tikhonov's regularization are discussed elsewhere (Sastry & Pujari 1997; Pujari 1998).

3D Analytic Signal Definitions

Roest, Verhoef & Pilkington (1992) and Nabighian (1984) have defined the 3D complex analytic signal of a potential field, M(x,y) respectively as

$$A(x,y,z) = \frac{\partial M}{\partial x}\hat{x} + \frac{\partial M}{\partial y}\hat{y} + \frac{\partial M}{\partial z}\hat{z} \quad (1)$$

$$A(x,y,z) = \left(\frac{\partial M}{\partial x} + \frac{\partial M}{\partial y}\right) + i\frac{\partial M}{\partial z} \quad (2)$$

where $\hat{x}, \hat{y}$ and $\hat{z}$ are unit vectors along coordinate axes. The amplitudes of analytic signal (AAS) as per Nabighian (1984) and Roest, Verhoef & Pilkington (1992) are respectively given by

$$|A(x,y,z)| = \sqrt{\left(\frac{\partial M}{\partial x} + \frac{\partial M}{\partial y}\right)^2 + \left(\frac{\partial M}{\partial z}\right)^2} \quad (3)$$

$$|A(x,y,z)| = \sqrt{\left(\frac{\partial M}{\partial x}\right)^2 + \left(\frac{\partial M}{\partial y}\right)^2 + \left(\frac{\partial M}{\partial z}\right)^2} \quad (4)$$

In a similar fashion, the other AS parameters, viz., Real (RIAS) and imaginary (IIAS) parts of inverse AS differ as per definitions of AS (eqn. 1 and 2).

METHODOLOGY

By drawing analogy between electrostatics and magnetostatics at governing equation level, it has been proved that analytic signal method, which has gained popularity in magnetic anomaly interpretation, can also be extended to the analysis of secondary DC pole-pole potential data. Accordingly, the following methodology is devised in the analysis of DC pole-pole potential data and interpretation is of basic four-step process:

1. Computation of secondary potential and its horizontal gradients (To meet the demand of field definition, M in equations 1 – 4)
2. Applying AS on the horizontal gradient of secondary potential and determination of different AS terms
3. Preparation of relevant Analytic Signal plots and
4. Interpretation

Accordingly, the needed plot parameters arising out of RES3AS are as follows:

a) RAS (Real part of AS)
b) IAS (Imaginary part of AS)
c) RIAS (Real part of Inverse of AS)
d) IIAS (Imaginary part of Inverse of AS)
e) AAS (Amplitude of Analytic Signal)

To meet the stated objectives, the following steps need to be undertaken:

1. The secondary pole-pole potential data distribution at shallow depth due to a buried point source impressed over the body center of buried 3D conductive/resistive inhomogeneities located within resistive/conductive host medium is computed using 3D hydrological modeling software, MODFLOW model. The horizontal gradients of secondary potentials are computed.
2. Prepare stabilized AAS, RIAS and IIAS contour plots as a result of applying RES3AS on horizontal gradient of secondary potential derived in step1.
3. Consider orthogonal profile pairs for each of AAS, RIAS and IIAS contour plots. For interpretation purpose, they need to be chosen along row and column direction (Position location of current source in plan) of current source on X-Y plane.
4. The real and imaginary component profiles (RAS and IAS) of analytic signal, AS as per step 3 identify the conductive body position. In case of Roest et al. (1992), the relevant vertical component plots exhibit symmetric and anti-symmetric property at the body center projection on X-Y plane at a shallow depth above the target(s). For corresponding Nabighian's case, similar feature can be noticed but here one deals with combined RAS plot instead of its components. The zero crossings of RIAS profiles confirm the body position.
5. For the definite current pole above the conductive body center as determined in Step 4, the profile pair (as Step 3) of IIAS infers lateral extent of the 3D body in two orthogonal (X- and Y-) directions. The lateral extents of the target are inferred from the plateau region of IIAS curve crossing from negative to positive values near zero line.
6. For current pole position above anomalous body center, consider profile pairs of AAS (as per Step 3) and apply Nabighian's depth rule to determine the depth to top of the body.

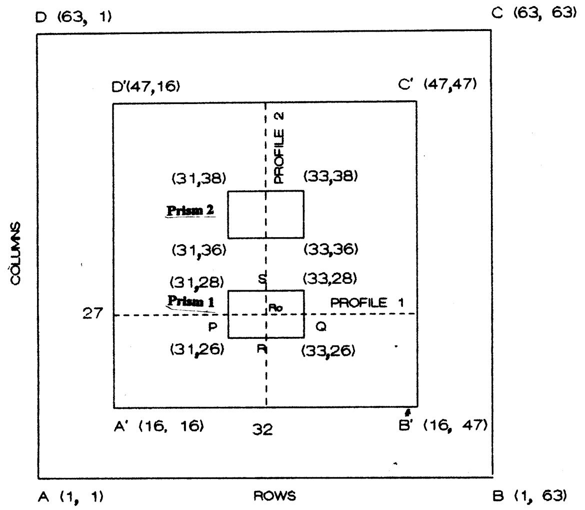
Numerical Experiments

Here, the two-prism model (Fig.1) is considered for inter-comparison of the performance of the available definitions for 3D AS. Table 1 outlines the dimensions, physical property distribution and current source details. Due to lack of 3D resistivity modeling packages at our disposal, forward modeling is carried out with the help of standard hydrological modeling package (Mc Donald and Harbaugh 1984) after considering the analogy between groundwater flow under steady state conditions and conduction of DC current (Pujari 1998). Similar efforts are made by Wolfe and Bodl (1997). After customary checks, the adopted 3D FDM mesh in MODFLOW is of size 63 x 63 x 9 nodes with the linear mesh covering 32 x 32 x 7 nodes, which accommodates the model, the current source and pole-pole receivers at depth. The secondary pole-pole distribution is computed as per earlier detailed methodology. The resulting secondary potential distribution is included in Fig. 2.

Table 1. Physical property distribution and geometric details of Two-Prism model (Model 1, Ref. Fig. 1) along with current source specifications

Current source strength: 1 A/m^2; Conductivity of half-space, σ_h= 0.01 S/m; Current pole is above the body center of Prism 1 (Ref. Fig. 1)									
Prism 1					Prism 2				
h_1 (m)	h_2 (m)	PQ(m)	RS(m)	σ (S/m)	h_1 (m)	h_2 (m)	PQ(m)	RS(m)	σ (S/m)
2	4	31-33	26-28	0.1	4	7	31-33	36-38	0.1

h_1, h_2 are depths to top and bottom of prisms; PQ, RS are lateral coordinates of prisms along Profile 1 and Profile 2 (Ref. Fig. 1) and σ is the conductivity of bodies.



Figue 1. Schematic plan view of conductive Two-Prism (Model 1) in the adopted linear portion of finite-difference mesh. Prism 1 and Prism 2 are the two conductive prisms. Profile 1 and Profile 2 are a pair of orthogonal profiles across Prism 1. In all numerical experiments results pertaining to these profiles will be considered for analysis. The numerals in the illustration refer to the finite-difference grid nodes.

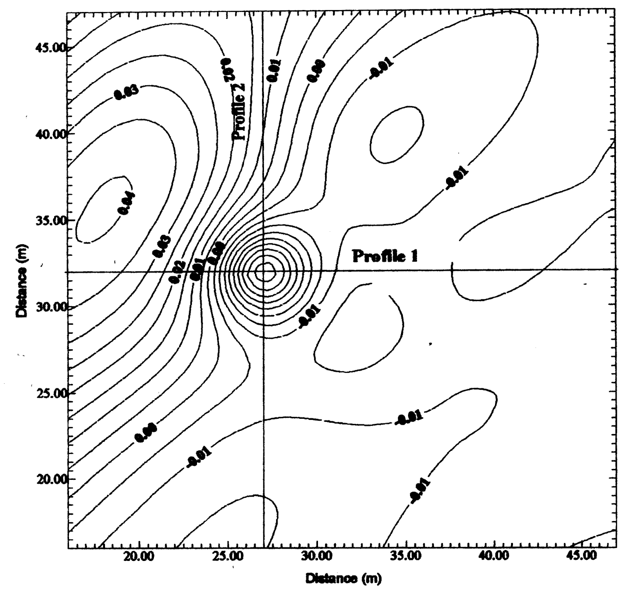


Figure 2. Input secondary pole-pole potential (error-free) map to multi-body model (Model 1, Fig.1) with the current source above the body center of Prism 1 at unit depth. The linear portion of finite-difference grid is displayed.

Table 2. Estimated parameters (h_1, PQ and RS) of Prism 1 (Ref. Fig. 1). Actual Body Parameters: h1=2m; Lateral Extent of Prism 1: Along Profile 2 (Rows, RS): 31-33m; Along Profile 1(Columns, PQ): 26-28m. Current Pole above body centre of Prism 1 (32,27,0) at one unit depth (m) below air-earth interface. Line R' and R indicate the body centre of Prism 1 (32,27,0) at one unit depth (m) below air-earth interface

As per Nabighian's (1984) definition for 3D AS							As per Roest et al. (1992) definition for 3D AS					
Estimated h_1=MN/ 2 (m)		Estimated Lateral extent (m)		Estimated Body Centre (m)		Estimated h_1=MN/ 2 (m)		Estimated Lateral extent (m)		Estimated Body Centre (m)	
Along Rows	Along Columns	Along Rows (RS)	Along Columns (PQ)	Along Rows (AS, RIAS) R'	Along Columns (AS, RIAS) R	Along Rows M'N'/2	Along Columns MN/2	Along Rows (RS)	Along Columns (PQ)	Along Rows (AS, RIAS) R'	Along Columns (AS, RIAS)R
2.5	2.0	31-33	26-28	32	27	3	3.0	31-33	26-28	32	27

RESULTS

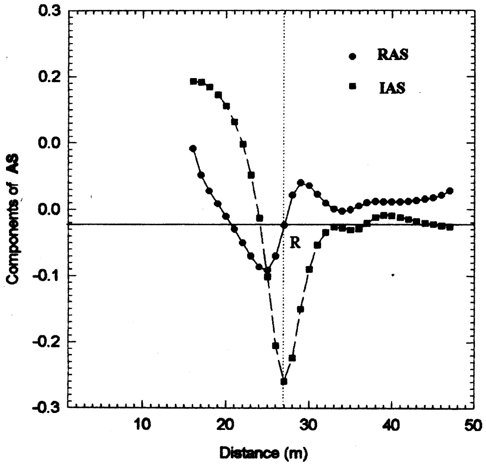
The different AS parameter plots along Profile 1 and Profile 2 (Ref. Fig. 2) are considered and analyzed. The results achieved thereby are included in Table 2. The position location of body center along Profile 1 and Profile 2 are attempted by combined RAS and IAS plots and RIAS plots. Figs 3, 4 and 5 serve the purpose. These plots suggest that Nabighian's (1984) definition is better in RIAS behaviour in view of a clear zero crossing.

The AAS plots (Fig.6) are utilized for deriving the depth of burial of Profile 1. The results included in Table 2 indicate that the depths inferred on the basis of Nabhghian's (1984) definition are closer to the actual ones compared to that of Roest, Verhoef & Pilkington (1992).

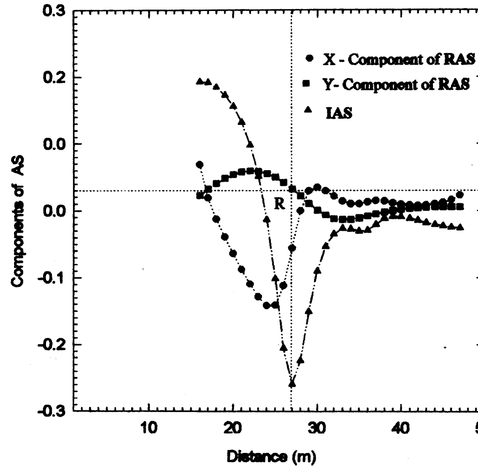
The IIAS plots relevant for both the definitions (Fig.7) infer the lateral extents of Prism1 along Profile1 and Prism2 exactly.

DISCUSSION

The combined RAS and IAS plots along either Profile1 or Profile2 are not very satisfactory in inferring the body center position, owing to the influence of neighbouring body. However, the performance of RIAS plots is far better in meeting this objective. The Nabighian's (1984) RIAS exhibits a clear zero crossing.

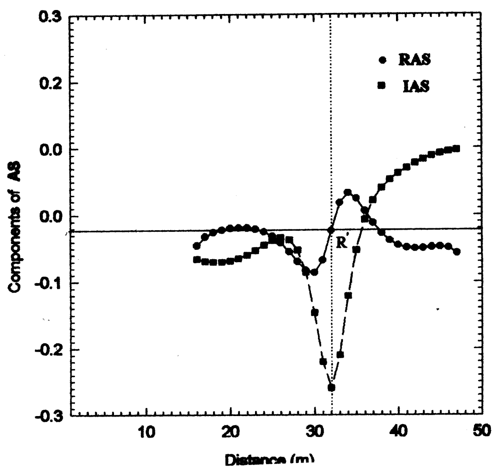


3(a)

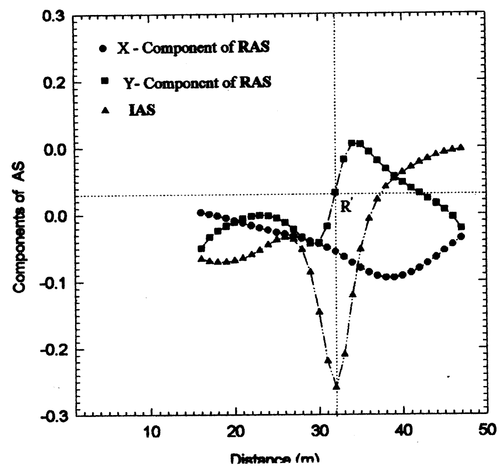


3(b)

Figure 3. RAS and IAS plots along Profile 1. a) Using Nabighian's (1984) definition and (b) Using Roest, Verhoef & Pilkington. (1992) definition

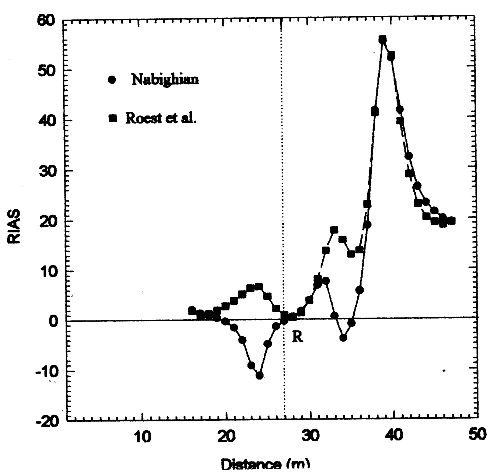


4(a)

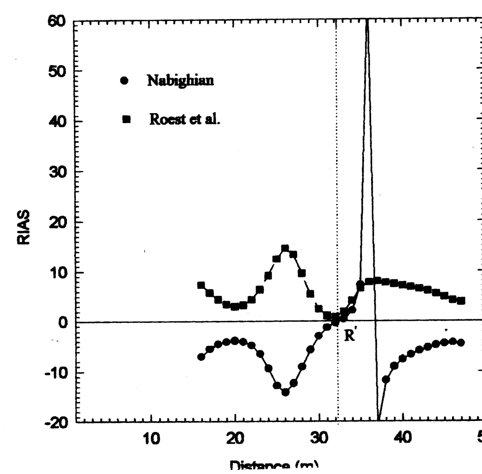


4(b)

Figure 4. RAS and IAS plots along Profile 2 : (a) Using Nabighian's (1984) definition and (b) Using Roest, Verhoef & Pilkington (1992) definition



5(a)



5(b)

Figure 5. RIAS plots using both Nabighian's (1984) and Roest, Verhoef & Pilkington (1992) definitions. Here R and R′ are the projections of body center (Prism 1, Fig. 1) on X-Y plane at one unit depth below air-earth interface : (a) along Profile 1 and (b) along Profile 2

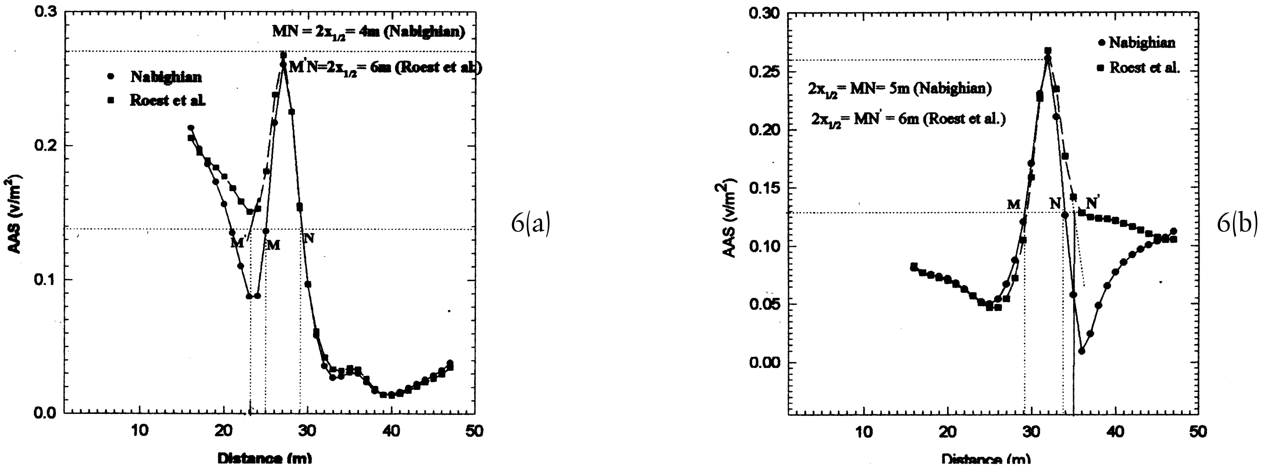


Figure 6. AAS plots using both Nabighian's (1984) and Roest, Verhoef & Pilkington (1992) definitions. Here R and R′ are the projections of body center (Prism 1, Fig. 1) on X-Y plane at one unit depth below air-earth interface: (a) along Profile 1 and (b) along Profile 2

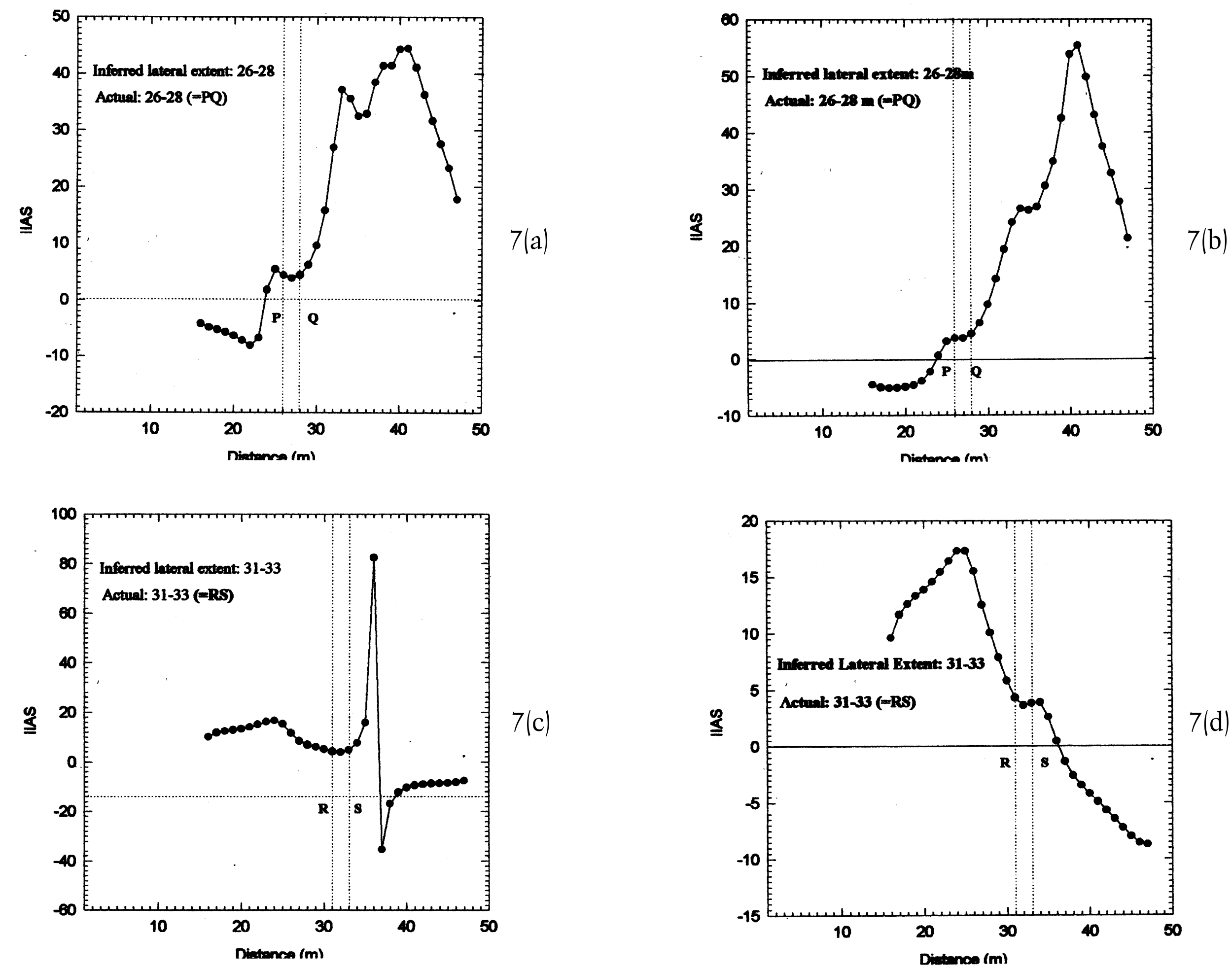


Figure 7. IIAS plots. Here RS and PQ are inferred lateral extents of Prism 1 (Fig. 1): (a) along Profile 1 using Nabighian's (1984) definition, (b) along Profile 1 using Roest, Verhoef & Pilkington. (1992) definition, (c) along Profile 2 using Nabighian's (1984) definition and (d) along Profile 2 using Roest, Verhoef & Pilkington (1992) definition

So, for a multi-body case, RIAS plots need to be relied more.

For depth estimates, AAS plots based on Nabighian's (1984) definition provide better estimates, while the lateral extent estimates by either of the definitions provide exact values.

A proper location of body center projection on X-Y plane at unit depth is facilitated by RAS and RIAS plots and for the current pole at such a position is a necessary precondition for the success of the proposed interpretation method. A systematic practical procedure can be evolved for this purpose.

Nabighian's (1972) depth rule used here is in fact valid for 2-D bodies only. More sophisticated depth rules are not warranted in 3D case, in view of the success achieved by this simple depth rule in 3D targets.

For 2D bodies, Nabighian (1972) has predicted theoretically that the lateral extent can be inferred from zeros of IIAS profile plot. In 3D case, it has been observed that the plateau region around zero of IIAS provides the lateral coordinates of the body along the orthogonal pair of profiles positioned at projected body center on XY-plane at unit depth.

In all the considered numerical experiments, the input data is error-free. However, RES3AS algorithm can meet the demand of error-prone input data also quite effectively.

The relative performances of this pair of definitions (Nabighian, 1984., Roest, Verhoef & Pilkington 1992) are also verified on a three conductive-body model consisting of three parallelepipeds of different dimensions, conductivities and depths of burial and the results achieved are similar to the reported two-prism model.

CONCLUSIONS

The Nabighian (1984) definition for 3D AS is found to be better than that offered by Roest, Verhoef & Pilkington (1992) in the considered example in estimating the position, depth of burial and lateral coordinates of the bodies. The numerical modeling provides a hope that this conclusion can even be valid under real geological situations also.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the help received from Dr. P.V. Seethapathi, Director, NERIWALM, Tezpur and Dr. M.K. Jose, Scientist, NIH, Roorkee in utilizing standard hydrological modeling software. Dr. Pujari gratefully acknowledges the financial help received from CSIR during this study.

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(Accepted 2004 September 24. Received 2004 September 12; in original form 2004 May 10)