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Application of Fuzzy graph theory in brittle plane network analysis - A potential method for carbon sequestration models

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ABSTRACT

Improved carbon sequestration (CCS) models with rocks as sinks require incorporation of uncertainty into the models. In such cases of uncertain geoscientific problems, fuzzy graph theory can be useful. Brittle shear plane network with indistinct shear planes is common in natural sheared rocks, and can be targeted for CCS. Due to non-unique possibility of continuity of P-planes, it is not possible to represent such networks as crisp graphs. We present few natural examples of the former type of P-planes in shear zones, and how fuzzy graph theory can represent the fracture network and fluid flow. The process involves assigning some sample numerical probability to represent the connectedness between the underdeveloped P-planes and the Y-planes. The presentation is a geometric exercise and does not extend to the genesis of the shear zones.

Keywords: Network, Fluid flow, Structural geology, Shear planes, Geoscientific uncertainty, Waste disposal, Carbon sequestration models

INTRODUCTION

Structural geological modeling requires representation of structures in some numerical form. When the structures are clearly decipherable, this is rather easy, for example, one can apply the graph theory (e.g., Sanderson et al., 2018; Mukherjee, 2019). Classical graph theory works when there is a full certainty that which nodes are connected by which vertices through edges.

Various brittle plane geometries develop in rocks at shallow crustal depths that undergo brittle shear (Figure 1a). In case the shear planes are clearly delineated (Figure 1b), one can use graph theory to represent the brittle plane network in terms of sigmoid shear bound by parallel planar shear planes commonly seen in sheared rocks. For example, in case of Figure 1b, the adjacency matrix is:

a	b	c	d	e	f	
a	0	0	0	0	1	0
b	0	0	0	0	0	1
c	0	0	0	0	0	1
d	0	0	0	0	0	1
e	0	0	0	0	0	1
f	0	0	1	1	0	0

Carbon sequestration (CCS) has recently been modeled in terms of extraneous CO_2 sources and CO_2 sinks in rocks and P-graph modeling approach has been undertaken (Chong et al., 2014). Fractured rocks especially coal has been targeted for CCS (e.g., Chen et al., 2020). However, since connectivity of fractures is uncertain at depth, establishing models on CCS becomes problematic (Figure 1 of Zhang et al., 2009). Pashin et al. (2008) presented a discrete fracture network model for CCS in coal. Such models would be possible when existence of the sink locations in the coal is fairly well understood. When the existence of the sinks is uncertain, models will nevertheless be required to be built in decision making. Other uncertainties can be *"unproven nature"* of the carbon sequestration technology, economic cost and life cycles of the CCS technologies and so on (Tan et al., 2010 and references therein). Researchers have identified CCS as one of the potential areas where graph theory concept needs more elaboration (e.g., Farid et al., 2021).

Anand et al. (2021) discussed various kinds of uncertainties in parameters in structural geological problems. The sigmoid planes can be indistinct due to *(i)* poor snap quality, *(ii)* they are indeed developed less prominently, or *(iii)* sometimes a part of them is below the ground surface hence not seen. For example, in seismic images, brittle planes may not always be clearly decipherable. This can be because of such brittle planes are below the seismic resolution (Misra and Mukherjee, 2018), and yet the modeler needs to make some presentation of the brittle plane network. In such cases, where the classical graph theory cannot represent the brittle plane network where uncertainty exists about continuation of fractures, fuzzy graph theory should be used.

The concept of fuzzy sets was introduced by Zadeh (1965), which later was well applied to solve several research problems that are uncertain in nature. Crisp set is a well-defined collection of distinct objects. If there is any uncertainty in the set boundaries of objects, the concept of fuzzy set is used. In other words, a fuzzy set is used to represent the degree of membership of any qualitative data. The crisp set is not able to work with scientific problems