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Monte – Carlo Simulations: Permeability Variation with Fractal and Pore Structural Parameters

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ABSTRACT

Often permeability is measured on rock samples in the laboratory or during well tests; however, such data is very sparse and takes a long time to generate. Permeability estimation from readily available porosity logs is an easier alternative approach. This paper proposes a new method of permeability estimation. The method relies on fractal theory and Monte – Carlo Simulations. Monte – Carlo simulations are performed to infer the dependence of permeability on pore size distribution. Unlike the empirical relations used to obtain permeability, present method generates random pore size distribution of the porous medium, which explains the fact that permeability depends on pore size distribution. Variation of permeability with pore-area fractal dimension, tortuosity fractal dimension and cluster mean size has been analyzed. The analysis infers that permeability is more sensitive to variations in cluster mean size compared to that of fractal dimensions. An attempt is made to define physical significance of fractal dimension of pore-area using fractal analysis of pore size distribution.

INTRODUCTION

Permeability is a key parameter for the production and management of hydrocarbon reservoirs as well as aquifers. Though the fluid content in the rock is proportional to porosity, the fluid flow depends on permeability, which is mostly predicted from effective porosity. However, even for a given effective porosity, permeability will be different for different rock types. This is because in addition to porosity other textural parameters like grain size, grain shape, sorting of grains also control permeability. These textural parameters affect tortuous nature of capillary pathways and arrangement of pores in the porous medium thereby making it complex to understand. In the past many authors related permeability to porosity in terms of empirical equations (Carman, 1956; Kozeney, 1927; Pape et al., 1999), which explain the results in terms of correlation coefficients. However, they do not explain the real physical situation existing in the porous medium. Fractal theory is applied to various complicated problems in earth sciences (Dimri, 2000b; Dimri, 2005; Dimri et al., 2005; Turcotte, 1997). Fractal theory is also found useful to explain complex nature of porous medium (Katz and Thompson, 1985; Dimri, 2000a; Dimri et al., 2012, Uma et al., 2014; Wheat craft and Tyler, 1988).

Monte – Carlo simulations for permeability of fractal porous media are formulated by Yu et al., 2005. According to them permeability is a function of cluster mean radius, relative particle size and fractal dimensions of pore area and tortuosity. In order to calculate permeability it is mandatory to select appropriate fractal dimensions of pore area and tortuous stream tubes and also cluster size, in the absence of measured information. In the present study permeability variation with respect to these parameters is analyzed to understand its behavior with variation in these parameters.

THEORY

A porous medium consists of tortuous capillary pathways. The tortuous length of the capillary (L_t) in terms of pore diameter (λ) and the representative length (L_0) is given by Yu and Chen (2002).

$$L_t(\lambda) = \lambda^{1-D_T} L_0{}^{D_T} \cdots\cdots\cdots (1)$$

Where, D_T is the tortuosity fractal dimension, with $1< D_T<2$, representing the extent of convolutedness of capillary pathways for fluid flow through a medium. Higher the value of D_T more will be the convolutedness or tortuosity. The limiting values of $D_T=1$ and $D_T=2$ correspond to a straight capillary and highly tortuous line that fills a plane (Wheatcraft and Tyler, 1988), respectively. The size distribution of pore diameters is another important property. Yu and Chen (2002) considered pores in a porous medium as analogous to the islands on earth or spots on engineering surfaces and obtained the following fractal scaling law.

$$N(L \geq \lambda) = \left(\frac{\lambda_{max}}{\lambda}\right)^{D_f} \cdots\cdots\cdots (2)$$

where N is the number of pores with diameter (L) greater than or equal to λ, D_f the pore area dimension, with $1< D_f<2$, is the fractal dimension of the intersecting pore cross sections with a plane normal to the flow direction. Eq. (2) suggests that when λ corresponds to the maximum pore size, λ_{max}, the number of pores greater than or equal to it is one. Conversely, when λ corresponds to smallest pore size, λ_{min}, number of pores will be maximum and the scaling law becomes

$$N_t(L \geq \lambda_{min}) = \left(\frac{\lambda_{max}}{\lambda_{min}}\right)^{D_f} \cdots\cdots\cdots (3)$$

Where, N_t is the total number of pores.

Based on these fractal scaling laws Yu et al., (2005) obtained probability expressions for pore diameter (Eq. 4) and permeability (Eq. 5) and performed Monte - Carlo simulations to estimate permeability of bi-dispersed porous medium (Yu and Chen, 2002). Small particles form as clusters in bi-dispersed porous medium. The space between clusters get form macro pores and within the cluster micro pores exist.

$$\lambda_i = \left(\frac{\lambda_{min}}{\lambda_{max}}\right)\frac{\lambda_{max}}{(1-R_i)^{1/D_f}} \cdots\cdots (4)$$

$$K = GA^{-(1+D_T)/2}\sum_{i=1}^{J}\lambda_i^{3+D_T} \cdots\cdots (5)$$

Where

$$\left(\frac{\lambda_{min}}{\lambda_{max}}\right) = \frac{\sqrt{2}}{d^+}\sqrt{\frac{1-\emptyset}{1-\emptyset_c}} \cdots\cdots (6)$$

$$\emptyset_c = 0.342\emptyset \cdots\cdots (7)$$

$$\lambda_{max} = \frac{\overline{R}_c}{2}\left[\sqrt{2\left(\frac{1-\phi_c}{1-\phi}-1\right)} + \sqrt{\left(\frac{2\pi}{\sqrt{3}}\right)\left(\frac{1-\phi_c}{1-\phi}\right)} - 2\right] \cdots\cdots (8)$$

Where

The approximated pore area of the unit cell is given by

$$A_p = \sum_{i=1}^{J}\pi\lambda_i^2/4 \cdots\cdots (9)$$

λ_{min} - Minimum pore diameter
λ_{max} - Maximum pore diameter
ϕ - Effective Porosity
ϕ_c - Micro porosity in the cluster
R_c - Cluster mean radius
d^+ - Ratio of cluster mean size and the minimum particle size
λ_i - Diameter of the i^{th} capillary tube chosen by Monte – Carlo simulations
A_p - Total pore area of a unit cell
R - A random number that lies between 0 and 1
A - Total cross sectional area of a unit cell

Since there are N_t pores in a unit cell, *J* in Eqs. (5) and (9) will be equal to N_t. In Eq. (3) $\lambda_{max}/\lambda_{min}$ is greater than one, thus total number of pores (N_t), increases with D_f. Readers may refer to Yu et al. (2005) and Uma et al. (2014) for detailed description of algorithm for determination of permeability using Monte - Carlo simulations.

RESULTS AND DISCUSSIONS

Permeability variations with tortuosity fractal dimension (D_T), pore area fractal dimension (D_f) and cluster mean radius (R_c)

Monte - Carlo simulations are run for different values of D_T by considering $R_c=0.3$ mm, $d^+=24$, $D_f=1.8$ (Yu et al., 2005). It is observed that permeability decreases with increase of D_T (Fig. 1(a)). As D_T increases from 1.1 to 1.3, the mean value of percentage reduction in permeability is 17. When all other parameters $R_c=0.3$ *mm*, $d^+=24$, $D_T=1.1$ are kept constant permeability decreases with increasing D_f (Fig. 1(b)). The mean value of percentage reduction for increase of D_f from 1.4 to 1.6 is 23. A small variation in Cluster mean radius (R_c) from 0.2 mm to 0.3 mm increases permeability by 53% (Fig. 1(c)). This indicates that permeability is more sensitive to Cluster mean radius (R_c) than to fractal dimensions.

DISCUSSIONS

The pore size distributions generated by Monte – Carlo simulations for a constant porosity with four different values of D_f are shown in Fig. 2(a) to 2(d). It is clear from these figures that with increase in D_f total number of pores are increased and their diameters are reduced. As we can see in Table1, when D_f increases the percentage number of pores having diameter less than 40 μm increases and those having diameter greater than 120 μm decreases. This analysis indicates that increased value of D_f increases the number of smaller scale/tiny pores, which increases flow resistance. The observation that permeability decreases with increase of D_f is supported by Pitchumani and Ramakrishnan (1999) and contradicts the results of Yu and Cheng (2002) in which an analytical expression is used to calculate permeability. Yu and Cheng (2002) state that permeability increases with increasing D_f because as D_f is increased, the total porosity or the pore area increases. For a given porosity with increase in D_f, increased number of pores leads to increase in simulated pore area (A_p) (Table 1). However, permeability decreases because the sum $\Sigma^J_{i=1}\lambda_i^{3+D_T}$ decreases. Smaller values of λ_i become negligible when they are raised to the power of $3+D_T$ and cannot contribute to the sum $\Sigma^J_{i=1}\lambda_i^{3+D_T}$. The variation of this sum is controlled by maximum value of simulated pore diameter, which again decreases with D_f (Table 1). The contribution of λ_{max} to total permeability ranges between 46% and 54% (Table 1). This in turn indicates that permeability is very sensitive to pore diameter. According to Yu and Cheng (2002), λ_{max} does not vary with D_f; it is calculated from analytical formula (Eq. (8)) and remains constant for a given porosity and particle size. This is the difference between the approach based

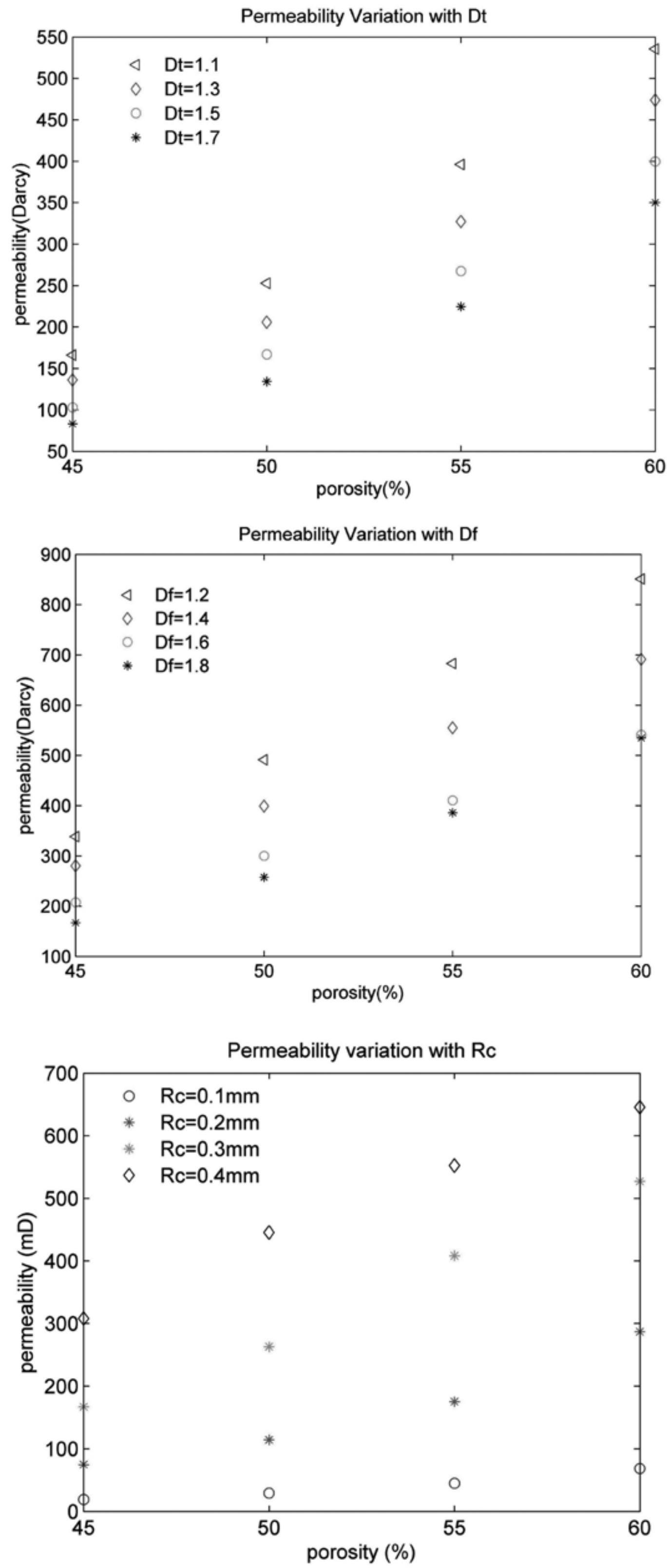

Figure 1. Permeability variations with tortuosity fractal dimension (D_T), pore area fractal dimension (D_f) and Cluster mean radius (Rc). Permeability decreases with increasing D_T, D_f and it increases with increase of Rc. The variation in permeability is more sensitive to changes in Cluster mean radius and it increases with porosity.

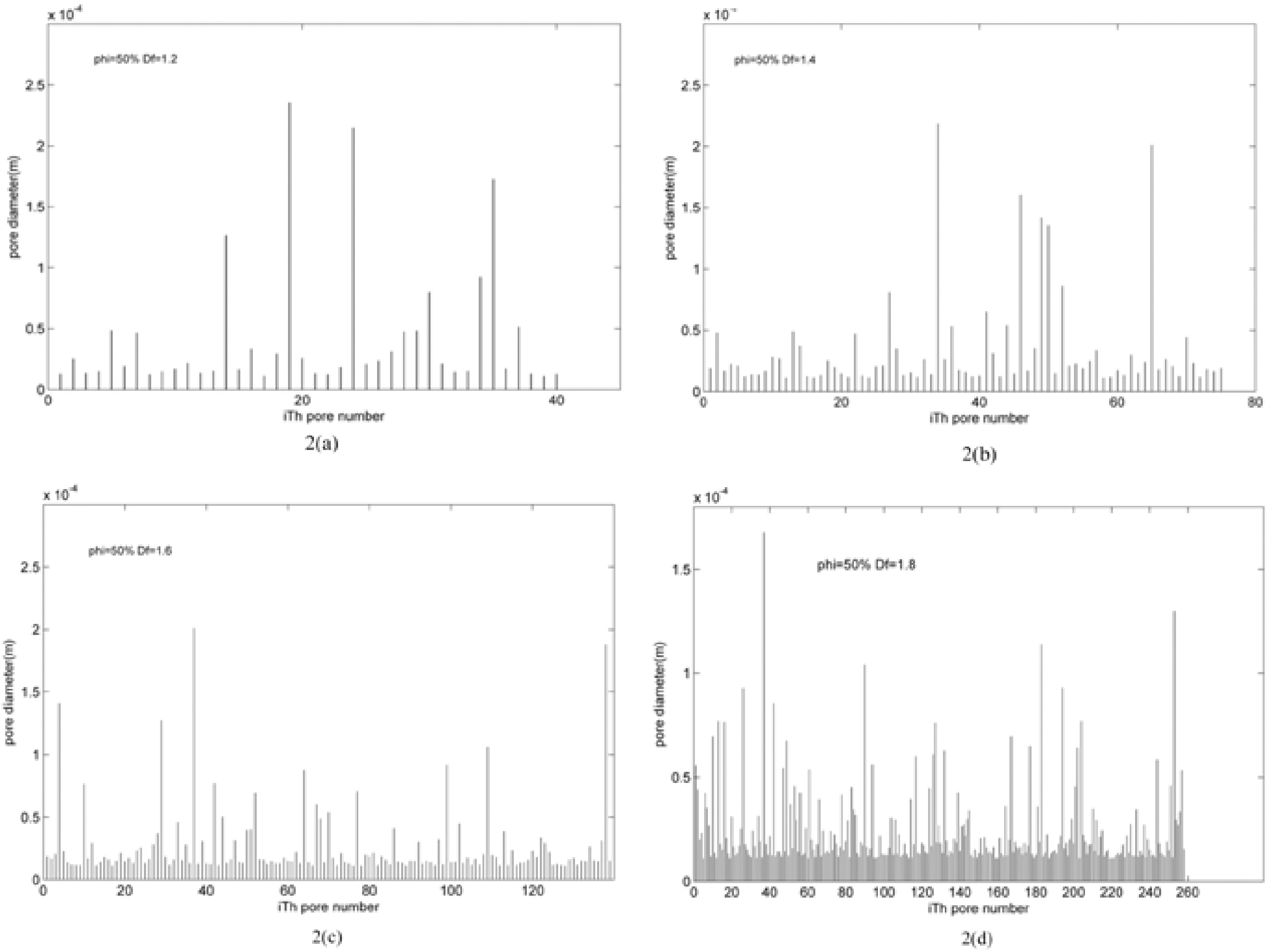

Figure 2. Fig. 2(a), 2(b), 2(c) and 2(d) illustrate the pore size distribution for a porosity of 50% and for D_f=1.2, D_f =1.4, D_f=1.6 and D_f=1.8, respectively. The random pore size distribution chosen by Monte – Carlo simulations replicates the irregular pore size distribution in porous media, provided porosity constant, with increase in Df total number of pores are increased and their diameters are reduced.

on analytical formula (Yu and Cheng 2002) and Monte - Carlo simulations (Yu et al., 2005). The negligible effect of smaller values of λ_i which represent tiny pores, towards the sum $\Sigma^J_{i=1}\ \lambda_i^{3+D_T}$ can be explained in terms of real physical scenario. The power $3+D_T$ takes care of tortuosity. Smaller scale tortuous capillaries cannot transmit fluid and their contribution to permeability is negligible, however, they can contribute to pore area and increase it. The existing analysis of Yu and Lee (2000) also showed that the smaller scale/tiny pores in porous media have impact negligible effect on total permeability. The analysis concludes that D_f =1 corresponds to flow through the unit cell with a single capillary tube and D_f =2 corresponds to a compact cluster with infinite number of tiny pores, which cannot permeate fluid.

An increment in the value of D_T makes capillary pathways more twisted, thus increases resistance for flow and decreases permeability. As cluster mean radius increases capillary pathways become less tortuous and wider, so permeability increases.

CONCLUSIONS

Permeability is very sensitive to variations in cluster mean size. With increase of pore area fractal dimension (D_f) permeability decreases because the number of smaller scale/tiny pores increase and thus flow resistance also increases. Consideration of this kind of complex real physical situation that exists in porous medium indicates superiority of the method over empirical relations. The observation that permeability decreases with D_f will help to select D_f value during permeability estimation. If the fractal dimensions and grain size of rock samples are known, permeability estimated by this method can be used as first-hand information even though well test data or laboratory measurements are not available.

Table 1. Variation in pore size distribution and permeability with increasing pore area fractal dimension (D_f) for a constant porosity of 50%, D_T=1.1& Rc=0.3 mm.

D_f	Simulated λ_{max} (μm)	Simulated λ_{min} (μm)	% No. of Pores having λ<40 μm	% No. of Pores having λ>120 μm	$\sum_{i=1}^{N_t} \lambda_i^{3+D_t}$ * 10^{-16}	Ap (m^2) * 10^{-6}	Permeability only due to λ_{max} (D)	Total Permeability (K) (D)	Percentage contribution of λ_{max} to total permeability (%)
1.2	218	11	82.5	12.5	21.5	0.270	228	490	46.5
1.4	209	11	84.0	6.7	17.7	0.282	196	404	48.5
1.6	191	11	86	0.7	17.7	0.300	158	290	54.4
1.8	177	11	88	0.38	11.2	0.376	124	256	48.4

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