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Modifying Hagen-Poiseullie's equation for an inclined porous media with varying porosity

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ABSTRACT

Mode of transport of fluid in soil is the basis for soil environmental engineering especially in transport of contaminants in subsurface groundwater. Volume flux (q) of water through media of different porosities in an inclined pipe for different angles of inclination (α) was determined. The patterns of flow were determined from q-α curve for two forms of porosity change. The porosity change $\Delta\varphi$ is positive when moving from aless permeable to amore permeable medium in an inclined pipe, while it is negative when moving from a more permeable medium to aless permeable medium in an inclined pipe. It was observed that (-$\Delta\varphi$) acts as a damping factor for turbulent flow, while (+ $\Delta\varphi$) increases the rate of flow vis-à-vis turbulent flow. Also, porosity difference, $\Delta\varphi$ was introduced to an already established Hagen-Poiseullie's equation in order to modify it for an inclined porous media of different porosities.

Key words: Volume flux, inclination angle, porosity, turbulent flow, permeable media.

INTRODUCTION

The greatest danger of groundwater pollution in subsurface including sewage sludge, leaking sewers, and polluted water from refuse disposal sites. Wherever any groundwater supply well is constructed, a viable groundwater measure must be taken to prevent contamination by pollutant (Hiscock et al., 1995). One approach is to control the rate of flow (or seepage) or change the direction of the flow of contaminated fluid by using appropriate sand layers of variable porosities (Leonard, 1962 and Silliman et al., 1998).

It should be appreciated that soil itself serves as a filter, and its ability to do so depends on its physical attributes such as permeability and porosity (Henry, 2003). Natural filters have been used as landfill liners to reduce the movement of contaminated fluid from solid waste landfill and waste water disposal into subsurface. (Boynton and Daniel, 1985; Foreman and Daniel, 1986; Benson and Daniel, 1990; Benson et al., 1994; Benson and Trast, 1995; Rowel et al., 1995, Boadu, 2000 and Henry, 2005). However, a graded filter serves better; it consists of layers of porous materials of different porosities or permeabilities in which the soil particles in a particular layer are coarser than that in the preceding layer (Cedergreen, 1976). It should be noted that the selection of a good graded filter as a protective layer or a seepage control medium depends on its properties, which can only be determined experimentally. Thus, there is a need for better understanding of the law which governs the flow of fluid through these media which are used as graded filter and modify it for an inclined porous media of different porosities.

Darcy's law

The application of Darcy's law enables hydraulic conductivity to be determined, from which permeability can be computed by Hubert King relation (Demenico and Schwartz,2000).

The results of Darcy's experiments indicated that the rate at which a fluid moves through a porous medium (Q) is proportional to the difference in hydraulic head of the water along the column and the characteristics of the porous medium and the column length.

This relationship is known as **Darcy's law**:

$$Q = -KA\frac{dh}{dl} \tag{1}$$

Where Q is the rate of water flow

Pressure Drop and Head Loss

A quantity of interest in the analysis of pipe flow is the pressure drop ΔP since it is directly related to the power requirements of the fan or pump to maintain flow. We note that dP/dx = constant, and integrating from $x = x_1$where the pressure is P_1 to $x = x_1+L$ where the pressure is P_2 gives

$$\frac{d\rho}{dx} = \frac{p_2 - p_1}{L} \tag{2}$$

Substituting into the V_{avg} expressions, the pressure drop can be expressed as

Laminar flow $$\Delta P = P_1 - P_2 = \frac{8\mu L V_{avg}}{R^2} = \frac{32\mu L V_{avg}}{D^2} \tag{3}$$

The symbol Δ is typically used to indicate the difference between the final and initial values, like $\Delta y = y_2 - y_1$. But in fluid flow, ΔP is used to designate pressure drop, and thus, it is $P1 – P_2$. A pressure drop due to viscous effects represents an irreversible pressure loss, and it is called pressure loss ΔP_L to emphasize that it is a loss (just like the head loss h_L, which is proportional to it).

Note that the pressure drop is proportional to the viscosity μ of the fluid, and ΔP would be zero if there were no friction. Therefore, the drop of pressure from P_1 to P_2 in this case is due entirely to viscous effects, and represents the pressure loss ΔP_L when a fluid of viscosity m flows through a pipe of constant diameter D and length L at average velocity V_{avg}.

In practice, it is found convenient to express the pressure loss for all types of fully developed internal flows (laminar or turbulent flow, circular or noncircular pipes, smooth or rough surfaces, horizontal or inclined pipes) as

Pressure loss: $$\Delta P_L = f\frac{L}{D}\frac{\rho V^2_{avg}}{2} \tag{4}$$

Where $\rho V_{avg}/2$ is the dynamic pressure and f is the Darcy friction factor,

$$f = \frac{8\tau_w}{\rho V^2_{avg}} \tag{5}$$

It is also called the Darcy–Weisbach friction factor. It should not be confused with the friction coefficient called the Fanning friction factor.

By solving for f gives the friction factor for fully developed laminar flow in a circular pipe,

Circular pipe, laminar: $$f = \frac{64\mu}{\rho D V_{avg}} = \frac{64}{\mathrm{Re}} \tag{6}$$

This equation shows that in laminar flow, the friction factor is a function of the Reynolds number only and is independent of the roughness of the pipe surface.

In the analysis of piping systems, pressure losses are commonly expressed in terms of the equivalent fluid column height, called the **head loss** h_L. Noting from fluid statics that ΔP = ρgh and thus a pressure difference of ΔP corresponds to a fluid height of h = ΔP/ρg, the pipe head loss is obtained by dividing ΔPL by ρg to give

Head loss: $$h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{V^2_{avg}}{2g} \tag{7}$$

The head loss h_L represents the additional height that the fluid needs to be raised by a pump in order to overcome the frictional losses in the pipe. The head loss is caused by viscosity, and it is directly related to the wall shear stress are valid for both laminar and turbulent flow in both circular tan noncircular pipes, but is valid only for fully developed laminar flow in circular pipes. Once the pressure loss (or head loss) is known, the required pumping power to overcome the pressure loss is determined from

$$W_{pump\,L} = \dot{V}\,\Delta P_L = \dot{V}_{\rho g h_L} = \dot{m}\,g h_L \tag{8}$$

where V is the volume flow rate and m is the mass flow rate.

For horizontal pipe: $$V_{avg} = \frac{(P_1 - P_2)R^2}{8\mu L} = \frac{(P_1 - P_2)D^2}{32\mu L} = \frac{\Delta P D^2}{32\mu L} \tag{9}$$

Then the volume flow rate for laminar flow through a horizontal pipe of diameter D and length L becomes

$$\dot{V} = V_{avg}A_c = \frac{(P_1 - P_2)R^2}{8\mu L}\pi R^2 = \frac{(P_1 - P_2)\pi D^2}{128\mu L} = \frac{\Delta P \pi D^2}{128\mu L} \tag{10}$$

This equation is known as **Poiseuille's law,** and this flow is called Hagen– Poiseuille flow in honor of the works of G. Hagen (1797–1884) and J.Poiseuille (1799–1869) on the subject.

Inclined Pipes

Relations for inclined pipes can be obtained in a similar manner from a force balance in the direction of flow. The only additional force in this case is the component of the fluid weight in the flow direction, whose magnitude is

$$W_x = W\sin\theta = \rho g\,\upsilon_{element}\sin\theta = \rho g(2\pi r dr dx)\sin\theta \tag{11}$$

where θ is the angle between the horizontal and the flow direction. The force balance now becomes

$$(2\pi r dr P)_x - (2\pi r dr P)_{x+dx} + (2\pi r dx\tau)_r - (2\pi r dx\tau)_{r+dr} - \rho g(2\pi r dr dx)\sin\theta = 0. \tag{12}$$

which results in the differential equation

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{dP}{dx} + \rho g\sin\theta. \tag{13}$$

Following the same solution procedure, the velocity profile can be shown to be

$$\mathrm{U(r)} = -\frac{R^2}{4\mu}\left(\frac{dP}{dx} + \rho g\sin\theta\right)\left(1 - \frac{r^2}{R^2}\right). \tag{14}$$

It can also be shown that the average velocity and the volume flow rate relations for laminar flow through inclined pipes are, respectively,

$$V_{avg} = \frac{(\Delta P - \rho g L\sin\theta)D^2}{32\mu L} \quad \text{and} \quad \upsilon = \frac{(\Delta P - \rho g L\sin\theta)\pi D^4}{128\mu L}. \tag{15}$$

which are identical to the corresponding relations for horizontal pipes, except that ΔP is replaced by ΔP - ρgL sin θ. Therefore, the results already obtained for horizontal pipes can also be used for inclined pipes provided that ΔP is replaced by ΔP- ρgL sin θ. Note that θ >0 and thus sin θ > 0 for uphill flow, and θ<0 and thus sin θ <0 for downhill flow.

In inclined pipes, the combined effect of pressure difference and gravity drives the flow. Gravity helps downhill flow but opposes uphill flow. Therefore, much greater pressure differences need to be applied to maintain a specified flow rate in uphill flow although this becomes important only for liquids, because the density of gases is generally low. In the special case of no flow (V = 0), we have $\Delta P = \rho g L \sin\theta$, which is what we would obtain from fluid statics.

MATERIAL AND METHODS

Sizeable quantities of these samples were brought to the laboratory after washing and rinsing in order to remove the organic particles and unwanted grains. Thereafter, the sand samples were sun dried and later placed in an oven for thirty minutes at temperature of 120^0 C. Afterwards, the samples were allowed to cool down, the stony particles were removed. Three different sieves of sizes 63,150 and212μm were used to sieve the available sand samples in order to obtain sample of different grain size.

Determination of porosity

The porosity of each sample was determined by volumetric approach. In the laboratory measurement of porosity, it is necessary to determine only two basic parameters (bulk volume and grain volume).

Bulk volume = grain volume + pore Volume

$$\text{Total porosity N} = \frac{porevolume}{bulkvolume} \tag{16}$$

In this research, bulk and grain or matrix volume were determined volumetrically by measuring 3ml of dried sand sample using a 10ml measuring cylinder. It was ensured that the measuring cylinder was tapped with a solid object and the sand inside gotre-arranged and compacted before the value of the volume was recorded.

Porosity can be determined as follows:

Volume of sand (bulk volume) = A (mL)

Volume of water = B (mL)

Volume of mixture of water and sand = C (mL)

$$\text{Therefore total porosity, } \varphi = \frac{Porevolume}{Bulkvolume}$$

$$= \quad \varphi = \frac{(A+\mathbf{B})-C}{A}(mL) \tag{17}$$

Determination of volume flux at different angles of inclination

The experimental setup consisted of a big transparent cylindrical pipe 108.5cm long with radius 2.23cm as inlet pipe and five small equal transparent pipe. Each of the outlet pipe was joined to the centre of the circular plastic plate on the top of the inlet pipe at different angles θ of 0^0, 20^0, 50^0, 70and 90^0 from the point normal or line. To serve as control experiment, water was allowed to flow through the empty inlet pipe and outlet pipe for a period of 60sec and the discharged volume of water at each outlet was collected and measured with measuring cylinder. This was done at different tilting angle or angle of inclination 5^0, 10^0, 15^0 and 20^0. Therefore, the inlet pipe and outlet pipes were filled with the same sample at a time and the volume of water discharged through each outlet was measured. This was measured in each outlet for different angle of inclination α and the same was repeated for other samples.

The sample in the inlet pipe was later changed in turn and the volume of water discharged in the outlet in different cases were collected and measured directly with measuring cylinder. The measuring flux q (ms^{-1}) or specific discharge was then computed from the volumetric flow rate Q (m^3s^{-1}) by dividing it with the cross sectional area $2.83 \times 10^{-5} m^2$ of the outlet pipe.

RESULTS AND DISCUSSION

Tables 1, 2 and 3 show the volume flux at different angle of outlets when fluid flows from more permeable medium to a less permeable medium for all inclination angles (cases 1, 2 and 3).Tables 4, 5 and 6 show the volume flux at different angles of outlet when fluid flows from a less permeable medium to a more permeable medium for all inclination angles (cases 4, 5, and 6).

Table 1. Volume flux at different angles of inclination α (case 1)

Angle of inclination α (degree)	Outlet 1 $q \times 10^{-6}$ (ms^{-1})	Outlet 2 $q \times 10^{-6}$ (ms^{-1})	Outlet 3 $q \times 10^{-6}$ (ms^{-1})	Outlet 4 $q \times 10^{-6}$ (ms^{-1})	Outlet 5 $q \times 10^{-6}$ (ms^{-1})
0	0	0	0	0	0
5	0.04	0.0165	0.0047	0	0
10	0.0483	0.0377	0.0188	0	0
15	0.0518	0.0483	0.0353	0.0106	0
20	0.1142	0.1095	0.488	0.0247	0

Table 2. Volume flux at different angles of inclination α (case 2)

Angle of inclination α (degree)	Outlet 1 $q \times 10^{-6}$ (ms^{-1})	Outlet 2 $q \times 10^{-6}$ (ms^{-1})	Outlet 3 $q \times 10^{-6}$ (ms^{-1})	Outlet 4 $q \times 10^{-6}$ (ms^{-1})	Outlet 5 $q \times 10^{-6}$ (ms^{-1})
0	0	0	0	0	0
5	0.00624	0.0188	0.0094	0	0
10	0.0907	0.0459	0.0283	0.0012	0
15	0.1248	0.0742	0.0612	0.013	0
20	0.1543	0.1272	0.073	0.0306	0

Table 3. Volume flux at different angles of inclination α (case 3)

Angle of inclination α (degree)	Outlet 1 $q \times 10^{-6}$ (ms^{-1})	Outlet 2 $q \times 10^{-6}$ (ms^{-1})	Outlet 3 $q \times 10^{-6}$ (ms^{-1})	Outlet 4 $q \times 10^{-6}$ (ms^{-1})	Outlet 5 $q \times 10^{-6}$ (ms^{-1})
0	0	0	0	0	0
5	0.0789	0.0236	0.013	0	0
10	0.1107	0.053	0.0436	0.0024	0
15	0.1684	0.086	0.0707	0.0247	0
20	0.179	0.1398	0.0836	0.0836	0

Table 4. Volume flux at different angles of inclination α (case 4)

Angle of inclination α (degree)	Outlet 1 $q \times 10^{-6}$ (ms^{-1})	Outlet 2 $q \times 10^{-6}$ (ms^{-1})	Outlet 3 $q \times 10^{-6}$ (ms^{-1})	Outlet 4 $q \times 10^{-6}$ (ms^{-1})	Outlet 5 $q \times 10^{-6}$ (ms^{-1})
0	0	0	0	0	0
5	0	0.0082	0.0094	0.0059	0.0012
10	0.0012	0.0212	0.00283	0.0071	0.0024
15	0.0035	0.0247	0.0436	0.0141	0.0047
20	0.0047	0.0165	0.0518	0.0306	0.0071

Table 5. Volume flux at different angles of inclination α (case 5)

Angle of inclination α (degree)	Outlet 1 $q \times 10^{-6}$ (ms^{-1})	Outlet 2 $q \times 10^{-6}$ (ms^{-1})	Outlet 3 $q \times 10^{-6}$ (ms^{-1})	Outlet 4 $q \times 10^{-6}$ (ms^{-1})	Outlet 5 $q \times 10^{-6}$ (ms^{-1})
0	0	0	0	0	0
5	0.0024	0.0071	0.0106	0.0106	0.0035
10	0.0035	0.0082	0.0294	0.0283	0.0067
15	0.0059	0.0082	0.0495	0.0448	0.0071
20	0.0071	0.0071	0.0542	0.0495	0.0141

Table 6. Volume flux at different angles of inclination α (case 6)

Angle of inclination α (degree)	Outlet 1 $q \times 10^{-6}$ (ms^{-1})	Outlet 2 $q \times 10^{-6}$ (ms^{-1})	Outlet 3 $q \times 10^{-6}$ (ms^{-1})	Outlet 4 $q \times 10^{-6}$ (ms^{-1})	Outlet 5 $q \times 10^{-6}$ (ms^{-1})
0	0	0	0	0	0
5	0.0047	0.0059	0.0118	0.013	0.0071
10	0.0094	0.0094	0.0306	0.0342	0.0094
15	0.0106	0.0118	0.0483	0.0506	0.0118
20	0.0141	0.0153	0.013	0.0683	0.0612

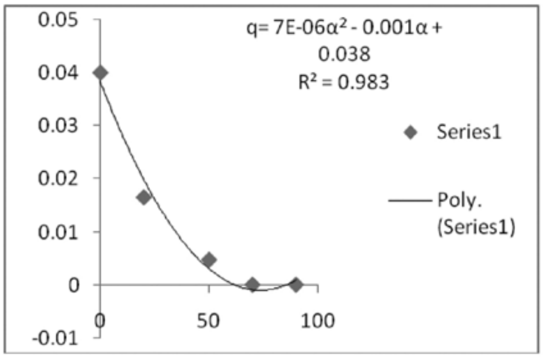

Figure 1. CASE 1 FOR $\alpha = 5^0$

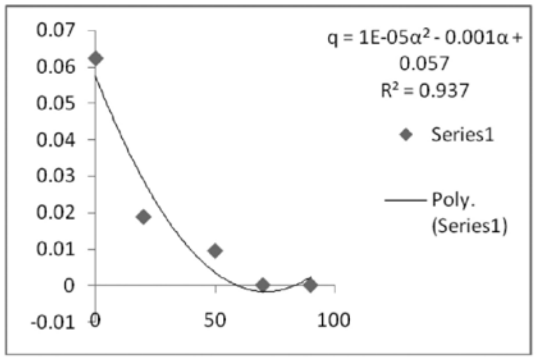

Figure 2. CASE 2 $\alpha = 5^0$

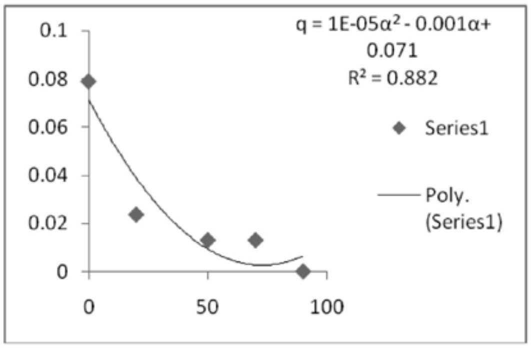

Figure 3. CASE 3 $\alpha = 5^0$

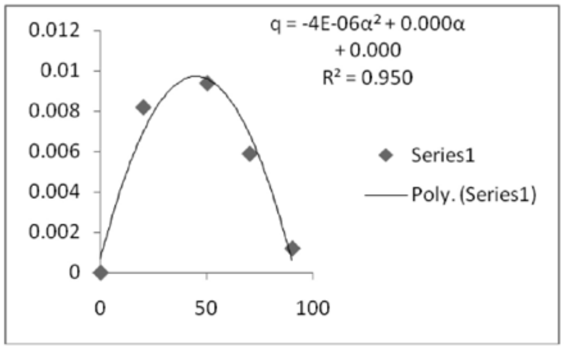

Figure 4. CASE 4 $\alpha = 5^0$

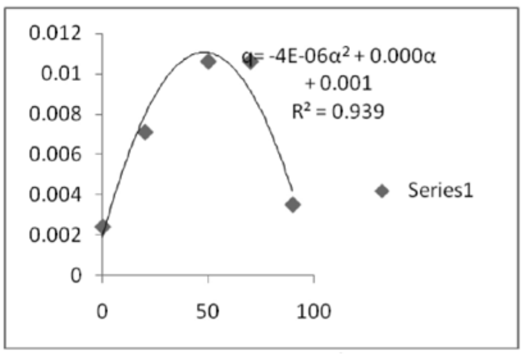

Figure 5. CASE 5 $\alpha = 5^0$

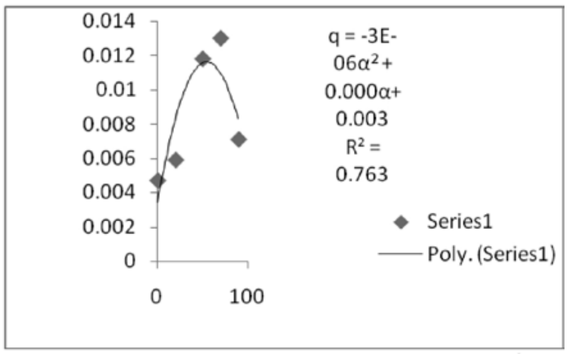

Figure 6. CASE 6 $\alpha = 10^0$

Figure 1 shows that volume flux is related with angle of inclination of polynomial of order 2, that is volume flux increases with increasing angle of inclination, with relation $q = 7E\text{-}06\alpha^2 - 0.001\alpha + 0.038$

Figure 2 shows that when fluid flows from a lower porous medium to a higher porous medium, the volume flux is related with angle of inclination of polynomial of order 2, with relation $q = 1E\text{-}05\alpha^2 - 0.001\alpha + 0.057$

Figure 3 shows that volume flux is related with angle of inclination of polynomial of order 2, that is volume flux increases with increasing angle of inclination, with relation $q = 1E\text{-}05\alpha^2 - 0.001\alpha + 0.071$

Figure 4 shows that volume flux is related with angle of inclination of polynomial of order 2, that is volume flux increases with increasing angle of inclination, with relation $q = -4E\text{-}06\alpha^2 + 0.000\alpha + 0.000$

Figure 5 shows that when fluid flows from a lower porous medium to a higher porous medium, the volume flux is related with angle of inclination of polynomial of order 2, with relation $q = -4E\text{-}06\alpha^2 + 0.000\alpha + 0.001$

Figure 6 shows that volume flux is related with angle of inclination of polynomial of order 2, that is volume flux increases with increasing angle of inclination, with relation $q = -3E\text{-}06\alpha^2 + 0.000\alpha + 0.003$

CASE 6 $\alpha = 10^0$

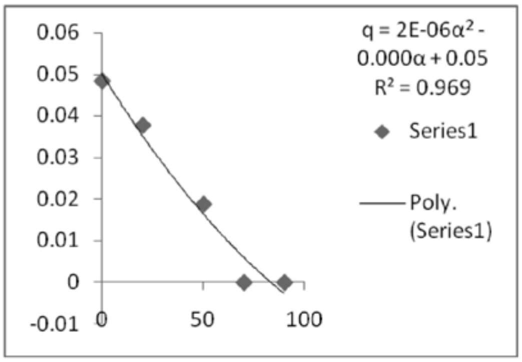

Figure 7. CASE 7 $\alpha = 10^0$

CASE 7 $\alpha = 10^0$

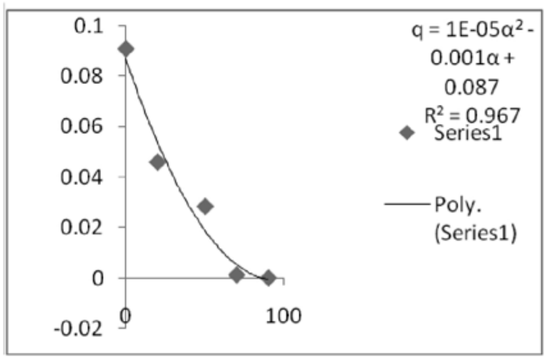

Figure 8. CASE 8 $\alpha = 10^0$

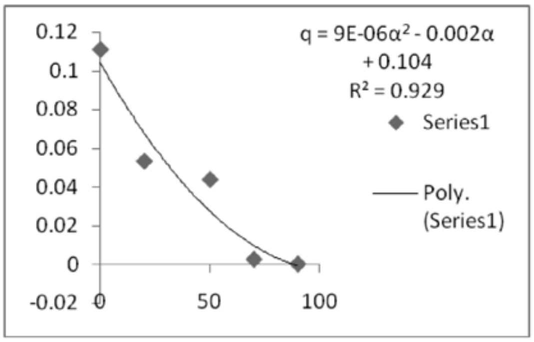

Figure 9. CASE 9 $\alpha = 10^0$

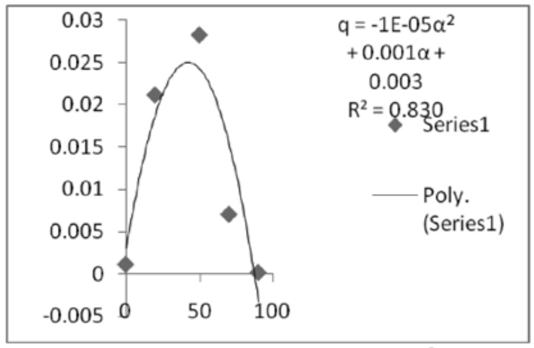

Figure 10. CASE 10$\alpha = 10^0$

CASE 10 $\alpha = 10^0$

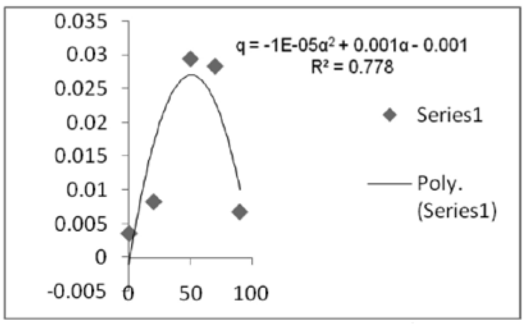

Figure 11. CASE 11 $\alpha = 10^0$

CASE 11 $\alpha = 10^0$

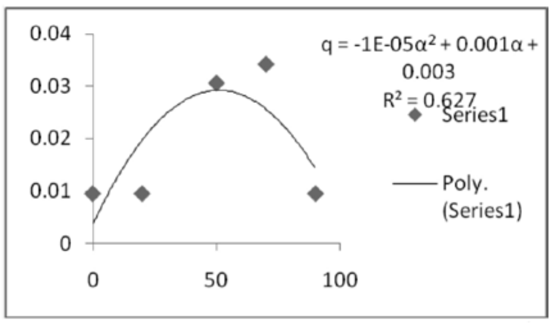

Figure 12. CASE 12 $\alpha = 10^0$

Figure 7 shows that when fluid flows from a lower porous medium to a higher porous medium, the volume flux is related with angle of inclination of polynomial of order 2, with relation, with relation $q = 2E\text{-}06\alpha^2 - 0.000\alpha + 0.05$

Figure 8 shows that volume flux is related with angle of inclination of polynomial of order 2, that is volume flux increases with increasing angle of inclination, with relation $q = 1E\text{-}05\alpha^2 - 0.001\alpha + 0.087$

Figure 9 shows that when fluid flows from a lower porous medium to a higher porous medium, the volume flux is related with angle of inclination of polynomial of order 2, with relation $q = 9E\text{-}06\alpha^2 - 0.002\alpha + 0.104$

Figure 10 shows that volume flux is related with angle of inclination of polynomial of order 2, that is volume flux increases with increasing angle of inclination, with relation $q = -1E\text{-}05\alpha^2 + 0.001\alpha + 0.003$

Figure 11 shows that volume flux is related with angle of inclination of polynomial of order 2, that is volume flux increases with increasing angle of inclination, with relation $q = -1E\text{-}05\alpha^2 + 0.001\alpha - 0.001$

Figure 12 shows that when fluid flows from a lower porous medium to a higher porous medium, the volume flux is related with angle of inclination of polynomial of order 2, with relation $q = -1E\text{-}05\alpha^2 + 0.001\alpha + 0.003$

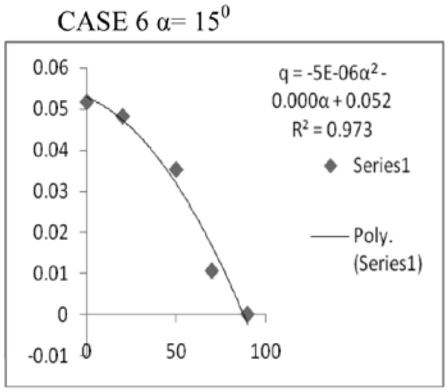

Figure 13. CASE 6 $\alpha = 15^0$

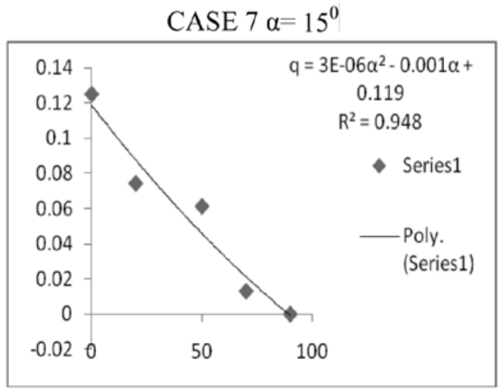

Figure 14. CASE 6 $\alpha = 15^0$

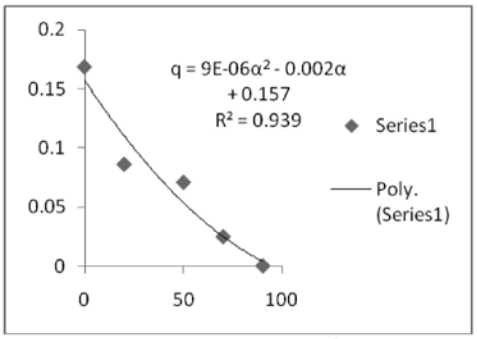

Figure 15. CASE 8 $\alpha = 15^0$

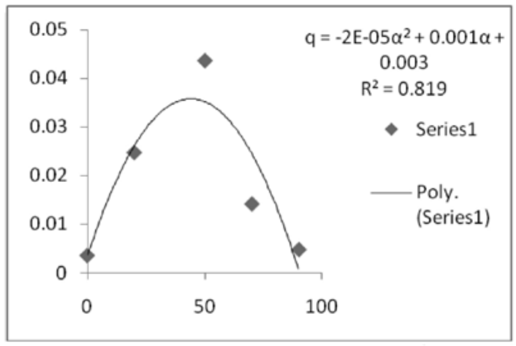

Figure 16. CASE 9 $\alpha = 15^0$

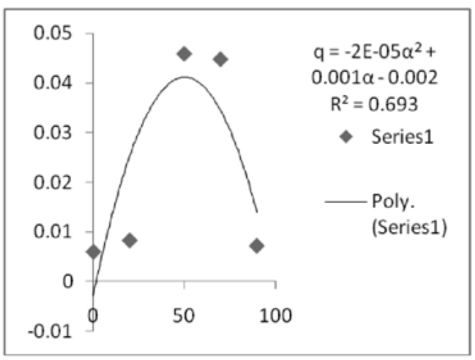

Figure 17. CASE 10 $\alpha = 15^0$

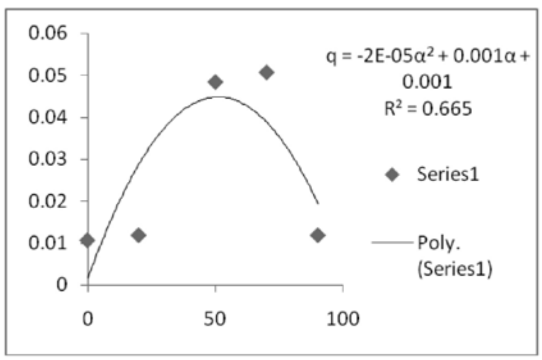

Figure 18. CASE 11 $\alpha = 15^0$

Figure 13 shows that volume flux is related with angle of inclination of polynomial of order 2, that is volume flux increases with increasing angle of inclination, with relation $q = -5E\text{-}06\alpha^2 - 0.000\alpha + 0.052$

Figure 14 shows that volume flux is related with angle of inclination of polynomial of order 2, that is volume flux increases with increasing angle of inclination, with relation $q = 3E\text{-}06\alpha^2 - 0.001\alpha + 0.119$

Figure 15 shows that when fluid flows from a lower porous medium to a higher porous medium, the volume flux is related with angle of inclination of polynomial of order 2, with relation $q = 9E\text{-}06\alpha^2 - 0.002\alpha + 0.157$

Figure 16 show that volume flux is related with angle of inclination of polynomial of order 2, that is volume flux increases with increasing angle of inclination, with relation $q = -2E\text{-}05\alpha^2 + 0.001\alpha + 0.003$

Figure 17 shows that when fluid flows from a lower porous medium to a higher porous medium, the volume flux is related with angle of inclination of polynomial of order 2, with relation $q = -2E\text{-}05\alpha^2 + 0.001\alpha - 0.002$

Figure 18 shows that volume flux is related with angle of inclination of polynomial of order 2, that is volume flux increases with increasing angle of inclination, with relation $q = -2E\text{-}05\alpha^2 + 0.001\alpha + 0.001$

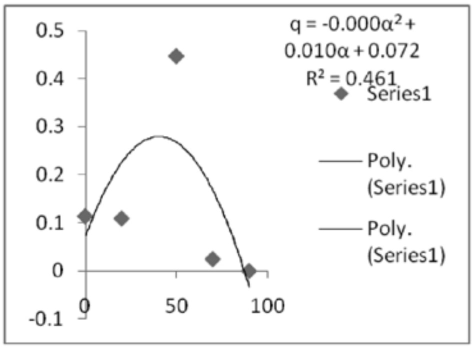

Figure 19. CASE 6 $\alpha = 20^0$

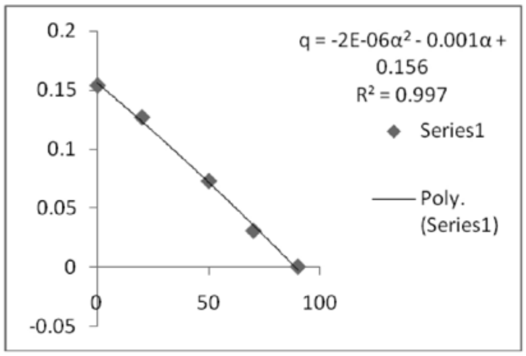

Figure 20. CASE 7 $\alpha = 20^0$

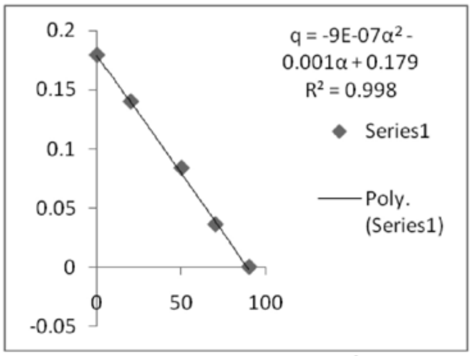

Figure 21. CASE 8 $\alpha = 20^0$

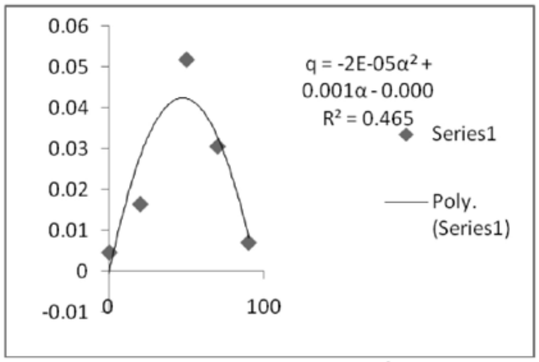

Figure 22. CASE 9 $\alpha = 20^0$

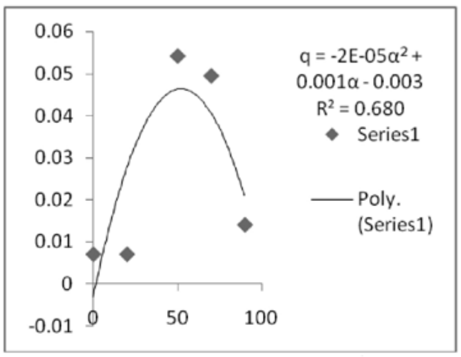

Figure 23. CASE 10 $\alpha = 20^0$

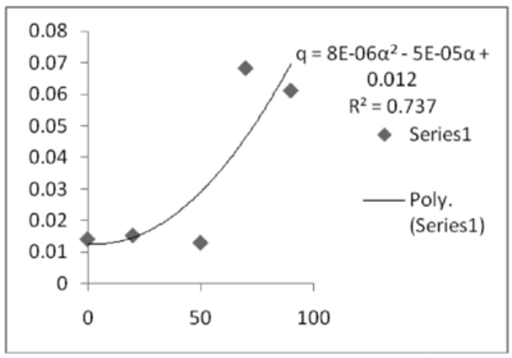

Figure 24. CASE 11 $\alpha = 20^0$

Figure 19 shows that volume flux is related with angle of inclination of polynomial of order 2, that is volume flux increases with increasing angle of inclination, with relation $q = -0.000\alpha^2 + 0.010\alpha + 0.072$

Figure 20 shows that when fluid flows from a lower porous medium to a higher porous medium, the volume flux is related with angle of inclination of polynomial of order 2, with relation $q = -2E\text{-}06\alpha^2 - 0.001\alpha + 0.156$

Figure 21 Show that volume flux is related with angle of inclination of polynomial of order 2, that is volume flux increases with increasing angle of inclination, with relation $q = -9E\text{-}07\alpha^2 - 0.001\alpha + 0.179$

Figure 22 shows that volume flux is related with angle of inclination of polynomial of order 2, that is volume flux increases with increasing angle of inclination, with relation $q = -2E\text{-}05\alpha^2 + 0.001\alpha - 0.000$

Figure 23 shows that when fluid flows from a lower porous medium to a higher porous medium, the volume flux is related with angle of inclination of polynomial of order 2, with $q = -2E\text{-}05\alpha^2 + 0.001\alpha - 0.003$

Figure 24 shows that volume flux is related with angle of inclination of polynomial of order 2, that is volume flux increases with increasing angle of inclination, with relation $q = 8E\text{-}06\alpha^2 - 5E\text{-}05\alpha + 0.012$.

Table 7. Summary of the porosity difference correlation coefficient

Cases	-Δϕ2	R^2	+Δϕ	R^2
α = 5°				
1	0.080	0.983	40.080	0.95
2	0.114	0.937	50.114	0.939
3	0.170	0.882	60.170	0.763
Cases	**-Δϕ2**	**R^2**	**+Δϕ**	**R^2**
α = 10°				
1	0.080	0.969	40.080	0.83
2	0.114	0.967	50.114	0.778
3	0.170	0.929	60.170	0.627
Cases	**-Δϕ2**	**R^2**	**+Δϕ**	**R^2**
α = 15°				
1	0.080	0.973	40.080	0.819
2	0.114	0.948	50.114	0.693
3	0.170	0.939	60.170	0.665
Cases	**-Δϕ2**	**R^2**	**+Δϕ**	**R^2**
α = 20°				
1	0.080	0.461	40.080	0.465
2	0.114	0.948	50.114	0.68
3	0.170	0.939	60.170	0.737

The pattern of flow can be inferred from the correlation R^2 for best fitting for each graph which indicates that as porosity change increases, the correlation coefficient (R^2) for polynomial for each cases were presented in table 7. The porosity changes in this work are of two forms. The first one is the porosity change when fluid flows from less permeable porous medium (φ_1) to more permeable medium (φ_2) while the other one is when fluid flows from more permeable medium (φ_2) to less permeable medium (φ_1). The two different forms of the porosity changes can be represented by ($+\Delta\varphi$) and ($-\Delta\varphi$) respectively. The first one leads to increases in flow rate, while the second one toflow rate in porous media decreases. This implies that, the $+\Delta\varphi$ promotes turbulent flow in porous media while $-\Delta\varphi$ promotes laminar flow in porous media. In order words, $+\Delta\varphi$ enhances turbulent flow while $-\Delta\varphi$ damped turbulent flow force.

From the plots of volume flux against outlet angles, it is observed that, volume flux tends towards linearity proportional to angle of outlets when fluid flows from more permeable medium to a less permeable medium for all inclination angles (cases 1, 2 and 3).The relationship between volume flux and angle of outlets follows a pattern of polynomial order 2, when fluid flows from a less permeable medium to a more permeable medium for all inclination angles (cases 4, 5, and 6).

Also, it is observed that for all inclination angles of α, as porosity change increases, the pattern of relationship of plots of volume flux against outlet angles is tending towards a polynomial of order 3 from order 2. This can be inferred from the correlation coefficient (R^2) of the plots of volume flux against outlet angles. This could be as a result of increase in flow rate caused by the increase in differences between the media porosities. This pattern is more pronounced in 10^0 and 15^0 inclination angles.

CONCLUSION

The purpose of this work is to establish an equation to determine fluid flow rate and volume flux in inclined heterogeneous porous media in subsurface. It is observed that there are different flow patterns in porous media for different porosities change ($\Delta\varphi$). The pattern of flow is tending more towards polynomial of order 3 as the porosity change ($\Delta\varphi$) increases. This can be inferred from the correlation R^2 for best fitting for each graph which indicates that as porosity change increases, the correlation coefficient (R^2) for polynomial of order 2 decreases.

Also, equation 15 above shows only the effects of gravity g, and pressure difference ΔP on the flow of fluid in an inclined pipe with no porous media. However, the effect of porosity difference, $\Delta\varphi$ of the porous media could be introduced to the problem whenever the pipe is filled with porous media of different porosities. This is an improvement on the Hagen-Poiseullie's equation. Thus, equation 15 becomes

$$\upsilon = \frac{(\Delta P \pm \Delta\phi + \rho g L \sin\theta)\pi D^4}{128\mu L} \quad (18)$$

The porosity change $\Delta\varphi$ is positive when moving from less permeable to more permeable medium in an inclined pipe, while it is negative when moving from more permeable media to less permeable medium in an inclined pipe($-\Delta\varphi$) acts as a damping factor for turbulent flow, while ($+ \Delta\varphi$) increases the rate flow vis-à-vis turbulent flow. This work could be of help in modeling the various mechanisms of water flow and contaminant transport along pathways in the soil and groundwater for contamination prevention and remediation.

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