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Subsurface thermal response to increase in air temperature with time dependent recharge

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ABSTRACT

Kumar et al., (2012) derived a solution to calculate the subsurface thermal response to time dependent air temperature with constant groundwater recharge using a Robin type boundary condition at the earth's surface relating the air temperature to surface temperature using a heat transfer coefficient. For time dependent recharge, analytical solution would be too complex except in a few simple cases. We thus use a numerical approach based on the finite element method to simulate the changes in the thermal response of subsurface to air temperature change under a step change in recharge. We illustrate the effects of time dependent recharge on the subsurface thermal regime for varying duration and size of step changes in recharge and with a heat transfer coefficient. Numerical results for an example show that the surface temperature rise declines in 6 yrs from the value for recharge of 0.5 m/yr lasting 10 yrs to that of recharge of 0.2 m/yr. This lag is found to be increasing with the duration of higher recharge and the size of the step.

Key words: Shallow subsurface temperature, Robin boundary condition, Finite element method, Time dependent recharge, time lag.

INTRODUCTION

Shallow subsurface temperatures depend on surface thermal perturbations and the subsurface thermal characteristics. Heat in the near subsurface is introduced mainly due to the sun's radiation on the surface. This heat is transferred into the subsurface via conduction and advection due to groundwater recharge. Annual/ decadal variations in air temperature and surface temperature are closely coupled with long term climate change. Several analytical models have been developed to solve the transient, one-dimensional heat transfer equation with both advection and conduction to calculate subsurface temperatures in response to decadal or seasonal variations in surface temperature. Seminal studies in this field were conducted by Suzuki (1960) and Stallman (1965). Anderson (2005), Constantz (2008), Saar (2011), Kurylyk et al., (2014) and Rau et al., (2014) have reviewed all the relevant literature on this subject and these review can be referred for earlier studies in this field. Many studies are based on Dirichlet type boundary condition in which air temperature and surface temperatures are tacitly equated. It has been observed by Beltrami and Kellman (2003) and Tsilingiridis and Papakostas (2014) that there is a lag between the air temperature and soil surface temperature signals. The relationship between air and surface temperatures is usually estimated through empirical relationships between these two time series (Figura et al., 2015) and then applied to predict subsurface temperatures from measurements of air temperatures using heat conduction theory.

Kumar et al., (2012) used a Robin type boundary condition in which the differences between air and surface temperatures are taken into account. In this work, groundwater recharge was assumed to be steady. However the recharge depends upon precipitation which is time dependent due to climate changes. It has been argued that warmer atmospheric temperature means more storage of vapour in the atmosphere, thus atmospheric moisture increases exponentially with atmospheric temperature. This can then lead to an intensified hydrologic cycle and thereby more groundwater recharge. Thus earlier heat transfer work needs to be extended to consider the effects of changes in recharge. Role of time varying recharge with Dirichlet type boundary condition with periodic variation have been considered in Shao et al., (1998), Keshari and Koo (2007) and Rau et al., (2015). Rau et al., (2015) considered periodic variation in the surface temperatures and showed how delays in amplitudes and phases due a step type changes in the recharge. Here references to other works advocating considerations of transient recharge are given which can be referred.

It is possible in principle to modify the analytical solution given by Kumar et al., (2012) to develop an analytical solution for a series of step changes in recharge. This constant recharge solution can be used to prescribe the initial condition for the next step change in recharge. But this initial boundary value problem would be too complicated for application of the Laplace transform method. In this case of time dependent recharge, it is better to resort to numerical methods. Numerical methods, such as finite element or finite difference methods can be applied to solve the governing advection diffusion equation for subsurface temperatures by reducing it to a set of algebraic equations obtained via discretization. The main objective of this contribution is to discuss changes in shallow subsurface temperatures due to time dependent recharge and a Robin type boundary condition using a finite element method.

Mathematical formulation of the problem

In the subsurface, temperature variations are represented by a one dimensional heat conduction-advection equation for homogeneous media. Transient thermal perturbations in porous subsurface subject to a vertical ground water recharge can be represented with the following advection-diffusion equation discussed extensively by Carslaw and Jaeger (1959):

$$(\phi(\rho C_p)_f + (1-\phi)(\rho C_p)_s)\frac{\partial T}{\partial t} = K\frac{\partial^2 T}{\partial z^2} - (\rho C_p)_f U(t)\frac{\partial T}{\partial z} + A(x,t,T) \quad (1)$$

The first term on the right hand side of the equation refers to heat conduction in the solid porous matrix with density 'ρ', specific heat 'C_p'and thermal conductivity 'K'. The second term corresponds to the heat advection, the property of fluid moving in the porous media with unsteady velocity 'U(t)', and '$A(z,t,T)$' is the volumetric heat source, we take it as zero here. The subscript 's' and 'f' refers to solid matrix (s) and fluid filled porous space (f). Also 'ϕ' is porosity, 'T' is temperature, 'z' is the depth from surface and 't' is time. Equation (1) is valid in case of local thermodynamically equilibrium, so that the temperature of the water is same as that of solid matrix. Thus the entering recharge water has the same temperature as the surface. Associated initial and boundary conditions are as follows:
Initial condition:

$$T(z,0)=T_0+az \qquad \text{at} = 0 \text{ for all } z > 0 \qquad (2)$$

A linear increase in temperature with constant temperature gradient is taken as the initial condition for the problem, which assumes that initial thermal regime is conduction-dominated and at steady-state. Here 'T_0' is the initial surface temperature, and 'a' is geothermal gradient. The boundary condition at the earth surface is taken as:

$$K\frac{dT}{dz} = H(T - T_A) \qquad at\ z = 0\ \ for\ all\ t > 0 \qquad (3)$$

$$T=T_c \qquad at\ z = 200m \text{ for all } t > 0 \qquad (4)$$

Here H is the heat transfer coefficient, and T_A is air temperature. Equation (3) is called a Robin type boundary condition. It depicts coupling of air temperature and surface temperature, and can be considered a mixture of Neumann and Dirichlet boundary conditions. T_c is the constant temperature of 19.4^{0}C applied on the bottom boundary taken from the initial condition.

The ground water recharge depends up on the precipitation and characteristics of the subsurface. The precipitation is partitioned into surface runoff and recharge. Since the ground water velocity is an important process for transporting the heat in the subsurface, its effect on subsurface thermal profiles is an important consideration. It is essential to consider time varying recharge to reflect natural environments and to accommodate changing precipitation. For that we use a simple recharge model which represents a case that with a step increase in precipitation, i.e. the recharge increases for a period and then reduces either due to changes in the precipitation. Thus we take the rate of recharge as:

$$U = U_0H(t) + U_1(H(t) - H(t–t_0))$$

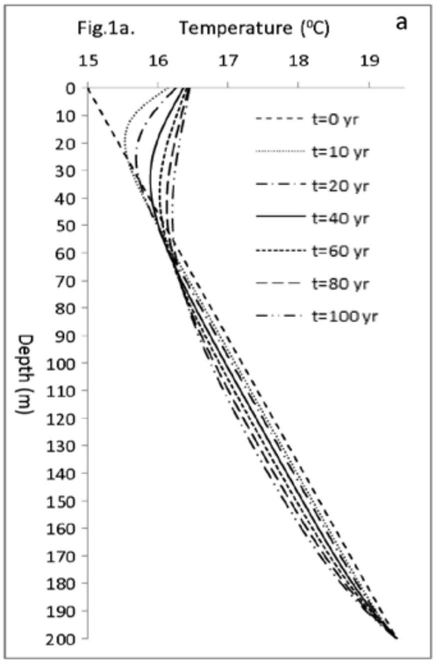
The step change occurs after a time period t_0 from its initial value U_0 to U_1. $H(t)$ is the Heaviside unit function. Since it is difficult to obtain an analytical solution for this governing equation, the numerical approach is developed. We obtain the numerical solution of this initial-boundary value problem using COMSOL Multiphysics software (COMSOL, 2015). This software was developed based on the finite element method. In this method, the domain will be discretized into small elements with user defined mesh. The numerical solutions is obtained by using the basis functions, which have the piecewise continuous derivatives. In this problem, the one dimensional domain has depth as 200 meters. The whole domain is divided into 200 extremely fine mesh elements to compute the solution. Simulations are performed for various values of H, U_1 and t_0 to discuss quantitatively the role of time dependent recharge. The values of various parameters appearing in the equation are given in Table 1.

Numerical Results and Discussions:

The temperature – depth profiles have been calculated for a step decrease in groundwater recharge from a constant 0.5 m/year for 10 years and then decreased to 0.2 m/ year throughout the remaining time period with initial and boundary conditions as given in Eqns. 2-4. Figure 1a shows temperature – depth profiles at different times, and Figure 1b shows the temperature anomaly profile [T (z, t)-T (z, 0)]. Figure 1a indicates that the temperature decreases up to a certain depth and then it begins increasing with depth. It is seen more clearly from the Figure 1b that after the subtraction of the initial condition, the temperature anomalies change decrease up to some depth and approaches zero at the bottom of the domain since the bottom boundary is kept as constant temperature. It is seen that the penetration of surface thermal disturbances influence subsurface temperatures more up to 50 to 60 m. The numerical results also show that the surface temperature changes and temperature anomalies decrease with increasing depth. The presence of negative anomalies is due to recharge. In the absence of recharge, all anomlies

Table 1. Value of parameters appearing in Equation (1-3)

Name	Expression	Description
T_0	15 °C	Surface soil temperature at zero time
T_A	16.5°C	Surface air temperature
K	2.5 W/(m °C)	Thermal conductivity
κ	6.1 m^2/yr	Thermal diffusivity
H	(0.2,0.4,1.0) W/(m^2 °C)	Heat transfer coefficient
a	0.022 ^{0}C/m	Geothermal gradient
t_0	(2,10,15) yr	Time step
U_0 (U_1)	0.5(0.2) m/yr	Ground water velocity



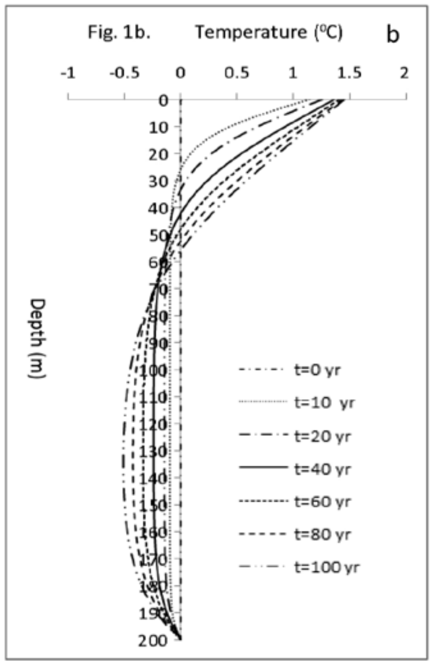


Figure 1a. Temperature depth profile for step change in ground water recharge(U_0(U_1)=0.5(0.2)) m/yr,H= 0.2 W/m^2 C with time step of 10 yr calculated for different times.
Figure 1b. Effective temperature anomalies for step change in ground water recharge(U_0(U_1)=0.5(0.2)) m/yr, H= 0.2 W/m^2 C with time step of 10 yr calculated for different times.

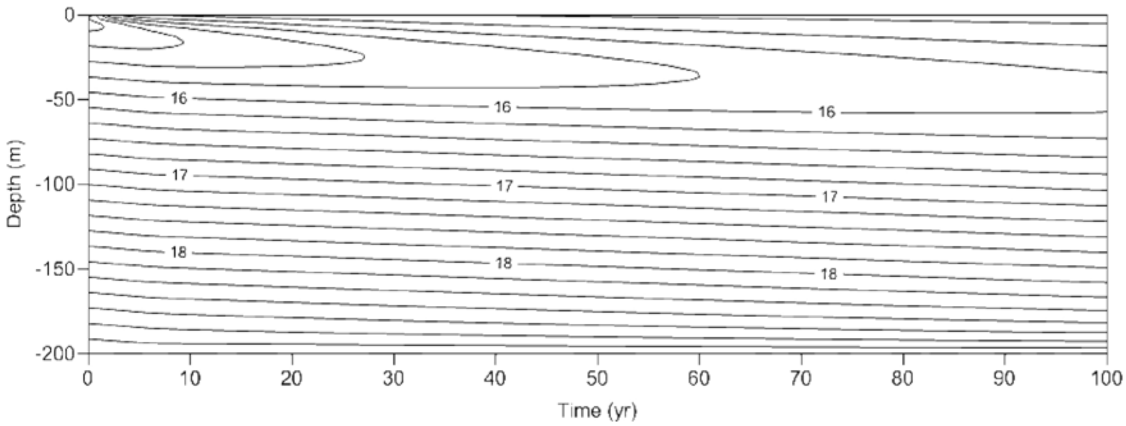


Figure 1c. Temperature (^{0}C) depth contours with intervals 0.2°C with H= 0.2 W/m^2 C for different times.

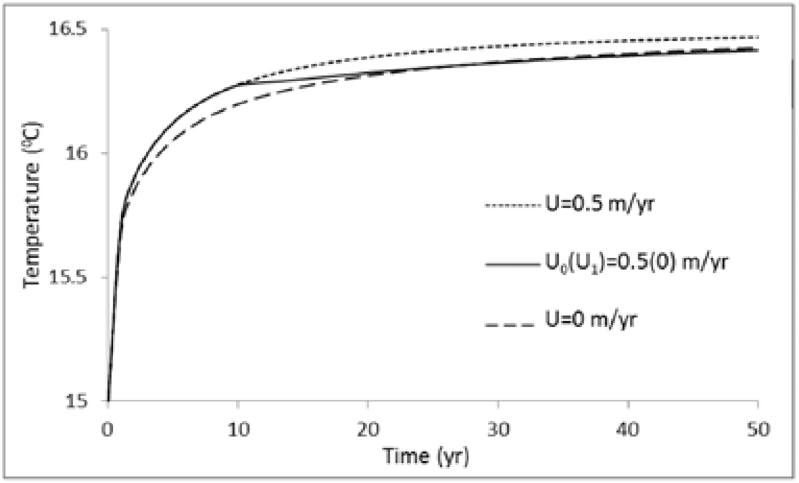


Figure 2a. Surface temperature with time (dashed lines for constant groundwater velocity, continuous line for step function)

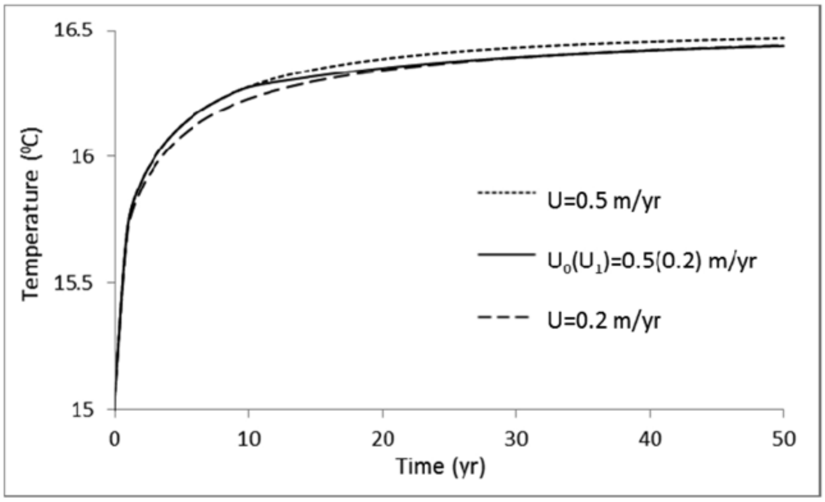


Figure 2b. Surface temperature with time (dashed lines for constant groundwater velocity, continuous line for step function)

would be postive. Furthermore, the temperature anomalies penetrate deeper zones with increasing time.

We have plotted subsurface temperature contours for step change in ground water recharge ($U_0(U_1)=0.5(0.2)$) m/yr, and heat transfer coefficient 'H' as 0.2 W/m^2 C (Figure 1c). The contour interval is 0.20°C. From the Figure, it is observed that temperatures are highly disturbed in the shallow subsurface (50-55 m depth), lasting over 100 years.

The effect of the step changes in the recharge rate is illustrated by calculating three time histories of surface temperatures. Two histories are for constant recharge of 0.5 m/yr and 0 m/yr (Figure 2a). The third history is for the recharge initially as 0.5 m/yr for 10 yrs and then reduced to 0 m/yr. It is seen that with such a step change in the recharge, the curve follows the uniform recharge of 0.5 m/yr and after a lag of nearly 8 yrs joins the curve for zero recharge. Figure 2b show the time history surface temperature for step change in the recharge from 0.5 m/yr to 0.2 m/yr at time of 10 yrs. In this case, the transtion also occurs from a large recharge value to a low recharge value, but this transition period is less than the earlier case.

For given initial value of recharge, there are three variables in this problem on which the subsurface temperatures depend. We shall now illustrate the changes in the surface and subsurface temperatures with changes in these three variables: U_1, t_0 and H.

The effect of step decrease in ground water velocity on surface temperatures is shown in Figure 3. Here the rate of recharge initially is 0.5 m/yr lasting duration of 10 yr and then decrease to different values of rate of recharge. It shows a sudden decrease in recharge will change the surface temperatures. The increase is slower for smaller values of recharge. We have also plotted in Figure 4a and 4b the changes in the subsurface temperatures with time for two values of U_1 as 0 and 0.4 m/yr. From the Figure 4a it is clear

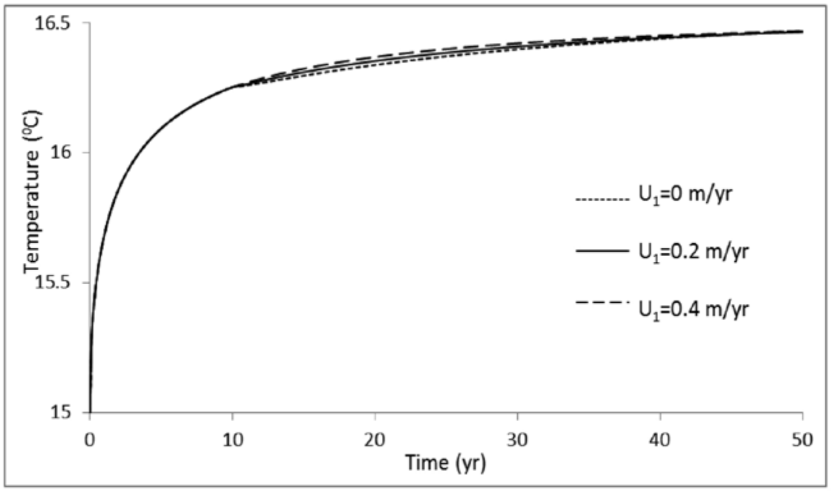


Figure 3. Surface temperature with time for different recharge rates for t_0=10 year.

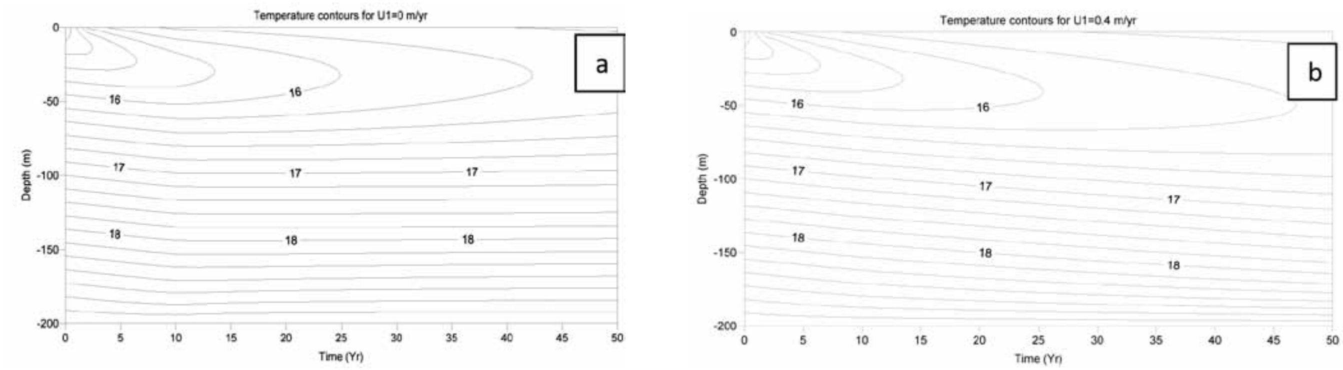


Figure 4. Isothermal contours for variation in step decreased ground water recharge with time with contour interval of 0.2 ^{0}C: (a) for U_1=0 m/yr (b) for U_1= 0.4 m/yr

that subsurface temperatures follow an increasing trend for 10 yr due to higher recharge and then these decreases. The effect is large up to a depth of 50-60 m.

The influence of duration of the higher surface recharge on surface temperatures for three different time steps is illustrated in Figure 5. This Figure shows that the increase in duration of the recharge will enhances the surface temperatures and the change persists after sudden decrease in the groundwater recharge. Surface temperatures drops slowly with time from higher recharge to lower recharge rate. From the Figure it is concluded that as the duration of higher recharge increases, transition time also increases.

The effect of variation in duration of ground water recharge on subsurface thermal structure is shown by plotting temperature contours for different time steps (t_0) in time and space coordinates. Figures 6a and 6b show temperature contours for t_0=2 yr and t_0= 15 yr. It is observed that subsurface temperature contours are more elongated with time for higher duration of initial higher rate of recharge. Contour of temperature 16.2^{0}C last longer by about 4 years.

The heat transfer coefficient depends on the interaction at the interface between the atmosphere and subsurface. On the earth's surface, the convective heat transfer coefficient couples land surface soil temperatures with surface air temperature using the Robin type heat flux condition on the surface boundary. Numerical results have been calculated for different values of heat transfer coefficient to quantify its effect on surface temperatures Figure 7. The results show that with increase of this coefficient, the surface temperatures increases faster and similarly the subsurface temperature anomalies will raise faster.

Evolution of subsurface temperatures for different heat transfer coefficient values are shown in contour Figures (8a) and (8b). Figure 8a show subsurface thermal perturbations for heat transfer coefficient as 0.4 W/m^2*K. With increase in the values of heat transfer coefficient there is faster perturbation of the subsurface. Increase from H=0.4 to 1.0, the contour for T=16.2^{0}C reaches earlier by about 3 years.

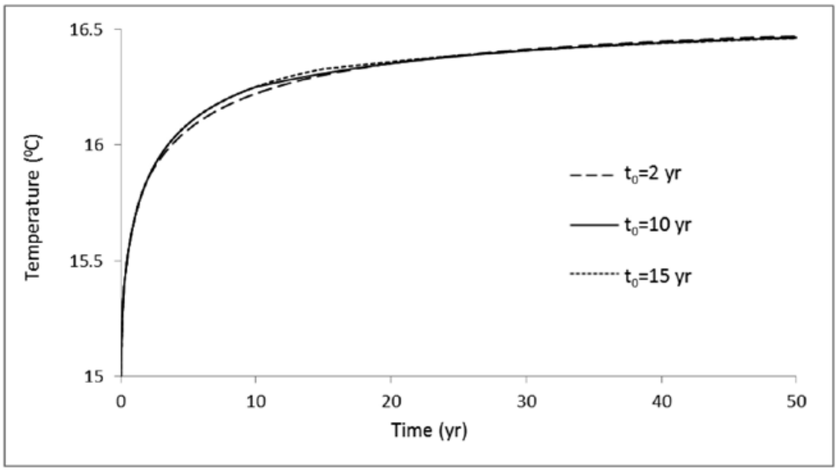


Figure 5. Surface temperature profiles for various values of time step t_0 (2yr, 10yr and 15yr) with $U_0(U_1)=0.5(0.2)$ m/yr.

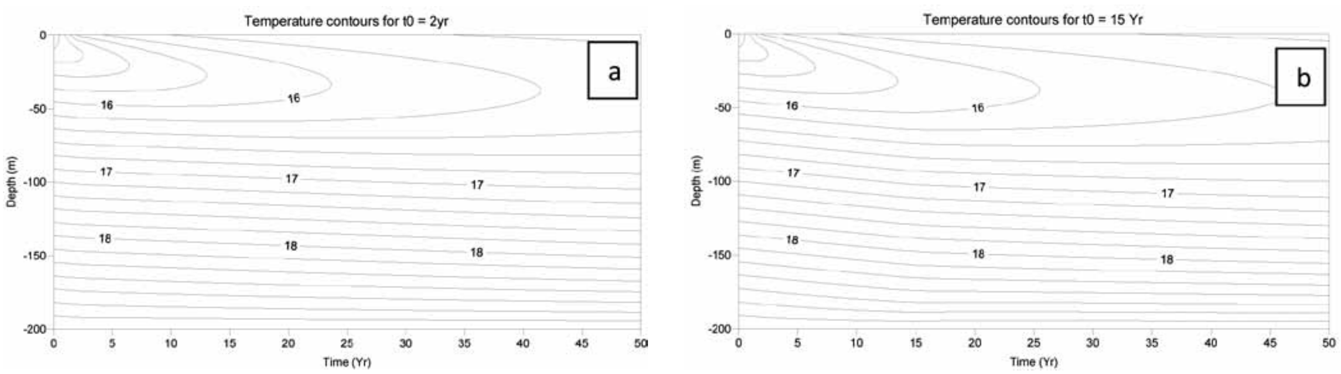


Figure 6. Isothermal contours for variation in duration of recharge with time with contour interval of 0.2 ^{0}C. (a) for t_0=2 yr (b) for t_0= 15 yr

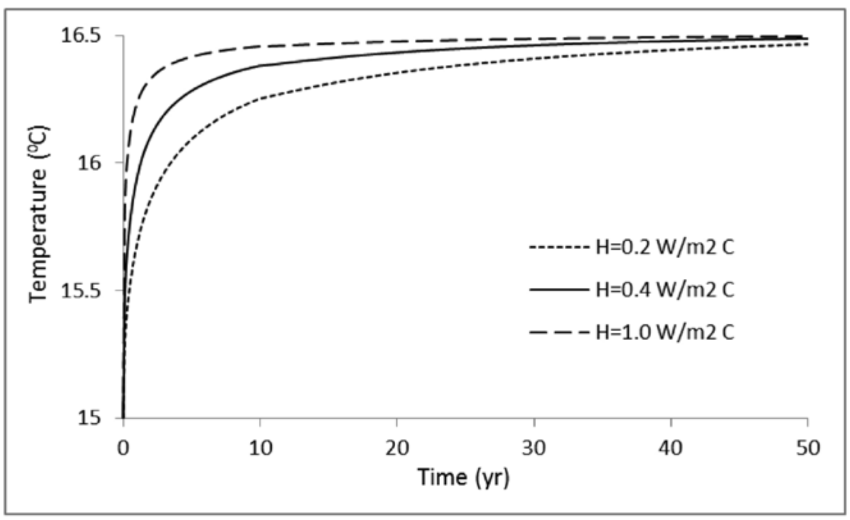


Figure 7. Surface temperature variation with time for different heat transfer coefficient (H) values.

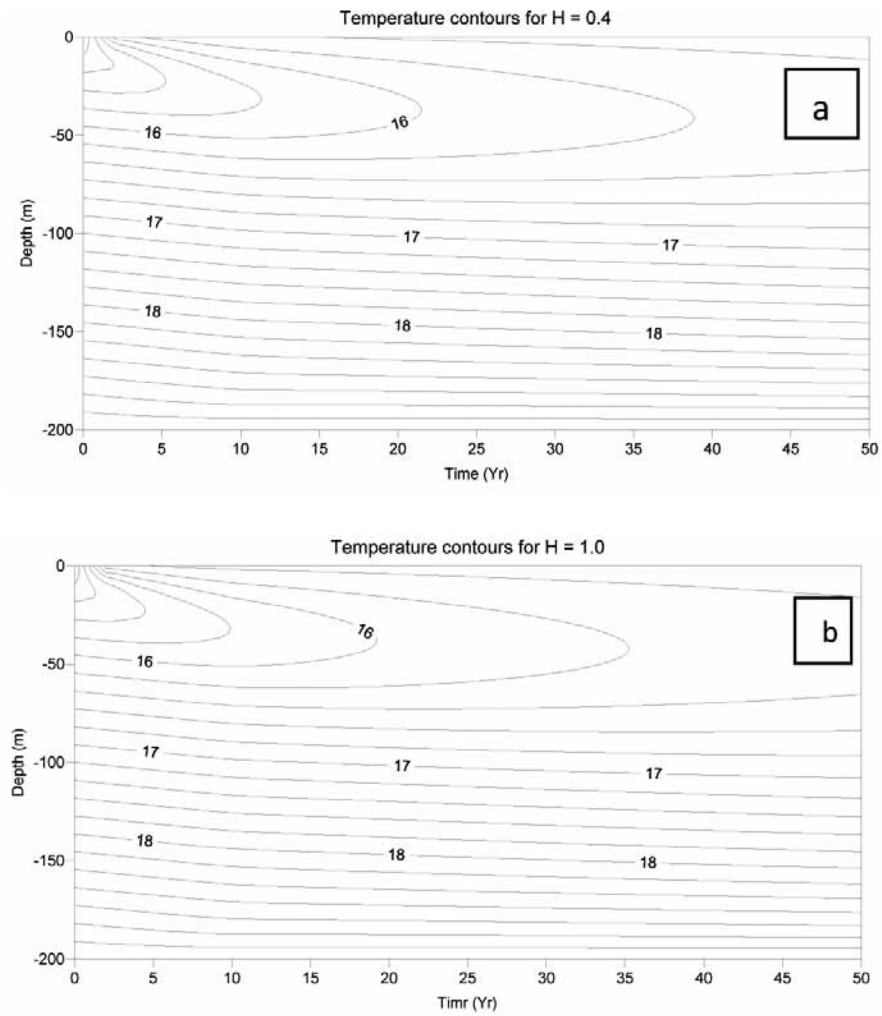


Figure 8. Isothermal contours for variation in heat transfer coefficient with time with contour interval of 0.2 ^{0}C. (a) for H=0.4 W/m^2 C (b) for H= 1.0W/m^2 C.

This numerical exercise can be repeated to show changes in the surface and subsurface temperatures for other values of the variables such as initial rate of recharge and its duration, final rate of recharge and the heat transfer coefficients.

CONCLUSIONS

We have obtained the numerical solution for shallow subsurface temperature using COMSOL Multiphysics for a model having time dependent recharge. Surface and subsurface thermal profiles for various values of step changes in the recharge, duration of higher recharge rate, and the heat transfer coefficient has been studied. Analysis of simulated temperature signals for sudden step decrease in recharge shows slower rise in the surface temperature to the prescribed air temperature. This pattern is seen in distribution of the temperature anomalies with depth. Surface temperatures for a given value of higher recharge decrease after a time period to join the surface temperature anomaly curve for lesser value of recharge. This transition time depends upon the duration of step change and lower level of recharge rates. It is also seen that duration of high recharge magnitude will enhance the transition time. Higher values of heat transfer coefficient lead to faster increase in the surface temperature with time. We would suggest that for obtaining surface and subsurface changes in the induced temperature to changes in the air temperatures, Robin type boundary condition is more reasonable than prescribing either constant temperature or heat flux condition on the surface. More numerical work should to be performed with more general time histories of groundwater recharge and different types of earth surface conditions. These results will be helpful in understanding borehole temperature data to assess the effect of climate change both in terms of changes in the air temperature and rate of recharge.

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