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Viscoelastic seismic modeling and Q estimation for an attenuating media+

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ABSTRACT

In seismic modelling experiments, the propagating seismic wave experiences attenuation when real media is simulated. However, for computational ease, we often ignore this effect and model the Earth as either an acoustic or an elastic medium. In this paper, we briefly discuss the importance of considering attenuation in seismic modelling, by using a viscoelastic wave equation. Further we also briefly discuss the effects of attenuation on seismic data and its incorporation in seismic modelling. We find that the attenuation has a large effect on synthetic seismogram, which should be properly addressed in advanced seismic processing and imaging methods.

INTRODUCTION

Attenuation is a property of a medium due to which the energy of a seismic wave can be dissipated in the form of heat and thus leading to a reduction in the wave amplitude. Factors and phenomena responsible for the attenuation can be broadly classified as either extrinsic, or intrinsic. In extrinsic attenuation, redistribution of energy leads to a reduction of amplitudes, while in intrinsic attenuation, the waves suffer energy loss due to conversion of mechanical energy into heat. Commonly observed wave phenomena like geometric spreading, scattering, leaky modes, etc. can be cited as examples of extrinsic attenuation and they do not contribute towards intrinsic attenuation. In general, seismic attenuation is quantified in terms of quality factor (Q), which can be defined as a ratio of stored energy to dispersed energy, as it measures relative energy loss per oscillation cycle of the seismic waves. In ideal scenario, Q is related to the physical state of the rock and it increases with increase in density and velocity.

Since attenuation results in a loss of seismic energy, the recorded seismic traces are required to be compensated for an exact of amount of Q. To achieve this, determination of a precise Q model is required, especially in the high attenuating regions like Deccan Volcanic Province (DVP), where thick basalt sequences cause high attenuation to the propagating seismic waves (Vedanti et al., 2015, 2018). In these areas, data processing using conventional methods may fail because of low signal to noise ratio, especially in the sub-basalt formations. In a synthetic study carried out by Malkoti et al., (2015), it is shown that due to attenuation, amplitudes of the late arrivals or amplitudes from deeper reflectors, are highly diminished in the generated seismogram, which makes processing and interpretation very challenging in sub-basalt layers. In recent past, attempts have been made to apply advanced techniques like full waveform inversion (FWI) to improve the seismic imaging. This technique needs 'complete wave field information' with precise amplitudes to obtain the accurate gradients and thus, it needs a Q structure of the domain. There are several techniques mentioned in the literature for estimation of Q from the acquired seismic data. Most of these techniques are based upon certain data attributes, such as spectral ratio technique which depends mainly on the amplitude, however, the seismic attenuation has a high influence on several other attributes known as the first order effect. Hence, considering the medium as an attenuating media, has its own consequences on Q estimation techniques and it is quite likely to obtain different values of Q from the same data using different methods (Tonn, 1991). To understand this problem of the estimation of Q, we first need to understand the theory of viscoelasticity and the aspects we should consider while incorporating attenuation in a seismic modelling experiment. Thus in this paper, we briefly discuss these aspects of incorporating attenuation in seismic modelling and demonstrate its need by using a simple Earth model.

VISCOELASTIC MODELS

Viscoelasticity is a property of a medium that exhibits both viscous and elastic characteristics when it undergoes deformation. In a pure elastic medium, the process of 'application and removal of the load' follows energy conservation; however, in a viscoelastic material, it involves energy loss. The energy is dissipated during the loading and unloading cycle and thus obtained hysteresis curve area can be used to estimate the attenuation or the quality factor Q. There are several approaches available in literature to model the quality factor, which includes simple damping, frictional models, complex moduli, time dependent moduli etc. (Carcione, 2007; Kjartansson, 1979; Liu et al., 1976; Tal-Ezer et al., 1990). However, in this paper, we preferred to follow more reasonable 'time dependent moduli' approach

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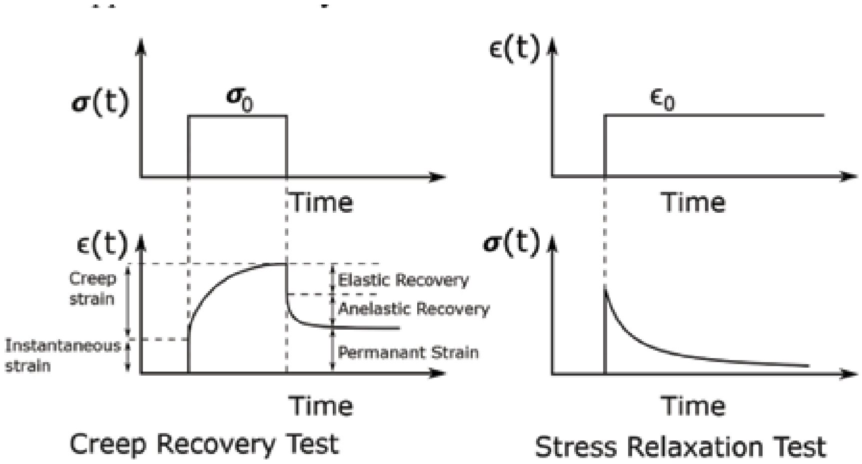

Figure 1. Characteristic functions for an ideal viscoelastic material.

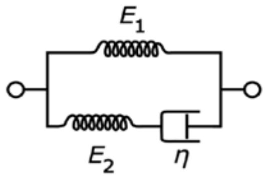

Figure 2. The Zener model with three elements (wiki).

for which it is required to define the basic characteristics of a viscous material.

Ideal viscoelastic material exhibits the characteristics functions (creep and stress-relaxation functions) as shown in Figure 1. These characteristic functions are obtained under the two kinds of tests, where the first test is, "Creep Recovery Test", and the second is "Stress Relaxation Test". In case of the former, a constant stress is applied on the sample for a fixed duration of time and created stress changes are observed over the body. In the latter test, a constant strain/deformation is applied on the body and then the stress variation with time is observed.

Different mechanical models can be defined to describe such a viscoelastic material. In these models, number of parameters may vary from 1 to N depending upon different configurations, which can be used to approximate the rheology of the material by satisfying the above defined tests functions. While doing so, we try to mimic the attenuation by using minimum number of elements to incorporate the correct amount of attenuation, while preserving the characteristics. Thus, we start with a simplest family of models i.e., the 1-element/parameter family that includes single springs or dash pots. In a pure elastic medium, there is no dissipation of energy, so it can be represented with a spring. However, a viscous material can lose the energy and hence it is modeled as a dashpot, which acts like a damper. Stress-strain relation for these two can be written as

$$\text{For spring,} \qquad \sigma = k\epsilon$$

$$\text{For Dashpot,} \qquad \sigma = \eta \frac{d\epsilon}{dt}$$

To represent the Earth more accurately, these two models shall be used in different combinations to form models with higher number of parameters. 1 spring and 1 dashpot model, is called as 2 parameters family. The spring and dashpot components can be arranged either in parallel or in series which are called as Kelvin-Voigt model and Maxwell model, respectively. These arrangements should properly model the characteristics shown in Figure 1. However, the Kelvin-Voigt model cannot model either creep function or stress-relaxation function correctly, which is a disadvantage while Maxwell model only has acceptable stress relaxation function. This limitation led to the inclusion of higher number of parameters such as 3-parameter model e.g., Zener model (Figure 2), also known as Standard Linear Solid (SLS) model, and 4-parameters model e.g., Burger model.

The most useful model to compute the time dependent moduli among these models is a Generalized Standard Linear Solid (GSLS) model for which the general stress-strain relation can be written as:

$$a_0\sigma(t) + \sum_{i=1}^{L} a_i \frac{\partial^i \sigma(t)}{\partial t^i} = b_0\varepsilon(t) + \sum_{i=1}^{L} b_i \frac{\partial^i \varepsilon(t)}{\partial t^i} \tag{1}$$

Where, a_i, b_i are constants for a linear material. The complex modulus (M^*) for a general viscoelastic material can be calculated by applying the sinusoidal varying

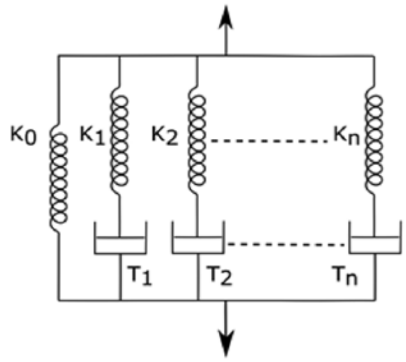

Figure 3. Standard Linear Solid model with Zener elements.

oscillating stress of given frequency, e.g., $\sigma(\omega)=\sigma^0 \exp(i\omega t)$. After certain amount of time the initial effect is negligible and the strain will also be in the form of $\varepsilon(\omega)=\varepsilon^0 \exp(i\omega t)$. The complex modulus of the material is given by Bland (1960); Christensen (2012).

$$M(\omega) = \frac{\sigma^0(\omega)}{\varepsilon^0(\omega)} = \frac{a_0 + \sum_{=1}^{L} a_i\, \omega^i}{b_0 + \sum_{i=1}^{L} b_i\, \omega^i} \tag{2}$$

Thus, the better approach to model time dependent moduli is to assemble many Zener models in parallel, which gives us a "Generalized Zener model" (Figure 3).

Viscoelastic wave formulation

There are many formulations available in literature to define the above mentioned viscoelastic materials in time domain. In this paper, we only discuss the memory variable approach which is based upon a SLS model (Carcione et al., 1988; Liu et al., 1976). As we know that the stress (σ_{ij}) strain (ϵ_{kl}) relationship for a viscoelastic medium can be written in the form of convolution integral as

$$\sigma_{ij} = \dot{\Lambda}(t) \star \varepsilon_{ij}(t)\delta_{ij} + \dot{M} \star \varepsilon_{ij} \tag{3}$$

Where Λ and M are the relaxation functions. A general relaxation function (Ψ) can be modeled with the help of SLS model comprised of L relaxation mechanisms (Carcione, 2007).

$$\psi(t) = M_R \left[1 - \frac{1}{L}\sum_{l=1}^{L}\left(1 - \frac{\tau_\varepsilon^l}{\tau_\sigma^l}\exp\left(-\frac{t}{\tau_\sigma^l}\right)\right)\right] H(t-t') \tag{4}$$

M_R is the relaxed modulus, H(t) is the heavy side function, τ^l_σ and τ^l_ε stress and strain relaxation times, respectively. The stress-strain and memory variable equations can be obtained by substituting the relaxation function (Eq. 4) into Eq. 3. Further, separate relaxation functions should be considered for the P-wave and S-wave. Thus, it will yield following relations:

$$\frac{\partial \sigma_{ij}}{\partial t} = \Lambda\, \dot{\varepsilon}_{k,k}\, \delta_{ij} + 2M\, \dot{\varepsilon}_{i,j} - \frac{1}{L}\sum_{l=1}^{L} r_{ij}^l \tag{5}$$

$$\frac{\partial r_{ij}^l}{\partial t} = -\frac{1}{\tau_\sigma^l} r_{ij}^l + \frac{1}{\tau_\sigma^l}\Lambda^l\, \dot{\varepsilon}_{k,k}\, \delta_{ij} + \frac{1}{\tau_\sigma^l} M^l(\dot{\varepsilon}_{i,j} + \dot{\varepsilon}_{j,i}) \tag{6}$$

Where, we have $\Lambda=\prod-2M$; $\Lambda^l=\prod^l-2M^l$, and $\Lambda_R=\prod_R-2M_R$. To simplify the equations we have assumed that, $\prod=\prod_R(1-\sum^L_{(l=1)} T_p{}^l)$; $M=M_R(1-\sum^L_{(l=1)} T_s{}^l)$; $\prod^l=\prod_R T_p{}^l$; and $M^l=M_R T_s{}^l$. Here $\prod_R$ and $2M_R$ are relaxed modulus for respective waves functions, $r^l{}_{ij}$, is known as the memory variable, and $T_p{}^l$ and $T_s{}^l$ stands for $\frac{1}{L}\left(1-\frac{\tau_{\varepsilon p}^l}{\tau_\sigma^l}\right)$ and $\frac{1}{L}\left(1-\frac{\tau_{\varepsilon s}^l}{\tau_\sigma^l}\right)$, respectively.

Eq. (5) and (6), along with the continuity equation, forms a complete set of equation for viscoelastic modelling. Thus the complete set of wave equation for 2D viscoelastic wave can be written in expanded form as follows.

$$\rho\frac{\partial v_x}{\partial t} = \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z} + \rho f_x \tag{7}$$

$$\rho\frac{\partial v_z}{\partial t} = \frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z} + \rho f_z \tag{8}$$

$$\frac{\partial \sigma_{xx}}{\partial t} = (\Lambda + 2M)\, \dot{\varepsilon}_{x,x} + \Lambda\, \dot{\varepsilon}_{z,z} - \frac{1}{L}\sum_{l=1}^{L} r_{xx}^l \tag{9}$$

$$\frac{\partial \sigma_{zz}}{\partial t} = \Lambda\, \dot{\varepsilon}_{x,x} + \Lambda\, \dot{\varepsilon}_{y,y} + (\Lambda + 2M)\, \dot{\varepsilon}_{z,z} - \frac{1}{L}\sum_{l=1}^{L} r_{zz}^l \tag{10}$$

$$\frac{\partial \sigma_{xz}}{\partial t} = M\, \dot{\varepsilon}_{x,z} + M\, \dot{\varepsilon}_{z,x} - \frac{1}{L}\sum_{l=1}^{L} r_{xz}^l \tag{11}$$

$$\frac{\partial r_{xx}^l}{\partial t} = -\frac{1}{\tau_\sigma^l} r_{xx}^l + \frac{1}{\tau_\sigma^l}(\Lambda^l + 2M^l)\, \dot{\varepsilon}_{x,x} + \frac{1}{\tau_\sigma^l}\Lambda^l\, \dot{\varepsilon}_{z,z} \tag{12}$$

$$\frac{\partial r_{zz}^l}{\partial t} = -\frac{1}{\tau_\sigma^l} r_{xx}^l + \frac{1}{\tau_\sigma^l}\Lambda^l\, \dot{\varepsilon}_{x,x} + \frac{1}{\tau_\sigma^l}(\Lambda^l + 2M^l)\, \dot{\varepsilon}_{z,z} \tag{13}$$

$$\frac{\partial r_{xz}^l}{\partial t} = -\frac{1}{\tau_\sigma^l} r_{xz}^l + \frac{1}{\tau_\sigma^l} M(\dot{\varepsilon}_{x,z} + \dot{\varepsilon}_{z,x}) \tag{14}$$

The relaxation times can be determined by minimizing the error between $Q(\omega)$ and given Q_0 (Blanch et al., 1995):

$$\phi = \int_{\omega_1}^{\omega_2}\left[Q^{-1}(\omega, \tau_\sigma^l, \tau_\varepsilon^l) - Q_0^{-1}(\omega)\right]^2 d\omega \tag{15}$$

where, $Q(\omega) = \frac{\mathrm{Re}[M^c(\omega)]}{\mathrm{Im}[M^c(\omega)]}$; $M^C(\omega) = \mathcal{F}\{\partial_t[\psi(t)]\}$ and $\tau^l{}_\sigma = \frac{1}{\omega_l}$ is the stress relaxation time.

These equations can be solved using the finite difference method. Here we use a synthetic data set to demonstrate the importance of the method.

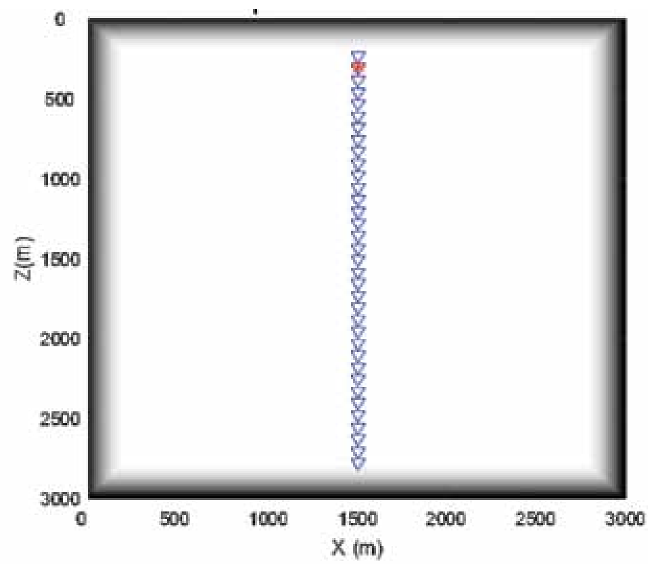

Figure 4. Arrangement of source (in red asterisk) and receiver (blue triangles) along with absorbing boundaries (along edges) as used in simulation.

Measurement of attenuation

In general attenuation measurements are carried out using the amplitude information from the seismic data. Thus, in this paper we only discuss the amplitude based spectral ratio technique, for Q estimation. The data used in this study was generated by designing a synthetic Vertical Seismic Profiling (VSP) survey. The VSP geometry used is shown in Figure 4, where we have laid the receivers vertically and placed the source at the top for the seismic wave simulation.

Spectral ratio technique

This method is very common among the geophysicists and has many variants. The fundamental principle of this method is to compare the spectral characteristics of the seismogram while assuming that the amplitude of a wave can be described by the following relationship:

$$A(\omega) = G(t)R(t)A_0(\omega)\exp\left(-\frac{\omega t}{2Q}\right) \tag{16}$$

where, A_0 is the initial amplitude of the wave which has reduced to A after travelling for time t in the given medium of attenuation characterized by Q. G and R represent the reduction in amplitude due to geometric spreading and reflectivity, respectively. Assuming two receivers placed some distance apart records the amplitude $A_1(\omega)$ and $A_2(\omega)$ respectively at time t_1 and t_2 respectively. We obtain

$$\ln\left(\frac{A_{2(\omega)}}{A_1(\omega)}\right) = -\frac{\omega \Delta t}{2Q} \tag{17}$$

Where, $\Delta t = t_2 - t_1$. When we plot the logarithm of amplitude ratio with the frequency, it represents the equation of straight line with a slope as equal to $\frac{\Delta t\,\pi}{Q}$, which can be in turn utilized to estimate the attenuation Q.

First order effect of attenuations

When a medium offers attenuation to seismic waves, certain phenomena come into play which can be understood as the first order effect of attenuation. Here, we describe the most important ones and their mitigation.

1. *Frequency dependence of* Q: The attenuation experienced by a seismic wave is dependent on the frequency and attenuation mechanism. In a seismic modelling experiment, it's advised to assume constant attenuation over the seismic frequency range (McDonal et al., 1958).
2. *Frequency dependence of the reflectivity*: Attenuation causes the reflectivity of the interfaces to become frequency dependent and thus can affect the attenuation estimation. Hence one must constrain the experiments to include only/nearly normal incident rays. It can be achieved using VSP geometry.
3. *Velocity dispersion causing the travel time difference (drift)*: As mentioned earlier, attenuation causes different frequencies to be attenuated differently. The higher frequencies attenuate faster in comparison to lower frequencies. This causes changes in seismic wavelet and it gets broader with depth. It also causes shifting of the peak amplitude and thus the events experiences a drift. This effect can be corrected or compensated by providing the appropriate time shift or by matching with well log.

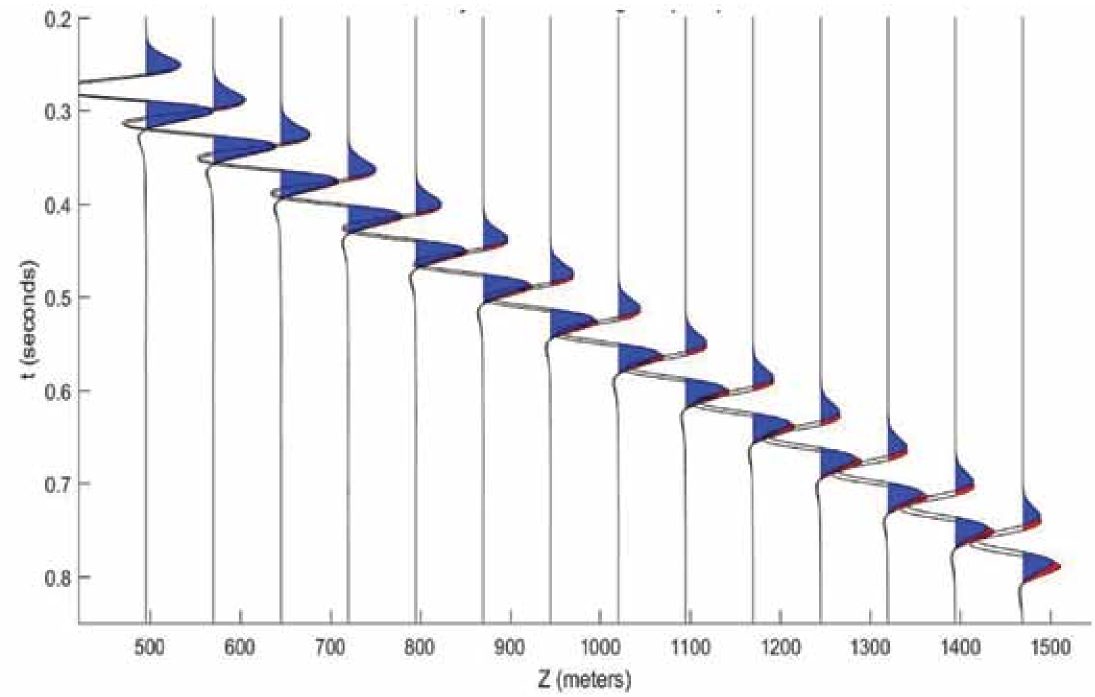

Figure 5. A zoomed section of the VSP gather generated using elastic wave formulation (in red color) and viscoelastic formulations (in blue color).

Table 1. List of parameters used in seismic simulation carried out for a viscoelastic media.

Parameter	*Value*
Model physical parameters	
Velocity, V_p	2000 ms^{-1}
Velocity, V_s	1700 ms^{-1}
Density, ρ	1900 kgm^{-3}
Quality factor, Q	70
Source Parmeters	
Source signature	Ricker
Central frequency, f_0	15Hz
Zero time offset/shift, f_0	0.07sec
Total time length, T	1sec
Simulation parameters	
Model size (x,z)	3km x 3km
Grid spacing, Δh	5m
Time step, Δt	0.1msec
Absorbing boundary nodes	40

Numerical Simulation of Seismic Wave Propagation in Viscoelastic media

Following the above mention concepts, we carried out a synthetic VSP modelling for the elastic as well as viscoelastic medium. The outputs of these simulations are compared in Figure 5. Some of the important parameters for the model and for the simulation are shown in the Table 1, along with their corresponding values. It can be seen that the receivers are arranged vertically as in VSP geometry (Figure 4). The simulation parameters, i.e. time step (*dt)* was taken according to stability condition and to minimize the grid dispersion, more than 6 grids/wavelength were used. To suppress the edge reflections, damping type absorbing boundaries (Cerjan et al., 1985) were applied on all the sides. The synthetic VSP seismogram generated by using viscoelastic formulation, considering attenuation in the media, is used for Q estimation. The results generated after taking care of all the above mentioned effects are shown in Figure 5.

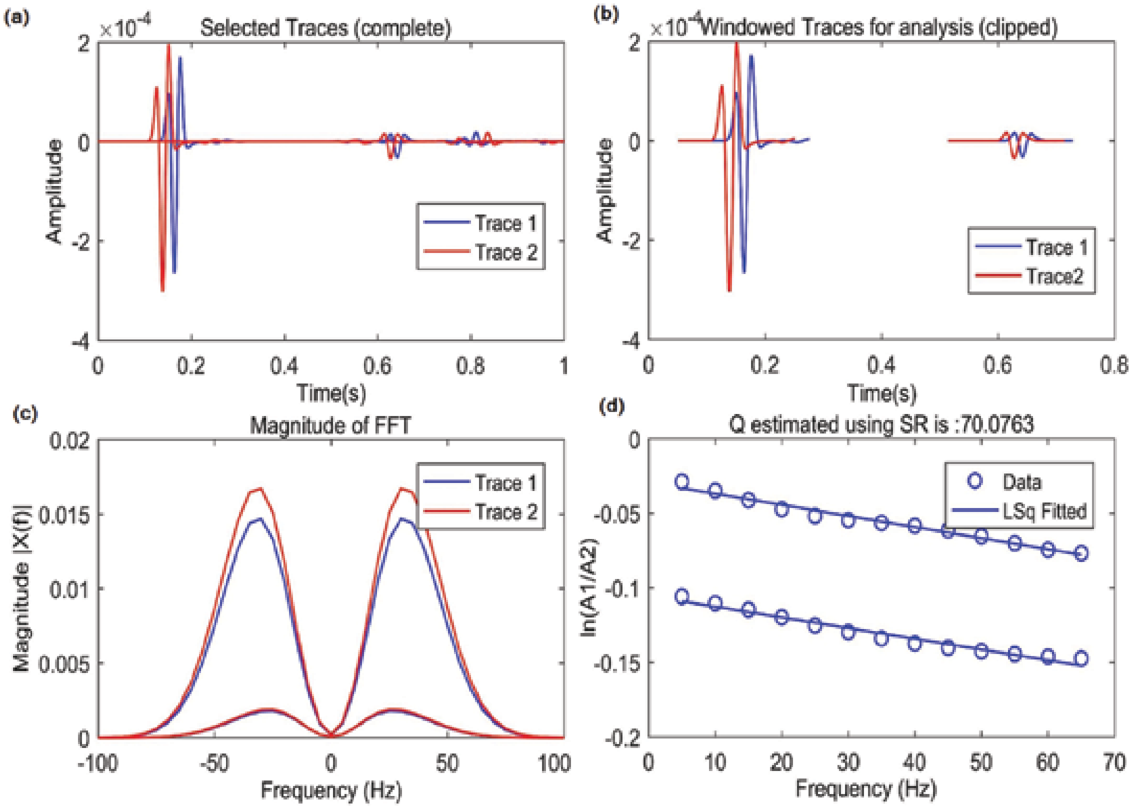

Figure 6. An example of attenuation estimation using spectral ratio. The complete traces selected for the estimation, (a) The clipped part used for the estimation at two places, (b) Fourier spectrum of the given clipped traces, (c) The least square fitting for the determination of slope to estimate Q, (d) Here 'Trace 1 refers' to elastic VSP trace and Trace 2 refers to viscoelastic VSP trace.

We have also used a two layered model and generated a VSP record for this model. Source and simulation parameters were same as for homogeneous. We have chosen two nearby locations to picked events (down going and upcoming) and computed the attenuation for down going as well as upcoming wave. Figure 6 demonstrates the key steps involved in attenuation measurement using the Spectral Ratio method for a two layer case. The steps are namely- selection of traces, clipping the waveform part, Fourier Transform of the clipped part, and estimation of attenuation using the slope of linear fit. For the seismic wave simulation, we have used the 'FDwave' package developed by Malkoti et al. (2018a, 2018b).

RESULTS

The Figure 5 shows a very simple experiment using homogeneous model to demonstrate the difference between the elastic and viscoelastic seismogram. The difference due to the first order effects is very prominent for late phases. This type of mismatch, if not taken care of, can lead to the erroneous results. In Figure 6, we have shown a successful application of this method for Q estimation. As we can see in this figure that considering the first order effects of attenuation and following the prescribed solutions, we can estimate the value of attenuation quite precisely.

CONCLUSIONS

In this paper we have discussed the behavior of an attenuating media and how to model it, using the theory of viscoelasticity, we have shown that the seismic attenuation has a large effect on synthetic seismogram, which was generated for a synthetic VSP survey. Further, we demonstrate an application of Spectral ratio technique to estimate precise value of Q using the synthetic seismic data. We have also discussed the first order effects of attenuation, which should be considered while incorporating seismic attenuation in the seismic modelling experiment. Further, details on the theory and seismic wave simulation algorithm being used are available in Carcione et al. (1988) and Malkoti et al. (2018a, 2018b) respectively.

Compliance with Ethical Standards

The authors declare that they have no conflict of interest and adhere to copyright norms

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