Performance evaluation of different interpretation techniques of vertical electrical sounding data

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ABSTRACT

Numerous interpretation techniques are available in the literature to interpret Vertical Electrical Sounding (VES) data and thus it becomes very difficult to choose one of them. A performance analysis of each method and its comparison with other techniques would aid the geophysicists to choose a better technique. An attempt has been made here to evaluate the performance and comparison of a few widely used interpretation techniques. The computer oriented interpretation techniques RESIST, IRESAN and IPI2Win have been subjected to rigorous analysis using input model parameters. These techniques are applied to interpret the theoretical three and five layered earth models generated for various combinations of resistivities and thicknesses and the output results are analyzed for their accuracy for all the three techniques. The analysis of their performance reveals that IPI2Win technique yields comparatively better results with minimum errors in the output of the interpreted parameters.

Key words: Vertical electrical sounding, Inversion algorithm, Percentage error, RMS error.

INTRODUCTION

The electrical method is one of the important geophysical methods in exploring the subsurface materials, particularly in shallow layers. The VES curves are usually interpreted using curve-matching procedures with the help of theoretical curves (Compagnie Generale de Geophysique, 1963; Flathe, 1963; Orellana and Mooney, 1966; Rijkswaterstaat, 1969) in conjunction with the auxiliary-point method of partial curve matching (Kalenov, 1957; Orellana and Mooney, 1966; Zohdy, 1965). Numerous techniques are available in literature for interpreting the VES data based on curve matching, inversion algorithm, forward algorithm and modeling to obtain the true resistivity and thickness of different layers. The utility of the master curves to interpret field data from various geological settings is rather restricted in the sense that the standard curves are available only for a limited number of layers of given resistivities and thicknesses. Quite often, it is found that the observed VES curve does not match with any of the available standard curves. In case of 5 or more number of layers, no standard curves are available for direct matching. In fact, even the experienced interpreter may find it difficult to obtain satisfactory curve matches, when he uses the auxiliarypoint method for geoelectrical sections containing layers with small effective-relative resistances or small effectiverelative conductance (Flathe, 1963; Zohdy and Jackson, 1973; Zohdy, 1974, Zohdy, 1975). The availability of inexpensive micro and personal computers has now made it possible to develop and implement algorithms, which can provide very efficient and accurate interpretation of VES data. Different computer programs like RESIST, IRESAN, Winsey, 1X1D, Zhond and IPI2 Win to interpret the

resistivity sounding data are available based on different algorithms. In this study an attempt has been made to evaluate the performance of three most widely used techniques via IRESAN, RESIST and IP12Win inversion algorithms for performance evaluation study. The percent errors in each parameter (ρ_1 , ρ_2 , ρ_3 ..., h_1 , h_2) for each technique, when applied to different models, are calculated.

WIDELY USED TECHNIQUES

Iterative Inversion

In this scheme, an initially guessed model is successively improved until the RMS (Root Mean Square) error is minimized to an acceptable level and the parameters remain stable (i.e., in successive iterations there is no improvement in the parameters), with respect to changes in the model. One of the simple ways to achieve this is provided by the Gauss method (Kowalik and Osborne, 1968), which requires accurate values of the partial derivatives of the model data with respect to their parameters. In this method, theoretical curves are computed from the initial guess model and matched with the observed values, which are improved iteratively until a best fit is achieved in the inversion process.

IRESAN

This program is developed by Das and Verma (1977), based on the steps of forward and inversion algorithms. While the theory for forward computation is largely reproduced from their work, the formulation for the inversion is based on the work by Jupp and Vozoff (1975). The computation of vertical electrical resistivity curves in the forward algorithm is based on theory of digital linear filters. The inversion algorithm is structured around a modified version of the program, developed by Jupp and Vozoff (1975). The numerical scheme is based on a modified Gauss-Newton-Marquardt method to solve a system of non-linear equations. Using a variety of stabilization methods, the solution is obtained by the Singular Value Decomposition (SVD) of Jacobean matrix of partial derivatives with respect to parameters.

RESIST:

This program was developed by Vander Velpen and Sporry (1993) to process data obtained with Wenner, Schlumberger and Dipole-Dipole array, which is built around three main procedures:

(i) Smoothing of noisy field data, accurate computation of apparent resistivity models and inversion of resistivity data on an iterative procedure, which includes a priori information of the model parameters, without generating convergence problems.

(ii) Data smoothing (single point correction, curve branch shifting, and eccentricity correction for Schlumberger configuration) is done on the screen. For the computation of apparent resistivity model curves, the linear filter was used.

(iii) The inversion of resistivity data is based on the "Marquardts-Levenberg" technique (Marquardt, 1963). To overcome the convergence problem in using a priori information of model parameters, the Marquardts-Levenberg inversion algorithm was modified, considering a probabilistic treatment of the field observations and the model parameters. This modification allows the program to quickly reach a solution.

IPI2Win

This is free download software with copy right to Bobachev et al., 1990-2002, Moscow State University, Geophysical faculty, Department of geophysics and distributed by Geo Scan-M Ltd, Moscow, Russia. (http://geophys.geol. msu.ru/ipi2win.htm). IPI2Win is designed for automated and interactive semi-automated interpretation of vertical electrical sounding and/or induced polarization data obtained with any of the most popular arrays used in the electrical prospecting and can be run on any computer with Windows 95/98 NT operating system.

The program pays special attention to the userfriendly interactive interpretation. Due to handy controls, the interpreter can choose one from a set of equivalent solutions, which fits well with both the geophysical data (i.e., best fitting error) and geological data (i.e., geologically sensible resistivity cross-section).

INTERPRETATION OF SOUNDING DATA

IPI2Win is capable of solving electrical resistivity 1D forward and inverse problems for a variety of commonly used arrays for the cross-sections with resistivity contrast within the range of 0.0001 to 10000 ohm-m. The forward problem is solved using the linear filtering. In comparison, the inverse problem is solved using a variant of the Newton algorithm of the least number of layers or the regularized fitting minimizing algorithm using Tikhonov's approach to solve incorrect problems. Priory information on layers depths and resistivities can be used for regularizing the process of the fitting error minimizing. The IPI2Win approach involving interactive semiautomated interpretation is preferable, since it considers both the effectiveness and the geological sense into consideration. Some of these, being rather of descriptive than quantitative nature, can hardly be introduced as formal parameters into the interpreting model. In this case, the interpreter's experience and geological erudition may occur to be of even greater importance, than the calculation accuracy. The best fitting two-layered model is automatically suggested for the initial interpreting model of the present sounding point. Model editing involves altering the quantity of layers (from 2 up to 30) by means of splitting or joining them and changing the properties of the layers on the screen. The layer properties can be edited at the desired cell of the table in the model window. The theoretical curve will be redrawn for the present values of the model parameters.

The inversion techniques may yield several models that converge to a reasonable solution, with least RMS error. The model that fits the observed curve may not be the correct solution, keeping in view of the principle of equivalence. Hence, a detailed study is necessary and the models that fit the observed data is also to be analyzed in terms of its priority to the real model (actual lithology). In this article we have attempted all these analyses.

METHODOLOGY

To test the performance of these three techniques, theoretical curves have been generated with Schlumberger array and subjected to interpretation. Schlumberger array is widely used for conducting Vertical Electrical Sounding surveys to explore the subsurface. Hence, the theoretical Vertical Electrical Sounding curves with Schlumberger array have been generated over three and five layer subsurface models for different combinations of resistivities and thicknesses. Six models for A, H and Q and eight models for K-type have been used for generating theoretical VES curves. Two models of HKH and KHA of five layers were also interpreted in this study (Figure 1).



Figure 1. Three and five layer models for interpretation.

All the three interpretation algorithms used in this study require initial guess model (Figure 2). The input theoretical model is updated by optimization technique, till a best fit between theoretical and observed (minimum RMS error) is achieved. To analyze the performance, errors $(0, \pm 5\%, \pm 10\%, \pm 20\%, \pm 30\%, \pm 40\%$ and $\pm 50\%)$ have been added to the theoretical parameters (resistivity and thickness) of the model and are given as input initial guess values for the techniques. It means that the error of the input model is known so that final output values can be compared for its accuracy. The convergence criteria (RMS error) for each technique are noted. The output parameters are compared with the actual parameters of the model curve. Errors and percent errors are computed as the difference between the theoretical and interpreted values. Thus, the percent errors are computed for all the error ranges introduced to the input models in all the parameters, such as resistivities and thicknesses for the three interpretation techniques.

RESULTS AND ANALYSIS

A-type curves

Six models of A-type theoretical curves, generated for different combinations of resistivities (ρ_i) and thicknesses (h_i): ρ_1 : ρ_2 : ρ_3 and h_1 : h_2 respectively are (1) 1:2.5:20 and 1:5 (2) 1:2.5:20 and 1:20 (3) 1:5:20 and 1:5 (4) 1:5:20 and 1:20 (5) 1:7.5:20 and 1:5 (6) 1:7.5:20 and 1:20. These theoretical VES curves are interpreted using RESIST, IRESAN and IPI2Win inputting the initial guess models with errors of 0, $\pm 5\%$, $\pm 10\%$, $\pm 20\%$, $\pm 30\%$, $\pm 40\%$ and $\pm 50\%$ added to the actual values. The output parameters corresponding to RMS error minimum are considered. The



Figure 2. Interpretation of VES curve over a three layered sub surface model by the three software programs (a) RESIST (b) IPI2 Win and (c) IRESAN.

difference between the actual theoretical parameters of the ρ_1 , ρ_2 , ρ_3 , h_1 and h_2 and output values of ρ_1 , ρ_2 , ρ_3 , h_1 and h_2 are computed and percent errors are calculated. The plots (Figure 3, A1 to A6) show -0.8 to 0.5 percent error in all the parameters (ρ_1 , ρ_2 , ρ_3 , h_1 and h_2) in the case of IPI2Win and the RMS error is also less than 0.05%. On the other hand, IRESAN also yields reasonable results with percent errors less than 1 % in all the parameters with the exception of a few cases (for resistivity ratios 1:5:20 and 1:7.5:20) where percent errors is around 5% in first layer resistivity and thicknesses. This is true for all the ranges of error in the initial input models. In the case of RESIST program, though the convergence criteria (minimum RMS error) is

achieved, the output values of the parameters are in large errors (maximum of \pm 50% particularly in first and second layers parameters), when the initial guess model is away (> \pm 30%) from the actual model. It can also be observed from all the plots of Figure 3, that in certain cases (when the thickness of second layer is more (1:20), the percent error in the output is small (<1.5%) for the over estimates (5%, 10%, 20%, 30%, 40% and 50%) of the initial input parameters. And for underestimates (-5%, -10%, -20%, -30%, -40% and -50%), the error percentage in the output increases with the increase of error in the input for all the parameters However, it can be confidently said, if the initial guess model is close to actual model (0, \pm 5%), the



Figure 3. Percentage error plots (A1 to A6) from three computer programs (IPI2Win, RESIST and IRESAN) of A- type model curve with different resistivity and thickness ratios. RL-1, RL-2 and RL-3 are percentage errors in resistivities of layers-1, 2 and 3and TL-1, TL-2 are percentage errors in thicknesses of layers-1 and 2.

percent error in output is very less ($<\pm10\%$), whereas in the case of IRESAN program, the percent error in output values is constant for all the subsurface parameters and for all the ranges of percent errors in the input models. In all the cases of models considered, the IPI2Win yields better results and percent error in output values is less than \pm 0.5% whatever may be the percent deviation of the input model from the actual one. The RMS error is also less than 0.05% for IPI2Win, less than1% for RESIST and less than 2% for IRESAN (Appendix).

H-type curves

Theoretical curves in H-type have been generated for six models of subsurface for different combinations of resistivities (ρ_i) and thicknesses (h_i). The models for (ρ_1 : ρ_2 : ρ_3 and h_1 : h_2) are (1) 1:0.2:10; 1:5 (2) 1:0.2:10; 1:20 (3) 1:0.5:10; 1:5 (4) 1:0.5:10; 1:20 (5) 1:0.8:10; 1:5 (6) 1:0.8:10; 1:20. Same procedure is followed as in the above case. The percent error plots of IPI2Win results (Figure 4, H1 to H6) show ± 1 percent in all the parameters (ρ_1 , ρ_2 , ρ_3 , h_1 and h_2) and the RMS error is also less than 0.102%.

In the case of IRESAN program, these plots show that the percentage errors are less than +2% for most of the layer parameters and for all input error ranges. However, in certain cases of large errors $(>\pm 10\%)$ in the initial model, the error in the output values is around $\pm 5\%$. In the case of resistivity ratio of 1:0.2:10 and thickness ratio 1:5, the percent error is +15% (Figure 4, H1 and H5) in second layer resistivity and thickness though the RMS error is less than 3% (Appendix). It is observed from the plots (Figure 4, H1 to H6) that the percentage errors are constant for of all parameters. In certain cases, if the initial guess model is close to actual model $(0, \pm 5\%)$, the percent error in output is very less $(\langle +2\% \rangle)$. It is noticed from the plots (Figure 4, H6), the output percentage error from IRESAN program is fluctuating if the resistivity contrast between first and second layers is less (1:0.8) and thickness of the second layer is more (1:20).

In the case of RESIST program, the output values reveal percentage errors less than one for all the error ranges of input values, but with the exception of the case where resistivity contrast between first and second layers is very less (1:0.8:10) (Figure 4, H5 and H6). In this case also, the percent error in first layer is less than $\pm 5\%$, that too for large errors $(\pm 50\%)$ of input model. RMS error is 0.4%. If the resistivity contrast is more between first and second layers and for large errors in input model, the output values are constant with percentage error ranging from -4.5 to 2.5. However, it can be confidently said, if the initial guess model is close to actual model $(0, \pm 5\%)$, the percent error in output is very less (<+0.5%). The maximum RMS error values for all the interpreted H-type curves for RESIST, IRESAN and IPI2Win are less than 0.1%, 2.85% and 0.1% respectively (Appendix).

K-type curves

K-type theoretical curves have been generated for eight models of subsurface for different combinations of resistivities (ρ_i) and thicknesses (h_i). The models for (ρ_1 : ρ_2 : ρ_3 and h_1 : h_2) are (1) 1:1.5:0.4; 1:5 (2) 1:1.5:0.4; 1:20 (3) 1:2:0.4; 1:5 (4) 1:2:0.4; 1:20 (5) 1:3:0.4; 1:5 (6) 1:3:0.4; 1:20 (7) 1:5:1; 1:5 (8) 1:5:1; 1:20. The percent error plots for IPI2Win results (Figure 5) show zero percent in all the parameters (ρ_1 , ρ_2 , ρ_3 , h_1 and h_2) for most of the models and in a few cases (Figure 5, K4 and K6), the percent error is less than $\pm 0.5\%$ in resistivity and thickness values for third layer. The RMS error is very less and never greater than 0.4% in any case.

IRESAN also yields reasonable results with percent errors less than +2% in most of the cases (Figure 5). If the thickness of the second layer is large (1:20) and resistivity contrast between first and second layers is also high (1:5:1), then an error of less than 5% is observed in the results of first layer resistivity and second layer thickness (Figure 5, K8). For all the eight models and for all the parameters considered have the same percent error in the output irrespective of the magnitude of error in the input model. The RMS error (Appendix) is less than one for all the model curves considered with the exception of K7 (Figure 5, K7) model for which RMS error is around 4.4%. In the case of RESIST program, it can be observed from figures 5, that K1 to K8, the percent error in the output is dependent on the input model error. For underestimates, if the error is large (-50%) in input, output error (-50%) is also large. For over estimates of input, the error percentage in output is less than 5% in all the parameters for the entire input model with the exception one case (1:1.5:0.4). In this case (1:1.5:0.4), the percent error in output increases (up to 20%) with increase of error in input. This may be due to small contrast in the resistivity values of first and second layers.

The RMS error values are in the range 0.1%-1.1%; 0.4%-6.1%; and 0.09% – 0.4% respectively for RESIST, IRESAN and IPI2Win algorithms (Appendix).

Q-type curves

Q-type theoretical curves generated for six models for $(\rho_1, \rho_2, \rho_3 \text{ and } h_1; h_2)$ are (1) 1:0.2:0.1; 1:5 (2) 1:0.2:0.1; 1:20 (3) 1:0.5:0.1; 1:5 (4) 1:0.5:0.1; 1:20 (5) 1:0.8:0.1; 1:5 (6) 1:0.8:0.1; 1:20. All the three techniques have been applied for obtaining the true values of resistivities and techniques. The percent error plots of output results with IPI2Win algorithm (Figure 6, Q1 to Q6) show -1.5 to 1 percent variation in all the parameters ($\rho_1, \rho_2, \rho_3, h_1$ and h_2) with the exception in a few cases where percent errors is around 4% in the thickness of second layer. The RMS error (Appendix) is also less than 0.42%.



Figure 4. Percentage error plots (H1 to H6) from three computer programs (IPI2Win, RESIST and IRESAN) of H- type model curve with different resistivity and thickness ratios. RL-1, RL-2 and RL-3 are percentage errors in resistivities of layers-1, 2 and 3and TL-1, TL-2 are percentage errors in thicknesses of layers-1 and 2.

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Figure 5. Percentage error plots (K1 to K8) from three computer programs (IPI2Win, RESIST and IRESAN) of K- type model curve with different resistivity and thickness ratios. RL-1, RL-2 and RL-3 are percentage errors in resistivities of layers-1, 2 and 3 and TL-1, TL-2 are percentage errors in thicknesses of layers-1 and 2.



Figure 6. Percentage error plots (Q1 to Q6) from three computer programs (IPI2Win, RESIST and IRESAN) of Q- type model curve with different resistivity and thickness ratios. RL-1, RL-2 and RL-3 are percentage errors in resistivities of layers-1, 2 and 3 and TL-1, TL-2 are percentage errors in thicknesses of layers-1 and 2.



Figure 7(a). Percentage error plots from three computer programs (IPI2Win, RESIST and IRESAN) of HKH- type model curve with different resistivity and thickness ratios. RL-1, RL-2, RL-3, RL-4 and RL-5 are percentage errors in resistivities of layer-1, 2, 3, 4 and 5 and TL-1, TL-2, TL-3, TL-4 are percentage errors in thicknesses of layer-1, 2, 3 and 4.

IRESAN yields reasonable results with almost constant errors varying from -10% to 10% except in the case where the resistivity contrast between first and second layers is 1:0.8 and second layer thickness is 5 times the first one. The percentage error in the output values of the thickness of layer 1 is in large error $(\pm 16\%)$ that too when the initial guess model is away (> $\pm 20\%$) from the actual model. This indicates uncertainty of the program for the guess parameters with low resistivity/thickness contrasts for input model. In the case of RESIST program, the percent error in most of the output parameters is less than +3% and never more than 5% with exception of Q5 and Q6 models (Figure 6, Q5 and Q6). In these cases also, the percent error in the output model for the first layer thickness is in proportion to the error in the input model (maximum of $\pm 30\%$). The RMS error values for RESIST, IRESAN and IPI2Win are respectively in the ranges 0.1% - 0.4%; 0.62% - 6.2%; and 0.18% - 0.42% (Appendix).

Five layer cases

Theoretical VES curves have been generated over five layer subsurface models. The two resistivity and thickness ratios considered for these models are (ρ_1 : ρ_2 : ρ_3 : ρ_4 : ρ_5 and h_1 : h_2 : h_3 : h_4) are (1) 1.0:0.2:1.0:0.1:10; 1:5:20:20 and (2) 1.0:10.0:0.4:2.0:20; 1:5:10:10. The two model curves have been interpreted using all the three interpretation algorithms after adding errors to the parameters of input model.

Analysis of HKH type curve (1.0:0.2:1.0:0.1:10;1:5:20:20) for different error ranges in input models, the percent errors in the output parameters of all the three techniques have been computed and plotted (Figure 7a). The percent error in resistivities for IPI2Win is almost zero or around $\pm 1\%$ for all the layer parameter except layer 4 where percent error increases with increasing error in input model. This may be due to large resistivity contrast (0.1:10) between layer 4 and 5. The percent error in thickness is also zero in case of thickness of layers 1, 2 and 3. In the case of the layer 4, the percent error is

around -15% for underestimates of input models. In the case of overestimates of input model percent error in the output increases. The percent is as large as around 40% when input model is in 50% error. In the case of IRESAN algorithm, the percent error in the output of resistivity parameters varies in the range of 4% to 20% for all the layers except for overestimates in resistivity of the layer 4, where the percent error increases to $\pm 40\%$ with increase of error in input model. The percent error varies between 5% and 30% in case of thickness for the layers 1, 2, 3 and 5, where as the percent error increases for thickness of the layer 4 with increase of error in input model. The analysis of RESIST algorithm shows that for overestimates of error in input model, the percent error in output values is less than 5% and for underestimates of error in input model, the percent error in output of resistivity values increases with the increase of error in input. But for resistivity values of layer 4 and 5, the percent in output increases with increase in the error of input model. The same inference is true in case of output of thickness parameters.

In analysis of KHA (1:10:0.4: 2:20; 1:5:10:10) (Figure 7b), IPI2Win algorithm shows (Figure 7b) a percent error in resistivity values of around $\pm 1\%$ in layers 1, 2, 3 and 5 and for all the error ranges of the input models. In the case of the layer 4, the percent error in the output is zero for all the ranges of error in input models. But when input model error is more than 30%, the percent error in the output substantially increases to very high values (~2500%). This may be due to large contrast (1:10:0.4:2:20) in the resistivities between 2nd, 3rd and 4th layers. In the case of thickness, the percent error in output values is in the range -0.2% to 28% for the entire range of input error models, with exception of layer 4, where the error percent is high around 50% for all the input error models.

In case of IRESAN, the percent error is around $\pm 5\%$ for all input error models in resistivity values of layers 1, 2 and 5 (Figure 7b). In case of layer 3 and 4, the percent error varies with error in input model. The same analysis is true in thickness parameter also, except for layers 3 and 4 which vary with percent errors in input models.



Figure 7(b). Percentage error plots from three computer programs (IPI2Win, RESIST and IRESAN) of KHA- type model curve with different resistivity and thickness ratios. RL-1, RL-2, RL-3, RL-4 and RL-5 are percentage errors in resistivities of layers-1, 2, 3, 4 and 5 and TL-1, TL-2, TL-3, TL-4 are percentage errors in thicknesses of layer-1, 2, 3 and 4.



Figure 8. VES data curve carried out at Jaddangi village and the lithology of the drilled well near sounding point. Interpreted results of the three programs are given table 1.

The analysis of RESIST algorithm yields less error (<5%) in the output of resistivity of layers 1, 2 and 5 for all the input error models with the exception of -50% for which the error in output is around -30%. In case of layer 3 and 4 with increase of error in input model, there is an increase in error in output values. The percent error in thickness for layer 1 and 2 is less than \pm 5% for overestimates in input error. For underestimates in input error of more than -30%, the output percent error is large (< \pm 40). In case of layers 3 and 4, the percent error increases with increasing error in input models.

EXAMPLES

Two field curves from different locations corresponding to different lithologies were interpreted with all the three techniques.

Example 1

A sounding carried out at Jaddangi village (17.4801°N, 82.1545°E) in East Godavari district covered by granite gneiss, was compared with the lithology of a drilled well at a distance of 196 m is taken for reference for comparing

interpreted results (Figure 8). The sounding curve is subjected to interpretation by the three programs using two initial guess models. From a comparison of the results (Table 1), it is observed that in spite of the large deviations in the two initial models, the final output values of resistivities and thicknesses for all the three layers for both the initial models are consistent in the case of IPI2Win algorithm and differ in the case of RESIST and IRESAN particularly with respect to the parameters of the layers 2 and 3.

Example-2

The Vertical Electrical Sounding data has been taken from the published work (Kedareswarudu et al., 2006) over Borra caves, Visakhapatnam district (Figure 9). The actual measured depth to the bottom of cave from surface is approximately 2.5 meters. The results of interpretation using three different algorithms and with two initial guess models with large deviations are shown in table 2. The interpreted results of IPI2Win show more or less same values in output layer resistivities and thicknesses though there is deviation in the two input models (Table 2). On the other hand, the interpreted results of RESIST and IRESAN show differences in the output parameters for the two input P. Venkateswara Rao, M. Subrahmanyam and D. Ratnakar



Figure 9. VES data curve carried out at Borra caves area. Interpreted results of the three programs are given table 2.

Table 1. Results and comparison of three inversion algorithms for the above sounding data carried out at Jaddangi (L-1, 2, 3 and 4 are layer numbers). VES curve is shown in figure 7. R=Resistivity, T=Thickness.

Guess Model	Gu	ess model va	alues	RESI	ST	IRES	AN	IPI2Win		
		R (Ω m)	T (m)	R (Ωm)	T (m)	R (Ωm)	T (m)	$R(\Omega m)$	T (m)	
	L-1	20	2	19	1.9	20.47	1.557	21.1	1.32	
1	L-2	5	5	5.6	4.4	4.94	3.085	6.95	5.48 14	
	L-3	25	10	26.2	9.7	21.24	13.19	28.3		
	L-4	300	-	303.4	-	663.55	-	2530	-	
	L-1	30	2	22.5	1.3	21.304	1.33	21.2	1.32	
	L-2	10	10	8.2	7.7	6.547	5.754	6.98	5.56	
2	L-3	40	20	47.3	17.2	42.788	15.59	28.7	13.6	
	L-4	500	-	517.5	-	619.84	-	1408	-	

Table 2. Results and comparison of three inversion algorithms for the sounding data carried out at Borra Caves (L-1, 2, 3and 4 are layer numbers). VES curve is shown in figure 8. R=Resistivity, T=Thickness

Guess model	Guess model values			RESIST	Γ	IRESAN	IPI2Win		
		R (Ωm)	T (m)	R (Ωm)	T (m)	R (Ωm)	T (m)	R (Ωm)	T (m)
	L-1	100	1	169.5	0.6	162.92	0.577	333	1.25
1	L-2	5000	5	4766.9	4.9	9072.186	2.576	16739	1.39
	L-3	500 5 252.7		252.7	9.8	227.175	8.909	139	4.85
	L-4	10000	-	7166.3	-	7455.228	-	4903	-
	L-1	300	0.5	205.8	0.7	204.981	0.728	333	1.25
0	L-2	10000	2.5	9284.7	2.4	9508.493	2.452	16668	1.39
2	L-3	200	10	238.1	8.4	225.709	8.832	154	5.37
	L-4	5000	-	5217.5	-	7315.92	-	4906	-

models. The measured depth to the bottom (~ 2.5 m) of cavern (revealed by high resistivity value is due to cavity) is in agreement with the results of IPI2Win (first and second layer thickness is 1.25+1.39= 2.64 meters).

CONCLUSIONS

From the analysis of all percent error plots of all the parameters, it is observed that IPI2Win yields less error

irrespective of the magnitude of percent error in the input model. However, IRESAN and RESIST methods also yielded good results in certain cases of models and in some parameters only and showed large errors in other parameters. All the three algorithms are very effective, when the input models are close to the actual ones. Choosing of an initial model very close to actual model (unknown) is very difficult. RESIST and IRESAN algorithms reveal that the contrast in resistivities and thickness between the subsurface layers plays a significant role in the interpreted results. If the contrast in the parameters (Resistivity/Thickness) of the model is large and also when the input model is in large error, the output parameters are in large errors. This analysis is true for all the cases of VES curves of the entire subsurface models. The IPI2Win algorithm yields better interpreted results even though the input model deviates largely from the actual model. The RMS error is never greater than 0.2% (Appendix) and much less in the case of five layer models. The RMS error is comparatively large for IRESAN software and is less than 5% (Appendix). RESIST software shows an RMS error around 0.1% for most of the models and in a few cases converges to a high value and never more than 0.9% (Appendix).

In cases of A and K type model curves (figures 3 and 5), RESIST algorithm software yields large percent errors, if the input model parameters are in large under estimates, whereas if the input parameters are in large over estimates, the percent errors in output are not much. For H and Q type curves (figures 4 and 6), this method showed errors which are in proportion to the deviation of the input model in the output. From the above study, it is inferred that the IPI2Win results are more reliable showing consistent output, in spite of large deviations in the initial guess models.

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Compliance with Ethical Standards

The authors declare that they have no conflict of interest and adhere to copyright norms.

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Madal	% E	ror RMS error value Model Error RMS error value			DMC ormon value	Madal	% E			% Model Erro		PMS error value			
Model	Error	RESIST	IRESAN IPI2WIN		RE	ESIST IRESAN IPI2WIN	J	Error	RESIST	IRESAN IPI2WIN	Vioder	EIIOI	RESIS	ΓIRESAN	IPI2WIN
A1	0	0	1.7441 0.0456	H1 0		0.1 1.9692 0.0451	K1	0	0.1	4.3881 0.141	K8	0	0.1	1.0442	0.19
	10	0.1	1.7438 0.0456	5		0.1 1.9639 0.0451		10	0.1	4.38/8 0.141		10	0.1	1.05/3	0.19
	20	0.1	1.7458 0.0456	20		$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		20	0.1	4.3896 0.141		20	0.1	1.0545	0.19
	30	0.1	1.7446 0.0456	30		0.1 1.9646 0.0451		30	0.1	4.3891 0.141		30	0.1	1.0515	0.19
	40	0.1	1.744 0.0456	4(0.1 1.9667 0.0451		40	0.1	4.3883 0.141		40	0.1	1.04/2	0.19
	-5	0.1	1.7438 0.0456	-5	<u> </u>	0.1 1.9681 0.0451		-5	0.1	4.3881 0.141		-5	0.1	1.0452	0.19
	-10	0.1	1.7443 0.0456	-10)	0.1 1.9639 0.0451		-10	0.1	4.3892 0.141		-10	0.1	1.051	0.19
	-20	0.1	1.7437 0.0456	-20		0.1 1.9639 0.0451	_	-20	0.1	4.3878 0.141		-20	0.1	1.0445	0.19
	-40	0.2	1.7438 0.0456		$\frac{1}{2}$	0.1 1.9639 0.0451 0.1 1.9639 0.0451	-	-40	0.1	4.3879 0.141	-	-40	0.1	1.0495	0.19
	-50	1.2	1.7438 0.0456	-50)	0.1 1.9639 0.0451		-50	1	4.3879 0.141		-50	0.1	1.056	0.19
A2	0	0	1.395 0.0385	H2 0	_	0.1 1.8514 0.0506	<u>K2</u>	0	0.1	0.645 0.245	Q1	0	0.1	4.3253	0.181
	$\frac{5}{10}$	0.1	1.3966 0.0385	5		0.1 1.8431 0.0506 0.1 1.8337 0.0506		10	0.1	0.649 $0.2450.6639$ 0.245		10	0.1	4.3221	0.181
	20	0.2	1.394 0.0385	20		0.1 1.834 0.0506		20	0.2	0.6456 0.245		20	0.1	4.3223	0.181
	30	0.2	1.3936 0.0385	30		0.1 1.8356 0.0506		30	0.2	0.6615 0.245		30	0.1	4.3229	0.181
	40	0.3	1.3924 0.0385	40		0.1 1.8362 0.0506		40	0.2	0.6463 0.245		40	0.1	4.324/	0.181
	-5	0.1	1.3907 0.0385	-5	<u> </u>	0.1 1.8379 0.0500		-5	0.1	0.6617 0.245		-5	0.1	4.3239	0.181
	-10	0.1	1.3928 0.0385	-10)	0.1 1.8421 0.0506		-10	0.1	0.6444 0.245		-10	0.1	4.3221	0.181
	-20	0.3	1.3902 0.0385	-20		0.1 1.834 0.0506	_	-20	0.2	0.6494 0.245		-20	0.1	4.3221	0.181
	-40	0.5	1.3902 0.0385	-40	$\frac{1}{2}$	0.1 1.8341 0.0500		-40	0.2	0.6449 0.245		-40	0.1	4.3237	0.181
	-50	0.9	1.3902 0.0385	-50)	0.1 1.8334 0.0506		-50	0.8	0.6438 0.245		-50	0.1	4.3221	0.181
A3	0	0.1	1.8501 0.0472	H3 0	_	0 2.8451 0.102	<u>K3</u>	0	0.1	6.0862 0.235	Q2	0	0.1	0.8828	0.209
	10	0.1	1.85 0.04/2 1.8514 0.0472	5		$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	-	10	0.1	6.0839 0.235		10	0.1	0.8945	0.209
	20	0.1	1.8495 0.0472	20		0.1 2.8452 0.102		20	0.1	<u>6.0842</u> 0.235		20	0.1	0.8874	0.209
	30	0.1	1.8495 0.0472	30	\square	0.1 2.8447 0.102		30	0.1	6.0851 0.235		30	0.1	0.9005	0.209
	40	0.1	1.849/ 0.04/2	40		0.1 2.8433 0.102		40	0.1	6.0855 0.235		40	0.1	0.8822	0.209
	-5	0.1	1.8498 0.0472	-5	-	0.1 2.8438 0.102 0.1 2.8439 0.102		-5	0.1	6.0841 0.235		-5	0.1	0.8842	0.209
	-10	0.1	1.852 0.0472	-10)	0.1 2.844 0.102		-10	0.1	6.0846 0.235		-10	0.1	0.9006	0.209
	-20	0.1	1.8495 0.0472	-20		0.1 2.8462 0.102	_	-20	0.1	6.0839 0.235		-20	0.1	0.8842	0.209
	-30	0.1	1.8511 0.04/2	-30		0.1 2.8464 0.102 0.1 2.8458 0.102		-40	0.3	6.0839 0.235		-40	0.1	0.8894	0.209
	-50	1	1.8495 0.0472	-50)	0.1 2.8464 0.102		-50	1.1	6.0839 0.235		-50	0.1	0.8822	0.209
A4	0	0.1	1.5362 0.0402	H4 0	_	0.1 2.5527 0.0654	K4	0	0.2	0.8934 0.368	Q3	0	0.1	6.2329	0.254
	10	0.1	1.54/6 0.0402	5		0.1 2.5533 0.0654 0.1 2.5542 0.0654		10	0.2	0.8954 0.368		10	0.1	6.2261	0.254 0.254
	20	0.1	1.5424 0.0402	20		$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		20	0.2	0.8934 0.368		20	0.1	6.2253	0.254
	30	0.2	1.5413 0.0402	30)	0.1 2.5526 0.0654		30	0.2	0.8934 0.368		30	0.2	6.2253	0.254
	40	0.3	1.5355 0.0402			0.1 2.5517 0.0654		40	0.2	0.8935 0.368		40	0.2	6.2254	0.254
	-5	0.5	1.5355 0.0402	-5	'	0.1 2.5517 0.0654 0.1 2.5521 0.0655	-	-5	0.2	0.8937 0.368		-5	0.2	6.2255	0.254 0.254
	-10	0.1	1.5377 0.0402	-10)	0.1 2.5518 0.0654		-10	0.2	0.9095 0.368		-10	0.1	6.2259	0.254
	-20	0.2	1.5351 0.0402	-20)	0.1 2.5536 0.0655		-20	0.2	0.895 0.368		-20	0.1	6.226	0.254
	-30	0.2	1.53/1 0.0402 1.535 0.0402	-30		0.1 2.5523 0.0654 0.1 2.552 0.0654	-	-30	0.3	0.8956 0.368		-30	0.2	6.2259	0.254
	-50	0.8	1.5351 0.0402	-50)	0.2 2.5516 0.0654		-50	0.9	0.8974 0.368		-50	0.4	6.2254	0.254
A5	0	0	1.5999 0.0471	H5 0		0.1 1.8246 0.0398	K5	0	0.1	3.304 0.0931	Q4	0	0.2	0.6509	0.42
	10	0.1	1.5988 0.04/1	5		0.1 1.8292 0.0398		10	0.1	3.3093 0.0931	_	10	0.2	0.66/4	0.42
	20	0.1	1.6049 0.0471	20		0.1 1.8403 0.0398 0.1 1.8278 0.0398		20	0.1	3.304 0.0931		20	0.2	0.6753	0.42
	30	0.1	1.5991 0.0471	30)	0.1 1.8261 0.0398		30	0.1	3.3046 0.0931		30	0.2	0.7057	0.42
	40	0.1	1.6036 0.04/1	40		0.1 1.8253 0.0398		40	0.1	3.3055 0.0931		40	0.2	0.6584	0.42
	-5	0.1	1.5988 0.0471	-5	<u> </u>	0.1 1.8249 0.0398		-5	0.2	3.3094 0.0931		-5	0.2	0.6544	0.42
	-10	0.1	1.5999 0.0471	-10)	0.1 1.8334 0.0398		-10	0.1	3.304 0.0931		-10	0.2	0.6803	0.42
	-20	0.1	1.5986 0.0471	-20		0.1 1.8276 0.0398		-20	0.2	3.3045 0.0931		-20	0.2	0.6929	0.42
	-40	0.1	1.5987 0.0471		$\frac{1}{1}$	0.1 1.82/1 0.0398 0.2 1.8268 0.0398		-40	0.2	3 3076 0 0931	-	-40	0.2	0.6504	0.42
L	-50	1.2	1.5987 0.0471	-50)	0.4 1.8268 0.0398		-50	0.8	3.3089 0.0931		-50	0.3	0.6505	0.42
A6	0	0	1.3254 0.0486	H6 0	_	0.1 1.58 0.0373	<u>K6</u>	0	0.1	0.456 0.173	Q5	0	0.1	2.685	0.251
<u> </u>	10	0.1	1.3353 0.0486	10		$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	-	10	0.1	0.4608 0.1/3		10	0.1	2.6832	0.251
	20	0.2	1.3302 0.0486			0.1 1.578 0.0373		20	0.1	0.468 0.173		20	0.1	2.6832	0.251
	30	0.2	1.3233 0.0486	30		0.1 1.5823 0.0373		30	0.1	0.4645 0.173		30	0.1	2.6836	0.251
	40	0.3	1.3301 0.0486 1.3234 0.0486	40		0.1 1.5831 0.03/3 0.1 1.5838 0.0373	+	40	0.1	0.4523 = 0.1/3 0.4585 = 0.173	+	40	0.1	2.6832	0.251
	-5	0.1	1.3227 0.0486	-5	-	0.1 1.5818 0.0373		-5	0.1	0.464 0.173		-5	0.1	2.6833	0.251
	-10	0.1	1.3285 0.0486	-10)	0.1 1.5883 0.0373		-10	0.1	0.5092 0.173		-10	0.1	2.6867	0.251
	-20	0.3	1.3209 0.0486	-20		0.1 1.5897 0.0373		-20	0.2	0.4765 0.173	-	-20	0.1	2.6837	0.251
	-40	0.6	1.3199 0.0486		śΗ	0.1 1.5825 0.0373	+	-40	0.2	0.4581 0.173	-	-40	0.1	2.6835	0.251
	-50	0.9	1.3199 0.0486	-50)	0.2 1.5779 0.0373		-50	0.5	0.4532 0.173		-50	0.1	2.6864	0.251
HKH	0	0.1	1.802 0.104	KHA 0	_	0.1 2.1783 0.0531	<u></u>	0	0.1	4.3958 0.153	Q6	0 5	0.1	2.1511	0.243
	10	0.1	1.8036 0.104	10		0.1 2.1780 0.0531 0.1 2.1789 0.0531	-	10	0.1	4.3938 0.153		10	0.1	2.1509	0.243
	20	0.1	1.8088 0.103	20)	0.1 2.1797 0.0534		20	0.1	4.3939 0.153		20	0.1	2.151	0.243
	30	0.1	1.8203 0.105	30		0.2 2.1807 0.0536		30	0.1	4.3939 0.153	_	30	0.1	2.1513	0.243
	40	0.1	1.8153 0.0699	40		0.2 2.1813 0.0546 0.3 2.1834 0.0556	+	40	0.1	4.3938 0.153	+	40	0.1	2.1512	0.243
	-5	0.1	1.7984 0.105	-5	1	0.1 2.1781 0.053		-5	0.1	4.3938 0.153		-5	0.1	2.1513	0.243
	-10	0.1	1.7972 0.106	-10)	0.1 2.1779 0.053		-10	0.1	4.394 0.153		-10	0.1	2.1518	0.243
	-20	0.1	1.7975 0.106	-20	+	0.1 2.1/5 0.0531		-20	0.1	4.3938 0.153		-20	0.1	2.155/	0.243
	-40	0.2	1.7961 0.100	-30	<u>;</u> -	0.2 2.1824 0.053 0.3 2.1811 0.053	+	-40	0.5	4.3963 0.153	1	-40	0.1	2.1508	0.243
	-50	0.9	1 7987 0 101	-50		0.4 2.1772 0.053		-50	11	4 3975 0 153		-50	0.1	2 1553	0 243

APPENDIX Table 1. RMS error values of three algorithms for different percentage errors in input models.